Forward hedging under price and production risk of wheat

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This paper estimates optimal hedging ratios for a Finnish spring wheat producer under price and yield uncertainty. The contract available for hedging fixes the price and quantity at the time of sowing for a delivery at harvest. Autoregressive models are used to obtain point forecasts for the conditional mean price and price volatility at harvest. Expected yield and yield volatility are estimated from the field experiment data. A range of coefficients of absolute risk aversion are used in the computations. The results suggest that yield volatility is large and it dominates the price volatility in the optimal hedging decisions of the Finnish wheat producers. The point estimate for the price and yield correlation is negative and has a large magnitude. Thus, a negative correlation between the price and the yield, as signalled by the point estimate, will decrease the optimal hedging ratio since the Finnish farmers do not have access to selling put options when they enter in a forward contract.

Key words: grain market, price, yield, risk management, forward contract

Introduction

The Agenda 2000 reform of European Common Agricultural Policy (CAP) decreased intervention prices for cereals. This allows for the European grain prices to move towards equivalent world market prices more freely than before. Fluctuations of world market prices will also be transmitted to the EU market and increase price uncertainty for the producers (Roche and Mcquinn 2001). Therefore, appropriate risk management tools such as forward contracts become more attractive to hedge against the increased price risk.

A quite large literature focuses on optimal hedging problems in the futures markets (see e.g. Lapan and Moschini 1994, Tomek and Peterson 2001). The standard in the studies on optimal hedging is that market is efficient such that the futures contract is allowed to be offset (liquidated) by an opposite contract before its maturity. The hedging problem of Myers and Thompson (1989) is also truly dynamic such that it allows for continuous adjustment in the hedging position.

A problem on generalizing these results to Finland is that the short run grain price movements may not be fully integrated to other markets, in which also the futures are traded (Kola and Taipale

2000). Another problem is that independent grain producers may not have a direct access to the derivatives market since transactions costs for entering these markets are too high for them. A "full" hedge, which is a combination of futures contracts and sales of put options is not feasible. Nevertheless, they can usually enter in a forward contract with a local grain dealer. In Finland, for example, Avena Nordic Grain (Avena) has offered forward contracts for the Finnish grain producers since the year 2000. In these contracts, the buyer (seller) is obliged to purchase (deliver) grains from (to) Avena at the agreed maturity date in the future at the fixed price.

The hedging problem of the Finnish grain producer is, at least for two reasons, a special and restricted version of the problems generally studied in the literature. First, the forward contract, they have access to, is incomplete compared to the liquid and highly standardized futures contracts. The non-tradable forward contract is irreversible and can only be terminated by the delivery. Therefore, there is no basis risk and the potential for pure speculation is negligible. Second, in the Northern wheat producing areas yield uncertainty is large, which moves weight from the price volatility to the yield volatility in the optimal hedging problem.

The goal of this paper is to estimate optimal hedging ratios for Finish spring wheat producers. Optimal hedging ratio is defined as the share of the expected yield sold through a certain forward contract. The contract is signed at sowing time for a fixed quantity and price for a delivery at harvest. We solve the hedging problem by an Mean Variance model (MV) and compare it with the Expected Utility model (EU), first derived into a similar hedging problem by Lapan and Moshini (1994). MV model is used because it is empirically tractable and it either coincides with the EU-model, or under fairly general conditions it results in only negligible approximation errors. Sufficient condition for MV model to coincide with the exact EU-model is that either an investor's utility function is quadratic, or the investor's expectation errors follow the normal distribution (Robison and Barry 1987).

Our empirical examination of optimal hedging focuses on a hypothetical spring wheat producer in the Southwest of Finland. We assume that the hedging and production decisions are made in April/May once a year, while the execution of the forward contract is performed in August of the same year. Therefore, we take 21st week of each year as the time at which production commitments are made and the forward contract is signed. Harvest and delivery take place 15 weeks later, at 36th week of the same year. Termination of the forward contract also occurs at harvest since it can be terminated only through the delivery.

Economic models: MV and EU-model

The goal of a farmer is to maximize his wealth at harvest, *W*, which is generated from the random income, y. The MV-approach is to approximate this wealth by the Certainty Equivalence (CE), defined as the difference between the expected income and the risk premium

$$W = E(y) - \frac{1}{2}A\sigma_y^2 \qquad (1)$$

where E(y) is the expected income and the term $\frac{1}{2}A\sigma_y^2$ is the Arrow-Pratt risk premium. It consists of the constant absolute risk aversion (CARA) coefficient, *A*, and from the variance of income, σ_y^2 (Pratt 1964, Arrow 1971). If the variance of income increases at a given income level, wealth is decreased as *A*>0.

Let $\tilde{q}$ be the random yield per hectare. The expected yield conditional on information (Ω) at sowing time (t) as

$$E_t[\tilde{q}_{t+s} \,|\Omega_t] = \mu_q \qquad (2)$$

where *s* is the length of the growing season. Conditional variance of the yield is denoted by σ_q^2. Similarly, we define $\tilde{p}$ as th e random spot price at harvest with expected value μ_p, defined as

$$E_t[\tilde{p}_{t+s} \mid \Omega_t] = \mu_p \qquad (3)$$

and conditional variance as σ_p^2. Information set Ω_t includes information available at sowing time, such as past prices and input use.

We define randomness of $\tilde{p}$ and $\tilde{q}$ to be additive such they can also be written as:

$$\tilde{p} = \mu_p + \tilde{v}_p \qquad \tilde{v}_p \sim N(0, \sigma_p^2) \qquad (4)$$

$$\tilde{q} = \mu_q + \tilde{v}_q \qquad \tilde{v}_q \sim N(0, \sigma_q^2) \qquad (5)$$

where $\tilde{v}_p$ and $\tilde{v}_q$ are jointly normally distributed errors with zero mean, variance σ_p^2 and σ_q^2, and covariance σ_{pq}.

Let h be the forward sale at the harvest per hectare at a fixed price p_f. The remaining output $(\tilde{q} - h)$ is sold at the spot market price at harvest. A hedge occurs when $0 \le h \le q$; speculation occurs when $h > q$ or when $h < 0$. The farmer's income comes from two sources: the uncertain earnings from the un-hedged output $\tilde{p}[\tilde{q} - h]$ and the certain returns, $p_f h$, from the output sold in the forward contract. Then, farmer income at harvest is

$$y = \tilde{p}[(\tilde{q} - h)] + p_f h = \tilde{p}\tilde{q} + h(p_f - \tilde{p}) \qquad (6)$$

and the expected income $E(y)$ is:

$$E(y) = E(\tilde{p}\tilde{q}) + h(p_f - \mu_p) \qquad (7)$$

where $E(\tilde{p}\tilde{q})$ is the expected value of joint normally distributed price and yield. Variance of income σ_y^2 is:

$$\sigma_y^2 = \sigma_{pq}^2 + h^2\sigma_p^2 - 2h\,\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q}) \qquad (8)$$

where σ_{pq}^2 is the variance of the product of the random price and yield, $\tilde{p}\tilde{q}$, and $\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q})$ is the covariance between $\tilde{p}$ and $\tilde{p}\tilde{q}$.

Substituting (7) and (8) into (1), we get:

$$W = E(\tilde{p}\tilde{q}) + h(p_f - \mu_p) - \frac{1}{2}A[\sigma_{pq}^2 + h^2\sigma_p^2 - 2h\,\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q})] \qquad (9)$$

The optimal hedge is given by the first order condition:

$$\frac{\partial W}{\partial h} = p_f - \mu_p - Ah\sigma_p^2 + A\,\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q}) = 0 \qquad (10)$$

and solving for the optimal hedge h we get:

$$h = \frac{\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q})}{\sigma_p^2} + \frac{p_f - \mu_p}{A\sigma_p^2} \qquad (11)$$

Here, the forward price is deterministic and rest of variables are unknown. Using (4) and (5) $\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q})$ is (See Appendix):

$$\mathrm{cov}(\tilde{p}, \tilde{p}\tilde{q}) = \mu_p r \sigma_p \sigma_q + \mu_q \sigma_p^2 \qquad (12)$$

where r is the correlation coefficient between $\tilde{v}_p$ and $\tilde{v}_q$. Substituting (12) into (11) and dividing both sides by μ_q yields in the optimal hedging ratio:

$$\frac{h}{\mu_q} = \left(1 + r\left(\frac{\sigma_q}{\mu_q} \middle/ \frac{\sigma_p}{\mu_p}\right)\right) + \frac{p_f - \mu_p}{A\mu_q\sigma_p^2} \qquad (13)$$

which can be separated into two components: "the hedging component" and "the speculative component". The speculative component is zero and drops out from the problem if either the forward market is unbiased ($\mu_p = p_f$), or the producer is infinitely risk averse ($A \to \infty$). In this case, the optimal hedge ratio would be identical to the minimum variance hedge ratio. If, in addition, either the price is independent from the yield ($r = 0$), or the yield risk is zero, the producer sells all of his expected yield through the forward contract ($h = 1$).

Clearly, when the forward price is an unbiased estimate for the price at harvest, the optimal hedge under yield uncertainty depends on the ratio of percentage yield volatility (μ_q/σ_q) to percentage price volatility (μ_p/σ_p), and correlation between the price and the yield (r). The correlation between the price and the yield is crucial for determining the optimal hedging strategy. When the correlation between the price and the yield tends to zero, MV model suggests that the hedge ratio approaches to unit regardless of farmer's risk attitude (Rolfo 1980, Newbery et al. 1981, Anderson and Danthine 1983). However, Losq (1982) along with Lapan and Moschini (1994) found in their EU-model that the optimal hedge, in general, is less than the

expected yield even when the price is independent from the yield.

Assuming a Constant Absolute Risk Aversion (CARA) in the utility function, an exact analytical solution to the hedging problem is possible even under the EU-model, as illustrated by Lapan and Moschini (1994). In their research, they considered both basis risk and production risk for the future hedging problem. In our case, the forward price is deterministic and, therefore, the basis risk does not exist. If the forward price is perceived to be the same as expected price, absent basis risk, the optimal hedge ratio derived from the EU-model is [1]:

$$\frac{h}{\mu_q} = \left(1 + r\left(\frac{\sigma_q}{\mu_q} \middle/ \frac{\sigma_p}{\mu_p}\right)\right) - A\frac{\mu_p \sigma_q^2}{\mu_q} \qquad (14)$$

here we can see that even when the yield and the price are independent (r = 0), the optimal hedge is still less than one. It is reduced by the product of risk aversion coefficient (A), the ratio between expected price and yield (μ_p/μ_q), and yield volatility.

Data

The weekly data on wheat cash (spot) price and intervention price are obtained from information centre of the Ministry of Agriculture and Forestry (TIKE). Both price series spans from January 1995 to 21st week of 2002 (Fig. 1). The original cash price series consists of 370 observations and 16 missing values. The missing values are replaced by the average of the preceding and following values in order to keep the continuity of data. The cash price reported in the data is the price at the warehouse of buyers, i.e. the price of the raw material to the buyer.

Growing seasons are 15 weeks and they are dated from the 21st week of the year to the 36th week of the year. In Figure 1, growing seasons are marked by vertical lines with marks of s.

The forward contracts are those offered by Avenakauppa and they are available for 2001 and 2002. These contracts are not tradable and they are always terminated by the delivery of the good. No penalty is imposed if farmer's yield is lower than expected and he can not deliver the good as much as agreed in the contract resulting from force majeure. Nevertheless, speculation through deliberate short selling is not allowed. The terms related to delivering the goods as much as agreed are for the most part based on trust. Avenakauppa company also publishes its forward prices publicly on internet and page 747 in the text-tv of MTV3 in Finland. The annual yield data of spring wheat are from MTT Agrifood Research Finland and they span the years from 1995 to 2001. The yield data consist of 40 observations based on experimental trials and they are conditional on different nitrogen applications (70–120 kg ha[-1]). The trials have been located in the Southwest Finland, where most of the wheat production takes place.

Estimation

Conditional mean process for the wheat price

The conditional mean process for the wheat price is modelled as an AutoRegressive (AR) time series model, which is the standard approach in estimating the parameters for the optimal hedging model (Dawson et al. 1999, Maurice and Kieran 2001). These models are generally found to perform well in forecasting future commodity prices conditional on current information (e.g. Muth 1961, Beck 2001).

A hypothesised relationship is that the Finnish wheat prices are mainly determined by their own lags, time trend, yield effect at harvest, the inter-

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[1] The lengthy derivation of the optimal hedge under the EU-model is given in Lapan and Moschini (1994) and it is not repeated here.
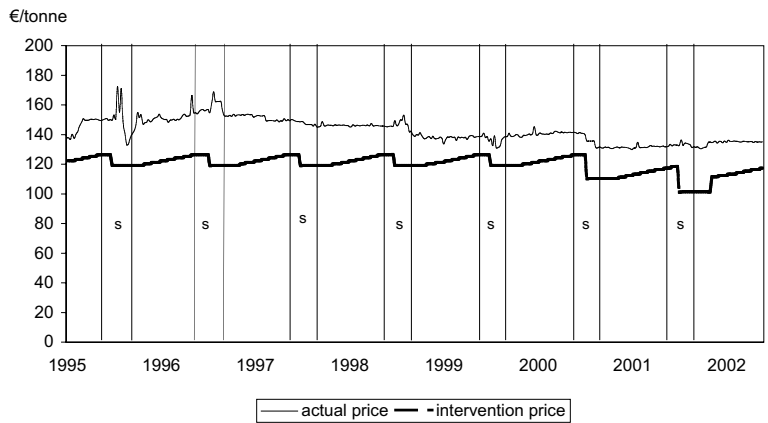
Fig. 1. Weekly marketing price and intervention price of wheat in Finland from 1995 to 2002.

vention price and a disturbance. A general model is denoted as AR(k) with k representing the number of lagged prices in the estimating equation. The model is:

$$p_t = \phi_0 + \sum_{j=1}^{k} \phi_{t-j} p_{t-j} + \alpha D_t q_\tau + \varphi p_t^i + \lambda t + v_{pt} \quad (15)$$

where p_t is the weekly price quotation for wheat; t is the time index. D_t is a dummy variable, and it receives value one at the time of harvest, i.e. at 36th week of each year and zero otherwise. q_τ is the yield in year τ. p_t^i denotes the intervention prices[2] at time t. α, ϕ_0, ϕ_{t-j}, φ and λ are parameters, and v_{pt} is the error term.

Time trend (*t*) is included in the model, because the observed price series show a declining trend over the estimated years. The yield variable (*q*) is included to test whether the annual yield has significant effect on the price at harvest. The estimating equation is further augmented by the intervention price for wheat (p_t^i), since the market prices are expected to, at least partially, respond to the changes in the intervention price. The intervention

price has had the largest changes in summer when the marketing years have changed.

One important simplifying assumption for Equation (15) is that of weak stationary of the wheat price series. For that purpose, we test for unit roots using the Augmented Dickey-Fuller (ADF test) (Dickey and Fuller 1981) and follow the sequential procedure of Dickey and Pantula (1987) for all the estimation period. A linear trend term is added as the wheat price has a noticeable downward trend over the period. The result, shown in Table 1 suggests that the null hypothesis of unit root is rejected at 1% risk level. Thus the data are informative enough and the ADF-test has power enough to reject non-stationarity in favour of stationary process around a deterministic trend.

Table 1. Augmented Dickey-Fuller test statistics. The full sample used in estimation.

Test specification	Test statistics
Without intercept	–0.21
With intercept but without drift	–2.13
With drift and intercept	–5.15**

Note: The number of observations is 401. Mackinnon critical values for rejection of hypothesis of a unit root at 1% significance are –2.58, –3.47, and –4.01 level for without drift, with drift, and with drift and trend respectively. A double asterisk (**) denotes significance at 1% level.

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[2] The intervention price is the price at which intervention agencies are obliged to purchase wheat from the market. The intervention price has gradually decreased between 1999 and 2002 in EU. For instance, at 37th week in 1999 the price was 119 euros per tonne and the same week in 2002 it was 101 euros per tonne.

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Table 2. OLS parameter estimates of AR(3) model. Estimated in Equation (15).[1]

Parameters	Unrestricted specification	Restricted specification
Intercept (ϕ_0)	22.290 (0.000)	24.899 (0.000)
Once lagged price(ϕ_1)	0.616 (0.000)	0.621 (0.000)
Twice lagged price (ϕ_2)	0.477 (0.019)	0.479 (0.018)
Third lagged price (ϕ_3)	–0.259 (0.020)	–0.260 (0.020)
Yield (α)	–0.087 (0.720)	–
Intervention price (φ)	0.028 (0.350)	–
Trend (λ)	–0.010 (0.000)	–0.010 (0.000)
Q(10) statistics	7.480 (0.680)	6.960 (0.729)
R-squared	0.910	0.910

Note: The numbers below the parameter estimates and in the parentheses denote P-values.
[1] OLS refers to Ordinary Least Squares estimation and AR(k) refers to Auto Regression with k being the number of lagged prices in the model.

Using the procedure of model identification of Box and Jenkins (1976), AR(3) is found to generate a dynamically complete specification such that the error follows a white noise. The parameter estimates in Equation (15) are displayed in Table 2. Noticeably, the P-values for parameters suggest that the intercept, the autoregressive term and the trend parameter differ from zero at 1% level. The price response with respect to the intervention price and yield is statistically insignificant. Q-statistic (Ljung and Box 1978) serving as a residual test suggests that the residuals yielding from Equation (15) follow the white-noise process. The predictive power of the equation is very high, the R^2 equals to over 90% as is the standard in similar price models.

Forecasting the conditional mean price at harvest

When the hedge position is opened at 21st week, we look forward and forecast the expected price 15 weeks ahead given the information at 21st week.

Because the marginal effects of yield and intervention price on the next period price are insignificant, we can estimate the expected value of price at harvest simply through the AR(3) price process by excluding the yield and intervention prices.

We use the minimum mean square error forecast method to predict the wheat price at harvest (Mills 1997, p. 53–55). A minimum mean *square* error forecast, denoted by $\hat{p}_{t+j}$, made at week t for the price at period $t + j$ for $j = 15$ can be estimated through the repeated substitution in Equation (15): Therefore, the forecasting conditional mean price at $t + j$ given the information at t could be generally described as.

$$E[p_{t+j}|p_t, p_{t-1}, p_{t-2}] = \begin{cases} p_{t+j}, & j \leq 0 \\ \hat{p}_{t+j}, & 0 < j \leq 15) \end{cases} \quad (16)$$

where $\hat{p}_{t+15}$ denotes the predicted price at harvest.

The predicted price at harvest in year 2002 is estimated at 129.84 euros per tonne. It does not differ significantly from 135 euros per tonne, offered in the forward contract of Avena. Thus, the results suggest that the market was unbiased.

Estimating conditional mean of yield

The expected yield at harvest is estimated using the annual data from 1995 to 2001 included in the MTT experimental trials. The data have 40 observations in total, consisting from 4 observations in 1996 and 6 observations per year for all other years. The yield Equation 5 is estimated conditional on information that is available for farmers at sowing when the hedging decision is made. The information includes location, soil quality, time trend and nitrogen fertiliser application. Because the location of the experimental sites and the soil types could not be estimated simultaneously in the same model we specified two models. The first model includes four dummy variables for location (five sites) excluding the soil types. The second specification includes five dummy variables for the soil types (six soil types) but excludes the location effect.

$$q(D^i, \tau, n) = a + \beta D^i + \gamma\tau + \pi n + v_q \qquad (17)$$

where q is output for the spring wheat ton/per hectare; D^i are the dummy variables indicating location ($i = 1$) or soil type ($i = 2$). They receive value 1 when the experiment represents certain location or soil type, and zero otherwise. Variable τ is the annual time trend. It equals to 1 in 1995, 2 in 1996, etc. Variable n denotes the nitrogen fertiliser application kg/hectare. a, β, γ and π are parameters, and v_q is error with zero mean and variance σ_q^2.

Table 3 lists the results of model including and excluding the dummy variables. The parameter estimates suggest that the yield exhibits a slightly decreasing trend and, as expected, the nitrogen application significantly increases yield (Table 3). Nevertheless, nitrogen application and the trend explain only less than 26% of annual yield variation. The dummy variables controlling for location of the field experiment or soil quality do not significantly improve the fit of the model. Thus, the results support the view that weather performs a crucial role in Finnish agriculture, and it contributes to a significant yield uncertainty. The average yield (μ_q), conditional on 90 kilogram nitrogen application, was estimated at 3.52 tonnes per hectare.

Covariance and correlation between the price and the yield

The correlation between the price and the yield is needed in the optimal hedging equations (Equations 13 and 14). Because the data used in estimating the price process in Equation (15) and yield process in Equation (17) exhibit different time frequencies, the following steps are used in estimating the correlation coefficient (r). First, we compute the predicted price for each year at harvest (36th week) and obtain the prediction error ($\tilde{v}_p$) as the difference between the observed and predicted

Table 3. Parameters in the yield equation (17).

Parameters	Dummy variables for location		Dummy variables for soil quality		Dummy variables excluded	
	Estimate	P-value	Estimate	P-value	Estimate	P-value
Intercept (α)	3.297	0.001	2.869	0.015	3.693	0.000
β_1	0.272	0.607	0.07	0.909	–	
β_2	–0.035	0.959	–0.14	0.847	–	
β_3	0.980	0.115	0.322	0.66	–	
β_4	0.588	0.373	0.984	0.16	–	
β_5	–	–	0.875	0.262	–	
Nitrogen (π)	0.014	0.107	0.018	0.075	0.0136	0.000
Time trend (γ)	–0.17	0.060	–0.147	0.118	–0.175	0.056
F-statistic	1.756	0.138	1.593	0.173	3.320	0.087
R^2	0.242	–	0.258	–	0.152	0.047

prices at harvest. Second, the prediction error for the yield $(\tilde{v}_q)$ is then computed as the difference between the observed and fitted values according to the yield function 17.

Because the normally distributed errors with zero mean $\tilde{v}_p$ and $\tilde{v}_q$ are jointly distributed by assumption, define ε as: $\varepsilon = [v_p, v_q]'$. The variance-covariance matrix of variation of price and yield can be written as $\Sigma = E[\tilde{\varepsilon}\tilde{\varepsilon}']$ of the random vector ε (Lapan and Moschini 1994):

$$\Sigma = \begin{bmatrix} \sigma_p^2, r\sigma_p\sigma_q \\ r\sigma_p\sigma_q, \sigma_q^2 \end{bmatrix}$$

where σ_p and σ_q denote the standard deviation of the price and the yield, and r denotes the correlation coefficient between v_p and v_q.

The correlation coefficient r is estimated at –0.28. The Pearson's correlation test suggests that the correlation coefficient is statistically significant at 10% significant level. Thus, a positive yield shock tends to decrease the price and a negative yield shock likely increases the price. The result is explained, for example, by transactions costs that drive a different wedge between the domestic and foreign prices depending whether the domestic market is in excess supply or excess demand.

Since the price and yield data have different frequency, we exploit interval estimate as a range of correlation values for the later use. With sample size 40 and mean value –0.28, the confidence interval for the population value of Pearson's correlation is $-0.56 \leq r \leq -0.02$[3] Clearly, the yield volatility is much larger than the price volatility with respect to the mean value of price and yield, respectively (2% of price volatility to 32% of yield volatility) In comparison, the average yield volatility in EU varies between 10 and 40% (Meuwissen et al. 1999a).

Table 4 summarizes the parameters we need to calculate the optimal hedge ratio as given by Equations (13) and (14).

Table 4. Summary of the estimated parameters needed in (13) and (14).

Parameters	
r	–0.28 (0.104)
σ_p	2.43
μ_p	129.84
Ratio σ_p/μ_p	1.87%
σ_q	1.12
μ_q	3.52
Ratio σ_q/μ_q	31.8%
P_f	135

Note: (.) is P-value of Pearson's correlation test.

Risk aversion and optimal hedge ratio

The appropriate value for the risk aversion parameter is very difficult to evaluate, as there has been little empirical evidence on the magnitude of risk aversion in Finland. Following Newbery and Stiglitz (p. 72–73), it is more usual to express the magnitude of risk aversion as coefficient of relative risk aversion, R_r, instead of coefficient of absolute risk aversion A. That is because R_r is dimensionless regardless of different level of wealth, while A is not dimensionless[4]. The relationship between these two coefficients is $R_r(y) = yA$, where y is the end of period income presented in equation (2). Anderson and Dillon (1992) proposed a rough and ready classification of degrees of risk aversion. Based on the relative risk aversion with respect of wealth, in the range 0.5 indicates hardly risk averse at all and about 4 very risk averse.

Martinez and Zering (1992) applied constant "Arrow-Pratt" index of absolute risk aversion to evaluate their optimal dynamic hedging model. The value of the absolute risk aversion were given to be 0.006, 0.007, and 0.008 and with respect to the total income of average 164.39$ per acre (1 acre approximately = 0.4 hectare) each year, the corresponding relative absolute risk aversion ap-

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[3] The result is computed by the formula $z' \pm z/\sqrt{N-3}$, where z' is Fisher's z' value converted from Pearson's r and z is statistics for normal distribution with 90% confident level. N represents the sample size

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[4] Since the coefficient of absolute risk aversion A depends on the units in which income is measured.

proximates 1. Accordingly, if we set up relative absolute risk aversion value between 1 and 10 with respect to minimum and maximum possible incomes per hectare, the corresponding coefficients of constant absolute risk aversion then vary approximately from 0.002 to 0.02.

Table 5 displays the optimal hedge ratios of Finnish spring wheat producers for values of the absolute risk aversion from 0.002 to 0.02 when the forward market is assumed to be unbiased. Clearly, increasing negative value of *r* tends to decrease forward contracts sales. In addition, it appears that only when the correlation between price and yield tends to zero can make forward contract sales attractive. In fact, MV model yields full hedge ratio when the price and yield are independent regardless of risk aversion degree, whereas EU model yields the optimal hedge ratios varying significantly for different values of *A*. The amount of forward sale clearly declines as risk aversion increases. The more risk averse, the less forward the farmers is willing to sell.

Since our empirical results weakly signal for a large negative correlation between the price and the yield, and yield volatility is relatively high, we simulated the MV-model conditional on alternative values for yield volatility and the correlation coefficient between the yield and the price. Figure

2 shows that the hedge ratio is a decreasing function of yield volatility and an increasing function of correlation coefficient. For instance, at the (insignificant) estimate –0.28 or the yield and price correlation (*r*), the forward contract is attractive to the farmer only if the volatility of yield is less than 7% of the mean yield (= 3.52 tonnes per hectare). Otherwise forward contract is attractive only if speculation is allowed and market is biased, or the farmer also has an access to options and can complement the forward contract by selling put options (Sakong et al. 1993, Moshcini and Lapan 1995). Yield risk increases farmer incentives to trade options even when the price offered in the forward contract is an unbiased estimate for the harvest price. The reason is that a farmer who has sold his expected yield forward at a fixed price is still exposed by the risk that lower than expected yield will decrease his revenue.

Concluding remarks

Both MV and EU model suggest that the correlation coefficient between the yield and the price play very crucial role in the optimal hedge ratio.

Table 5. Optimal hedge ratios in MV- and EU-models conditional on different value of r and alternative values for risk aversion (simulated in Equations 13 and 14).

	Correlation coefficient (r)			
	r = –0.56	r = –0.28	r = –0.02	r = 0
MV-model	–8.52	–3.76	0.66	1.00
EU-model when A equals				
0.002	–8.61	–3.85	0.57	0.91
0.003	–8.66	–3.90	0.52	0.86
0.004	–8.71	–3.95	0.47	0.81
0.005	–8.75	–3.99	0.43	0.77
0.006	–8.80	–4.04	0.38	0.72
0.007	–8.84	–4.08	0.34	0.68
0.008	–8.89	–4.13	0.29	0.63
0.009	–8.94	–4.18	0.24	0.58
0.01	–8.98	–4.22	0.20	0.54
0.02	–9.45	–4.69	–0.27	0.07

*MV does not depend on risk attitudes when the futures price is perceived to be unbiased.
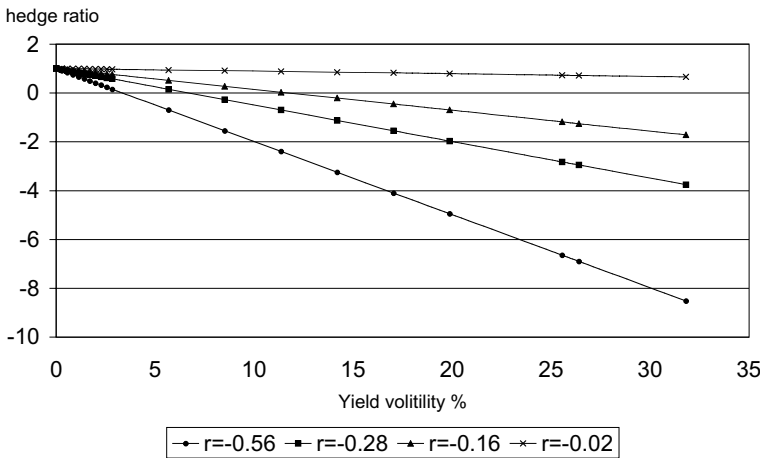
Fig. 2. Optimal hedge ratio conditional on yield volatility and the correlation coefficient between price and yield in the MV-model (Equation 13).

Our results suggest that domestic yield and the domestic price of wheat are negatively correlated. The result is consistent with earlier literature. The total demand for food changes only moderately from year to year, while supply can fluctuate considerably due to weather conditions (Rolfo 1980, Newbery et al. 1981, Losq 1982, Lapan and Moshini 1994, Tirupattur and Hauser 1996, Mahul 2003). The negative covariance between price and yield create a partial "natural hedge"[5], which weakens the role of income risk reducing from forward contracts (ceteris paribus). When the natural hedge takes place, the optimal hedge is always less than expected output.

Hedging effectiveness declines further as the yield variability increases. Thus, the optimal hedge ratio decreases as the yield variability and production risk increase. The finding is very important in such a country as Finland, where the yield uncertainty dominates price volatility. As a result, forward contracts could be less attractive in Finland than some other consistent production areas such as USA, France and Germany.

If the high natural hedge and high yield volatility take place simultaneously, forward sale may not be an attractive risk instrument any more if the producer does not have access to other derivatives, such as options. Instead of forward sale, producers would even wish to buy forward as to ensure the output at harvest. This problem may confront the Finnish wheat growers and traders, because the yield uncertainty dominates price uncertainty, and farmer access to derivatives markets is likely to incur high transactions costs.

Even more generally, high yield risk provides a rationale for the use of other risk instruments along with forward. Mahul (2003) showed futures and crop yield insurance are complements. When either option or crop yield insurance is available, forward contracts become more attractive to the producers.

It is likely that also the European intervention program that sheds a lower bound for the wheat price and truncates the price distribution from below, decreases the optimal futures and options position in a similar fashion as highlighted by Hanson et al. (1999). The truncation effects are left here, nevertheless, topic for further research.

We conclude that under the Finnish production conditions and under the European intervention programs, where the yield risk dominates the price risk, forward contracts alone do not provide sufficient means for efficient hedging. The solution to the problem is to increase farmer access to other risk derivatives such as options and further develop domestic yield insurance.

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[5] A negative yield-price correlation means that a farmer's income is less variable from year to year than it would be otherwise, thus being called "natural hedge".

Random variables $\tilde{p}$ and $\tilde{q}$ can be written as follows:

$$\tilde{p} = \mu_p + \tilde{\tau} \qquad \tilde{\tau} \sim N(0, \sigma_p^2)$$
$$\tilde{q} = \mu_q + \tilde{v} \qquad \tilde{v} \sim N(0, \sigma_q^2)$$

where $\tilde{\tau}$ and $\tilde{v}$ are jointly normally distributed errors with zero mean, variance σ_p^2 and σ_q^2 with correlation coefficient r.

Therefore, the expected value and $\tilde{p}\tilde{q}$, variance of $\tilde{p}\tilde{q}$ and covariance between $\tilde{p}\tilde{q}$ and $\tilde{p}$ can be evaluated as:

$$E(\tilde{p}\tilde{q}) = E[(\mu_p + \tilde{\tau})(\mu_q + \tilde{v})] = \mu_p\mu_q + r\sigma_p\sigma_q$$

$$\begin{aligned}
Var(\tilde{p}\tilde{q}) &= Var[(\mu_p + \tilde{\tau})(\mu_q + \tilde{v})] \\
&= \mu_p^2\sigma_q^2 + \mu_q^2\sigma_p^2 + (1 + r^2)\sigma_p^2\sigma_q^2 + 2pqr\sigma_p\sigma_q
\end{aligned}$$

$$\begin{aligned}
Cov(\tilde{p}\tilde{q}, \tilde{p}) &= E(\tilde{p}^2\tilde{q}) - E(\tilde{p}\tilde{q})\mu_p \\
&= E[(\mu_p + \tilde{\tau})^2(\mu_q + \tilde{v})] - E[(\mu_p + \tilde{\tau})(\mu_q + \tilde{v})]\mu_p \\
&= \mu_p r\sigma_p\sigma_q + \mu_q\sigma_p^2
\end{aligned}$$

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