

On the Role of Geometry in Formal Design

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Introduction

In recent decades, new methodologies have emerged in architectural design that exploit the computer as a design tool. This has generated a varied set of digital skills and a new type of architectural knowledge. This means, architecture is taking part in an “intellectual revolution [that] is happening all around us, but few people are remarking on it. Computational thinking is influencing research in nearly all disciplines, both in the sciences and the humanities. It is changing the way we think.”¹ A good example of such reshaping of discipline-immanent thinking by means of computation is the paradigmatic shift in sciences like physics or biology caused by the introduction of the computer as the primary tool for simulating and modelling natural processes (Figure 1). Since the 1950s, this has resulted in a successive modification or even replacement of reductionism as the predominant paradigm of research by a systemic, bottom-up understanding. It is not surprising that architects became interested in these systemic models of nature due to related new methods of organization and form-generation provided by computers and appropriate software. As a result, over the past decade, systemic notions and concepts from science have diffused into architectural discourse and are currently being explored for design purposes.

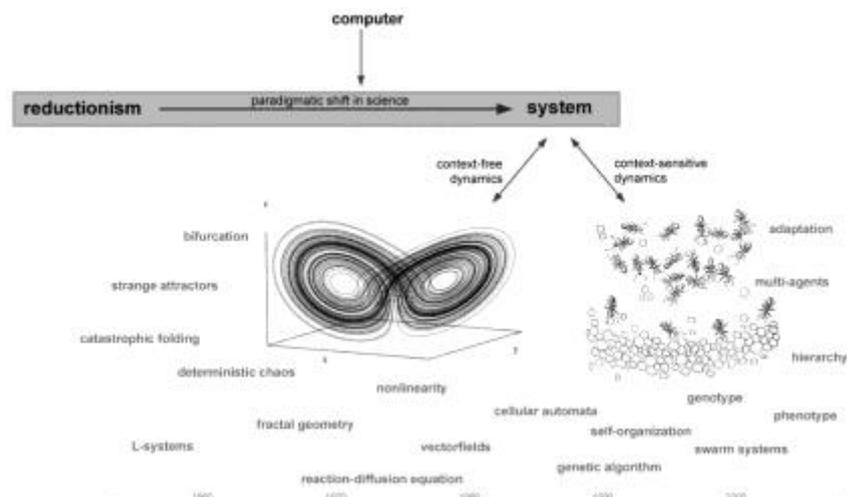


Figure 1: Paradigmatic shift in science from reductionism to systemic thinking and related concepts of exploration that have been taken up in digital architectural design with some delay

The systemic perspective of digital design and the interest in formation processes in science has also caused a renewal of the notion of materiality in architectural design.ⁱⁱ This time, however, materials are not viewed from a phenomenological perspective as it has been the case in architectural discourse of the 1980s.ⁱⁱⁱ Rather, focus is on the use of quantifiable material properties and their influence on architectural form and construction processes.^{iv} The reason is that quantifiable entities can easily be described within a computational environment and, therefore, enable parametric and algorithmic design approaches.^v Mathematics, hence, plays a key role in this current shift towards materiality in architecture. It is the mediator between form and matter, between architectural design and engineering.

Because of this close relation between quantification and materiality it does not surprise, that current computational approaches to architecture are dominated by an engineering paradigm: the use of mathematics as precise tool for the description of various material phenomena and its application for questions of predictability of behavior, the economic use of means, and for reasons of efficiency. Thus, within digital architectural design mathematics is mainly related with the notion of simulation, optimization and performance. This specific understanding of the role of mathematics within design often results in a typological fixation with respect to structural design and emphasize in the exploration on the spectra of possibilities within a well-defined typology rather than an exploration of new structural and architectural solutions for a given problem.^{vi}

As a result of this it can be observed that architects interested in computational means increasingly work like engineers but not as designer, or as Le Corbusier has put it: “Engineers: that means analysis and calculation; Constructor: that means synthesis and creative action”. In a similar way, Hanif Kara has coined the notion of design engineer as an “emphatic model that requires inhabiting the mind of the architect ... while thinking with the knowledge of the engineer.”^{vii} As this ability for transdisciplinary transgression is rare, digital trained architects often end up in so-called geometry units of larger offices, or as specialized consultant and technical support within the design process.^{viii} But very rarely do digital designer end up as responsible design architects. It is this development that marks the starting point of the subsequent rethinking of the role of mathematics within contemporary design thinking.

Mathesis

The reduction of mathematics to a tool for the description of quantifiable relationships does not reflect the essence of mathematics. Already Edmund Husserl had pointed out that the scientific method of quantification of causal relationships is an emptying of mathematical thinking.^{ix}

Etymologically, mathematics has its roots in the Greek *ta mathemata*, which means what can be learned where learning, *mathesis*, is about the recognition of the unchanged, the stable, of the Being in a world of constant Becoming.^x For Martin Heidegger “this genuine learning is an extremely peculiar taking, a taking where one who takes only takes what one basically already has. ... The mathemata, the mathematical, is that ‘about’ things which we already know. Therefore we do not first get it out of things, but, in a certain way, we bring it with us.”^{xi} *Mathemata*, therefore, is bound to the human perception and its ability to identify reoccurring pattern.

Mathematics is the science of patterns! It is about the examination of numerical patterns, patterns of shape, patterns of motion, and patterns of behavior. They can be real or imagined, visual or mental, static or dynamic, qualitative or quantitative.^{xii} *Mathemata* is the human search for patterns as a means of orientation. A fact that Penelope Maddy has pointed out in her exploration of mathematics: “Imagine the purely physical world. This would have to be a giant aggregate composed of all the physical stuff in the universe. There is nothing

nonphysical in this, but most philosophers prefer a less amorphous characterization; they begin with all physical objects, or all particles, or all space-time points. ... To add even this small amount of structure - the differentiation of the amorphous mass into individuals of some kind - is already to broach the mathematical.^{xiii}

Order in nature, therefore, is not only based on the laws of physics but also very much the result of perception as an active act of filtering and structuring sensorial information. This means, order is the result of the human capacity to constantly organize and structure perceived information for the sake of orientation. With other words, the world is a construct whose main reference point is the here and now of the body. It is from the body that surrounding reality is perceived, structured, and accessed.^{xiv} In its original meaning, mathematics is about relating the body with the world around. As such, *mathemata* is about orientation.

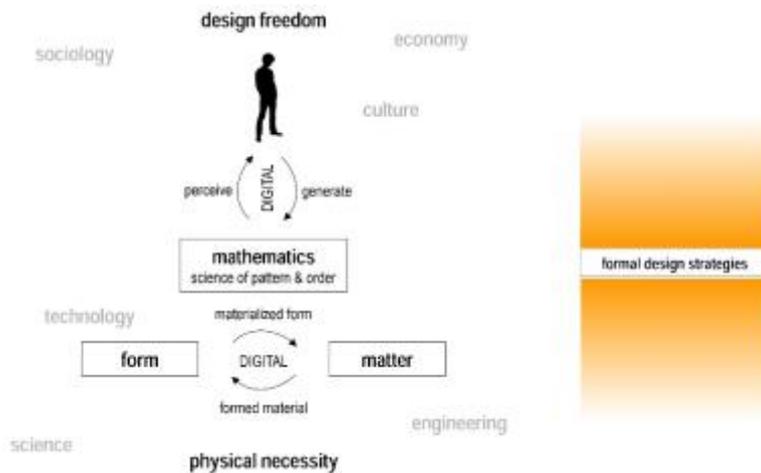


Figure 2: a more general understanding of mathematics can open up the potential for formal design methods as bridge between the technical and the artistic

Because of this, mathematics as a discipline primarily is not a collection of technical tools for precise description and forecasting but rather a way of thinking in structures and organizational pattern. What the discussion above also shows is that contrary to common belief mathematics is not about absolute truth but rather a man-made construct that helps us as humans to formulate and discuss ordering principles and with it the possibility to order and organize the world in which we are living, mentally but also physically. And this characterization as activity of mental and physical ordering is true for architecture, too. Architecture is a man-made construct and a form of expression of the order and organization of our *Lebenswelt*, as Edmund Husserl has called it, of our mental and physical Being-in-the world.

Formal Design Methods

With this in mind, architectural design and its inherent question for spatial order and organization in has to be seen as a mathematical issue, as a question of *ta mathemata* with the mathematical as a way to understand, on one hand, the interaction of form and matter in the physical world and at the same time as a means of human expression. It is in this field between the necessity of physics and the freedom of design that mathematics functions as mediator. This means, mathematics is not limited to the description of the quantifiable causalities in the realm of the physical. But it is an essential part of the design exploration (Figure 2). It is through the architectural organization of form and matter that we as humans are getting aware of related patterns of organisation like social, political or cultural relationships that frame human life.

Formal design strategies in digital architecture, therefore, should not only be discussed with respect to the technical content and requirements but should also

be seen as operations that influence the perception of the underlying configuration. The building up of for example associative geometries and parametric variations are steps in the construction of forms and relationships between parts that introduce information through organization of pattern, through differentiation and disruption of order.

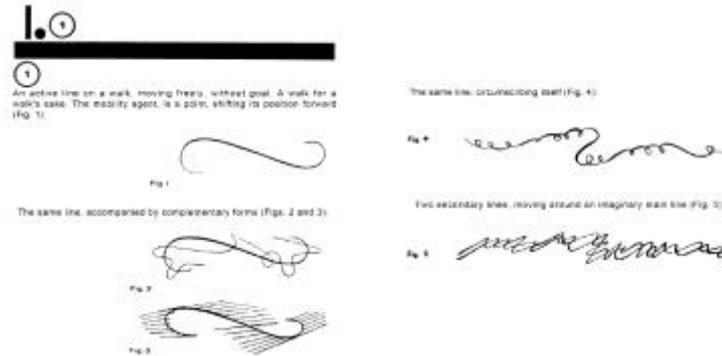


Figure 3: illustration of dynamic visual principles by Paul Klee: the line as trace of a moving point and the change of visual expression of the line by means of simple transformations

Such understanding of digital design as construction of information relates back to similar attempts of exploration of form at the Bauhaus and especially to the systematic teaching of a design theory by Paul Klee in his famous *Pedagogical Sketchbook* (Figure 3).^{xv} Klee used it to introduce students into the dynamic principles of visual art that is into principles of *Gestaltung*, into principles of formation.^{xvi} Such visual concepts are often acquired through experience. But Klee argued that reflection upon the visual grammar opens up the possibility for a more conscious process of creation.^{xvii}

And this argument stays true even more today. Contemporary engineering-driven incorporation of formal method of computation into the design process has introduced the problem of communicability into the discipline of architecture. A problem that is already common to science and engineering. These disciplines have a strong dependency on methods of abstraction and techniques of higher mathematics. Contrary to science or engineering, however, architectural design does not only aim at functionality of its outcome but also at some iconographic readability, of information by gestalt. This does not mean that architects should avoid digital design techniques all together. But it means that formal design methods have to be evaluated not only from a technical point of view but also from a point of view of human perception, similar to Klee's approach to *Gestaltung*. What is required is a human-centered perspective of computation.^{xviii}

Spatial Diagrams

This request is the starting point of an ongoing design research that returns to the question of comprehensibility of architectural form and explores the possibility of computational design process guided by human perception. Such a coupling is based on the simple observation that a low-dimensional space of parametric variation of a configuration can be scanned efficiently by humans if a family of variations is looked at simultaneously (Figure 4). This way, the logical-analytic design tool of formal description transforms into a visual design tool that enables intuitive-creative thinking.^{xix} The field of variations thereby functions in a way comparable to the imprecision of sketching which evokes visual-spatial thinking. By comparison within the parametric field of configurations, emerging spatial phenomena can be identified more easily and linked to a specific parametric setting. This enables the controlled use of spatial phenomena and their combination within the design process.

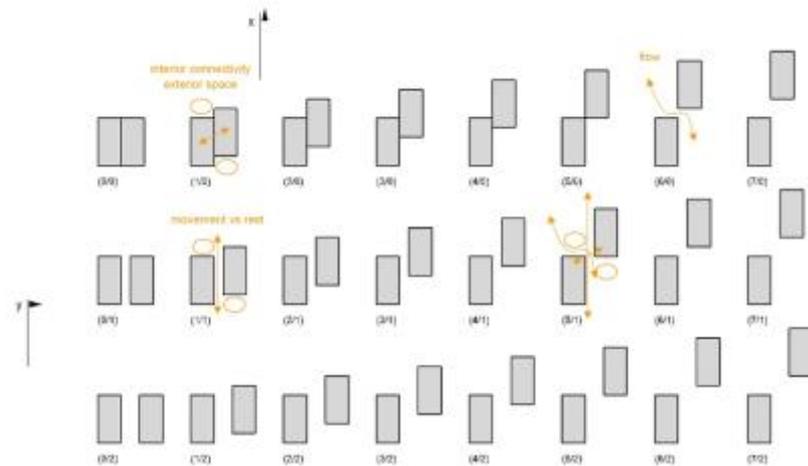


Figure 4: visual identification of spatial phenomena within a field of parametric variation of a simple spatial configuration

Such linking between the computational and the perceptible provides the basis of a design process that has its origin in the detailed study of simple primary forms and their possible variation (Figure 5). These studies are exploring potential spatial transformations that is they are explorations of spatial dynamics and transitions aiming at the production of operative diagrams of spatial conceptions.^{xx} As Rudolf Arnheim has already observed in his famous *Dynamics of Architectural Form* “space is created as a relation between objects. These relations persist in perceptual experience. ... There are many aspects of experience of which we are not explicitly conscious that nonetheless tinge our awareness in important ways. The visual relations between objects are of this kind. Space between things turns out not to look simply empty.”^{xxi} The resulting spatial diagrams, therefore, can be seen as the condensed description of the underlying field condition given by the parametric variation, of the “form between things” as Stan Allen has defined it.^{xxii}

It is the dynamic of space captured in the spatial diagram, its rhythmic change, its speed of flow, its directionality that defines its architectural potential, its usability for specific program and organizational schemes. In other words, within this design approach it is not a given program that drives the creation of a specific architectural form but rather the architectural form that defines the appropriate program. It is the spatial dynamics of form that informs the architecture and defines its functionality. The generative force of the spatial diagram, however, is not rigid. Rather it is a topological description of a set of relationships that has not solidified into a fixed architectural expression yet. This immanent flexibility enables an adaptation of the internal structure of the diagram to a given context by means of translation of the external conditions into formative sets of geometric rules that help to actualize the spatial diagram based on the inherent dynamic of space.

In contrast to Lynn’s urban strategy of NURBS-based deformation of a given geometric configuration this design approach can be viewed as actualization: an abstract spatial diagram is translated into an architectural concept through configurational concretization like for example the interpretation and adaptation of a spatial diagram as schematic sectional drawing. The concretization and contextualization, thus, happens on an intellectual level inspired by perception and visual thinking and not so much by an algebraic transformation driven by quantifiable input data. The resulting architectural conception is still rather schematic and requires further elaboration. It is still abstract but through the additional level of architectural interpretation a path for design development has been opened up that enables and guides further concretization.

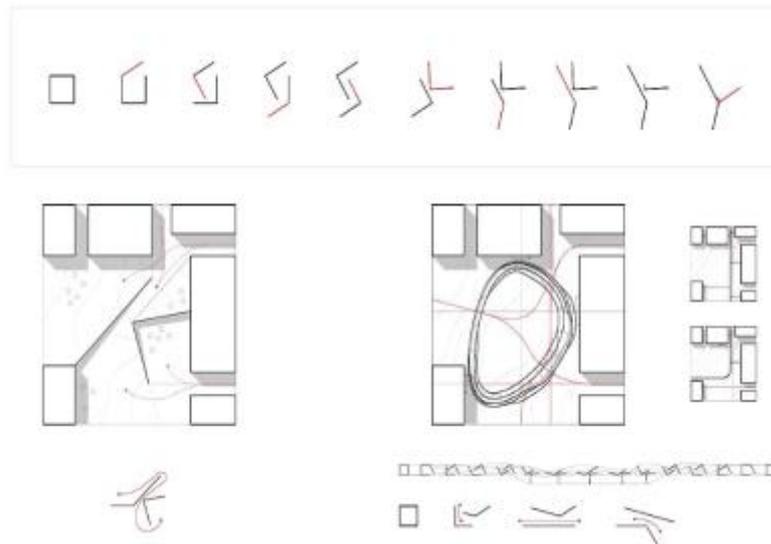


Figure 5: spatial diagram based on a sequence of parametric variations and contextualization of the diagram in a prototypical urban environment (Formal Design Studio, Aalto University, fall 2015, student: Hanna Jahkonen)

First Principles

Central to this further concretization of the spatial diagram are structural considerations because the physical presence of architecture, its corporality requires structural integrity. “Every physical being, living and non-living, has to support its materiality against the various forces that are imposed upon it by its environment, such as gravity, wind or atmospheric pressure. Philosophically speaking, the materiality of physical beings can be thought of as embodiment of two intrinsic coincident principles: primary matter itself and its form, its gestalt in space. Both principles are intricately interwoven, and in the physical world one cannot occur without the other: no material is without form and no form exists without materialization.”^{xxiii} Structural design, thus, is used to actualize the spatial conception and give it a physical, tangible expression.

In this regards, structural design is not understood as composition of building elements but rather as flow of forces through space and the interaction of this flow with form and matter, with a material system that tries to give shape to an underlying spatial idea. Such a structural thinking is necessarily not bound to typological fixation but aims for topological flexibility that can cope with the topological softness of the spatial diagram. Because of this, the force distribution in space is not simply visualized using contemporary computational tools like FEA but rather it is constructed step-by-step in a geometric manner based on simple vector-based operations that have their origin in plasticity theory.

Contrary to the widely used elasticity theory, plasticity theory evaluates the structural behavior of a material system, not with respect to the usability based on elastic deformation, but rather, with respect to its load bearing capacity based on the plastic behavior of the material. This approach enables a decoupling of material effects, kinematics and equilibrium^{xxiv}, and the application of the First Fundamental Theorem of Limit Analysis makes it possible to reduce the question of structural design to equilibrium solutions during the conceptual phase of the design process.^{xxv} These equilibrium solutions are based on resultant forces of underlying stress fields, and can be represented by inscribed strut-and-tie models and corresponding tension and compression forces. With this, plasticity theory provides a solid theoretical foundation for a simple vector-based method of structural design that has its origin in Karl Culmann’s *Die Grafische Statik* from 1866, and is able to illustrate the reciprocal relationship between shape and stress in load-bearing elements.^{xxvi} In addition, the formal character of this description of the flow of forces enables the direct implementation of a parametric

strut-and-tie geometry that maintains major structural behavioral characteristics under transformation.

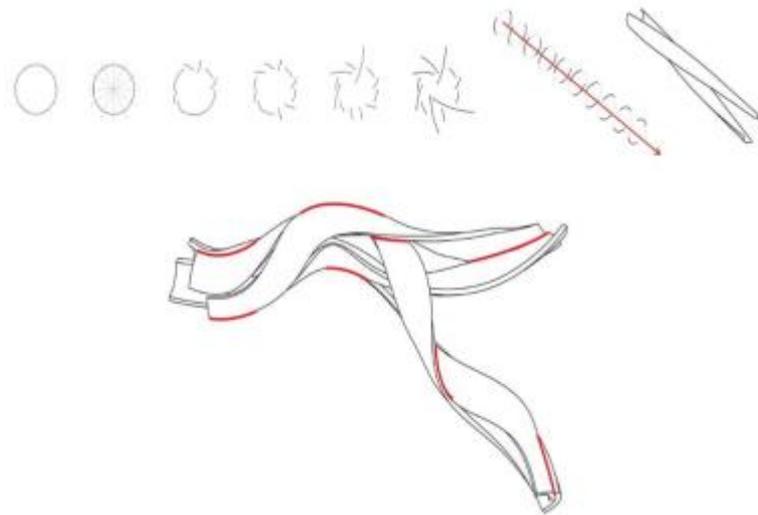


Figure 6: actualization of spatial diagram through geometric stiffening of surface and spatial network interaction (Formal Design Studio, Aalto University, fall 2015, student: Salvador Hernandez Gazga & Laura Zubillaga)

Due to the focus on the flow of forces through space and the topological flexibility of the flow and of the geometric diagrams, the actualization of the spatial concept can easily be freed from the use of predefined structural systems and buildings elements. What gets more important are basic structural principles that can provide an adequate framework for the concretization and the physical manifestation of the spatial diagram. Thus, the conceptual idea of a space as dynamic center that opens up in a fluid way to its surrounding finds its expression in a twirling sheave of concrete strips that stiffen itself locally through the activation of curvature and globally through a weaving strategy that ensures a network of connection points for structural interaction (Figure 6). This means structural design is seen as an active part of the design process out of form and matter emerges according to the underlying spatial concept. Furthermore, structural thinking is not understood anymore as abstract calculus and as analytic activity but much more as synthetic form of thinking that is tangible and aims at a corporal expression of concepts which results in a proto-architectural proposal (Figure 7).



Figure 7: an urban park (student: Hanna Jahkonen (top left)), a cliff-diving platform (Salvador Hernandez Gazga & Laura Zubillaga (top right)), a plaza (student: Karita

Rytivaara & Heikki Myllyniemi (bottom left)), and a museum (student: Fahimeh Fotouhi & Xianwen Zheng (bottom right)) as proto-architectural proposal (Formal Design Studio, Aalto University, fall, 2015)

Conclusion

The discussed design process describes an approach to conceptual design which understands formal methods not as rigid tools but rather as flexible diagrams. Such a perspective enables an integrative design approach that can help to activate knowledge from science and engineering as supportive part of the design process already at an early phase. The possibility of integration is based on an opening up of the underlying mathematics to creative interpretation induced by an understanding of structures and organizational patterns as visual grammar. From a teaching perspective this reading of structures results in a paradigmatic shift that can be described best in three related moves: from precision to principle, from typology to topology, and from computation to construction.

Strive for precision is omnipresent in applications of mathematics. But precision is not of too much relevance in the conceptual phase of design. Rather it is about directing appropriate fundamental design decisions that will determine the future design development to a large degree. This requires flexibility and the ability to constantly recombine and adapt design schemes and basic concepts. That is why working with principles is of greater relevance at this stage of the design process.

Working with principles immediately implies a rather soft and malleable topological understanding of relationships between entities. Fixed scheme of solutions, typologies, are of less importance in an open and creative process. Typologies are stabilizing factors in a design process. Especially in an integrative approach such standardized schemes can form an obstacle. Topological flexibility, thus, is a key element. It also implies that not the final form is the main driver of the design process but rather the process of formation, i.e. the process of coherent integration of information.

Because of this required flexibility, computational approaches and the underlying well-defined and deterministic procedures often can play only a supportive role in the conceptual phase of the design process. What is of more importance at that stage, therefore, is a building up of the inherent logic of the design, it is the construction of the organizational pattern. Constructability, thus, plays the major role in the conceptual phase. And it is constructability that relates back to the importance of mathematics for design: mathematics emphasizes geometrical construction and problem solving.^{xxvii} “Rather than reiterating ontologies of sameness, modern mathematics produces difference through new constructions.”^{xxviii} And it is this new construction, this new organizational patterns that an integrative design aims at.

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References

- ⁱ Bundy, Alan: *Computational Thinking is Pervasive*, Journal of Scientific and Practical Computing, 2007, 1 (2), 67-69
- ⁱⁱ Oxman, Rivka & Oxman, Robert: *Theories of the Digital in Architecture*, Routledge, 2014
- ⁱⁱⁱ See for example Norberg-Schulz, Christian: *Genius Loci, Towards a Phenomenology of Architecture*, Rizzoli, 1980 or Pallasmaa, Juhani, 1996. *The Geometry Of Feeling, A Look at the Phenomenology of Architecture*, in Nesbitt, Kate (ed.): *Theorizing A New Agenda For Architecture: An Anthology of Architectural Theory 1965–1995*, Princeton Architectural Press, 1996, 448–453
- ^{iv} See for example a series of issues from AD like *Emergence: Morphogenetic Design Strategies*, 2004 or *The New Structuralism*, 2010 or *Material Synthesis: Fusing the Physical and the Computational*, 2015
- ^v Kotnik, Toni: *Digital Architectural Design as Exploration of Computable Functions*, International Journal of Architectural Computing, Vol.8, No.1, 2010, 1-16
- ^{vi} An example for such a typological fixation are popular software applications like Rhinovault or RhinoMembrane, both plug-ins for Rhinoceros.
- ^{vii} Kara, Hanif: *On Design Engineering*, AD Architectural Design, Vol. 80, No. 4, 2010, 46-51
- ^{viii} This development has resulted in the upcoming of new types of specialized consultancy practices like *Design to Production*, specialized in digital fabrication, or *Evolute*, specialized in panelization.
- ^{ix} Husserl, Edmund: *Die Krisis der europäischen Wissenschaften und die transzendente Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*, 1936
- ^x Compare Platon in his creation story *Timaios*
- ^{xi} Martin Heidegger: Modern Sciences, Metaphysics, and Mathematics, in Krell, David F.: *Martin Heidegger: Basic Writing*, Harper Collins, 1992, 271 – 305
- ^{xii} Devlin, Keith: *Mathematics: The Science of Patterns*, Scientific American Library, 1994
- ^{xiii} Maddy, Penelope: *Realism in Mathematics*, Oxford University Press, 1990
- ^{xiv} Zec, Peter: *Orientierung im Raum*, Mabeg, 2002, 17 – 19.
- ^{xv} Klee, Paul: *Pedagogical Sketchbook*, Praeger, 1953
- ^{xvi} There is a close relation between Klee's principle of Gestaltung and results made in experimental Gestalt psychology at the beginning of the 20th century. For more details see van Campen, Crétien: *Early Abstract Art and Experimental Gestalt Psychology*, Leonardo, Vol. 30, No. 2, 1997, 133-136
- ^{xvii} For a contemporary collection of the visual rules see for example Leborg, Christian: *Visual Grammar*, Princeton Architectural Press, 2006
- ^{xviii} While most computing-based design tends to emphasize the formal aspects of architecture, overlooking space and its users, early computational design approaches first spearheaded in the UK in the 1960s and 1970s tended to be focused on behavioral and occupational patterns. Over the last decade, a new generation of design research has emerged that has started to implement and validate previous investigations into spatial computation. See for example Derix, Christian & Isaki, Åsmund: *Emphatic Space: The Computation of Human-Centric Architecture*, AD, Vol 84, No 5, 2014
- ^{xix} Gänshart, Christian: *Tools for Ideas: An Introduction to Architectural Design*, Birkhäuser, 2007, 121
- ^{xx} In the given situation, the underlying parametric description ensures the operativeness of the diagram. For a detailed definition of the notion see Kotnik, Toni & D'Acunto, Pierluigi: *Operative Diagramatology: Structural Folding for Architectural Design*, Proceedings of the Design Modeling Symposium, Berlin, Springer, 193-203
- ^{xxi} Rudolf Arnheim: *The Dynamics of Architectural Form*, University of California Press, 1977, 17
- ^{xxii} Allen, Stan: "Field Condition" in Davidson, Peter & Bates, Donald L: *Architecture After Geometry*, AD, No.127, 1997, 24-31
- ^{xxiii} Kotnik, Toni & Weinstock, Mike: *Material, Form and Force*, AD Architectural Design, Vol. 82, No.2, 2012, 104-111
- ^{xxiv} See for example Marti, Peter: *Theory of Structures*, Ernst & Sohn, 2013
- ^{xxv} Muttoni, Aurelio / Schwartz, Joseph / Thürlimann, Beat: *Design of Concrete Structures with Stress fields*, Birkhäuser, 1996

^{xxvi} Lachauer, Lorenz & Kotnik, Toni: *Geometry of Structural Form*, Proceedings of Advances in Architectural Geometry, Springer, 2010, 193-203

^{xxvii} Lachterman, David Rapport: *The Ethics of Geometry: A Genealogy of Modernity*, Routledge, 1989, vii

^{xxviii} Mertins, Detlef: Bioconstructivism, in Spuybroek, Lars: *NOX: machining architecture*, Thames & Hudson, 2004, 360-369