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4π = 12.5? – THE PROBLEMS IN THE VITRUVIAN HODOMETER

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Abstract

In the tenth book of his work (10,9,1–4) Vitruvius describes a hodometer, a device meant to measure the mileage of a road. It has been questioned whether the hodometer of Vitruvius was ever built for actual use, but the reconstructions made by A. Sleeswyk prove that it was technically realizable. However, the Vitruvian mathematics cause a significant problem for its usage – either one of the most prominent names in the history of engineering didn't have a clear conception of the value of π or the passage of his text got corrupt before the archetype of all the remaining manuscripts got formed. The first option seems inacceptable and in the worst-case-scenario its practical consequences would have led to every mile measured by the hodometer being c. 26.55 Roman feet[1] too long.[2] The second option is hard to verify even with a study of the manuscripts, but an explanation is searched for to clear Vitruvius's name.

Introduction

Vitruvius gives the description of the functioning principles of the hodometer in

[1] = c. 7.85 m.

[2] The terms *foot* and *mile* in this paper refer to Roman foot and mile, not to the foot and mile still in use in the Anglo-American world. The metric equivalent of the Roman foot used in this paper is 29.57 cm (see G. Lugli, *La tecnica edilizia romana*, Roma 1957, 189–90) hence the exact metric equivalent of the mile in this paper is 1478.5 m (see for example Vitr. 10,9,4).

the tenth book of his work. [3] He tells that it is used "to be able to know the distance traveled".[4] The function of the device shows that in principle it resembles the odometers or taximeters used in modern-day vehicles. It is probable that the machine was used in Roman road building to measure the mileage of the roads.[5]

This paper focuses on an inaccuracy found in Vitruvius's description. The parameters provided by him show that either Vitruvius was not aware of the value of π or the passage of his text containing the description got corrupt somewhere between Vitruvius's death and the formation of the archetype of all the extant manuscripts. The problem, although not widely studied, has been noted before. It has been suggested for example that instead of the value 3.125 (from the formula in the title), accepted by several critical editions of Vitruvius's work, the actual Vitruvian value of π was in fact 3.[6] For this reason this paper concentrates in paleographical and philological questions related to the passage, trying to clarify what might have happened to it. A study of the manuscript tradition will help in getting closer to what could have caused a possible posthumous misconception of Vitruvius's words.

A. Sleeswyk's reconstruction and the possible Archimedean origins of the hodometer

A. Drachmann, in his handbook *The mechanical technology of Greek and Roman antiquity*, shows skepticism towards Vitruvius's hodometer judging it as an unre-

[3] Vitr. 10,9,1–4. The editions examined for the original text are F. Krohn (ed.), *Vitruvii De architectura libri decem*, Leipzig 1912; L. Caillebat (ed.), *Vitruve, De l'Architecture, livre X*, Paris 1986. The text is from the latter.

[4] More precisely: *qua in via raeda sedentes vel mari navigantes scire possimus, quot milia numero itineris fecerimus* (Vitr. 10,9,1). The description of the nautical hodometer is given in Vitr. 10,9,5–8.

[5] C. Wikander, "Weights and measures", in J. P. Oleson (ed.), *Oxford Handbook of Engineering and Technology in the Classical World,* Oxford 2008, 759–69, 766–7.

[6] See J. Pottage, "Vitruvian Value of π", *ISIS* 59 (1968) 190–7. It seems however that Pottage hasn't read the text in Latin. Also E. Stone's (E. Stone*, Roman surveying instruments* [University of Washington Publications in Language and Literature 4:4], Seattle 1928, 215–42, 219) comprehension of the passage is incomplete. They both for example take for granted that the form in manuscripts considering the diameter of the wheel of the hodometer is *pedum quaternum et sextantis.* However, this is necessarily not the case, as will be seen.

alizable armchair invention, remarking that the dimensions of the gears in the device become impossible to realize in practice.[7] Skepticism towards the Vitruvian hodometer is expressed also by P. Fleury mainly due to challenges in gearing.[8] Nevertheless, alone the famous Antikythera mechanism shows that building gears of small dimension was possible in the Classical World. Also the reconstructions of the hodometer made by A. Sleeswyk first in 1981 on the basis of both Vitruvius's description and Leonardo da Vinci's failed attempts show that in practice the machine was realizable.[9] Sleeswyk argues that the genius responsible for the invention of the hodometer was originally Archimedes in the mid-3rd century BC, when Roman roads got their first milestones. The observations made by M. Lewis give further support for the Archimedean origin of the device.[10] Sleeswyk points out that Vitruvius may not ever have seen an actual hodometer because he starts his description by saying that he now starts to write about an invention made by ancestors, adding a notice made already by Drachmann, that throughout the description Vitruvius is using the subjunctive instead of the indicative mood.[11] This brings a feeling of Vitruvius making a summary of a hodometer manual to the reader. In other words Vitruvius is possibly only repeating what he has read in his Greek source.

7 A. Drachmann, *The mechanical technology of Greek and Roman antiquity,* Copenhagen 1963, 157–9.

8 P. Fleury, *La mécanique de Vitruve*, Caen 1993, 206–12.

9 See for example A. Sleeswyk, "Vitruvius' odometer"*, Scientific American* 245 (1981) 158–71. Following Sleeswyk's groundbreaking reconstruction O. Lendle presented improvements to the Sleeswyk-Vitruvian hodometer in his paper "Vitruvs Meilenzähler (De Arch. 10.9.1–4)" (in W. Görler – S. Koster (eds.), *Pratum Saraviense*, Stuttgart 1990, 75–88, 84–8).

10 Sleeswyk 1981 (n. 9 above), 168–71; M. Lewis, *The surveying instruments of Greece and Rome*, Cambridge 2001, 135–6. To the arguments on behalf of the Archimedean origin of the hodometer presented by Sleeswyk and Lewis should be added that in his treatise right after the hodometer (10,10->) Vitruvius continues with *ballistae* and *scorpiones* – catapults are one of the most often praised Archimedean inventions. On the other hand right before the passage on the hodometer (=10.7–10.8) the inventions discussed by Vitruvius are commonly attributed to Ctesibius (the water organ and the water pump) – considering the description of hodometer written by Heron later in the 1st century AD thus also an Alexandrian origin of the device could be proposed. Another point are the words *Transfertur nunc cogitatio scripturae* Vitruvius uses in the beginning of the description: would he say so, if he were to continue with an invention made by Ctesibius?

11 Sleeswyk 1981 (n. 9 above), 158.

However, given the prominence of Vitruvius as an architect and engineer it seems unlikely that he wouldn't have ever seen a hodometer, keeping in mind the fact that the reign of Augustus witnessed the rebuilding of some major Roman roads such as Via Flaminia and Via Salaria.[12] T. Howe also remarks how Vitruvius's choice of words, *ratio non inutilis,* in the beginning of his description might point out to the fact that the device was actually in use at the time when Vitruvius wrote his description.[13]

The machine

Basically the hodometer was a device set in a cart drawn by horses or pushed forth manually on the road line. In order to understand better the mechanism of the hodometer, getting acquainted with Vitruvius's words is necessary. It is also useful to read Vitruvius's account with an eye on Sleeswyk's reconstruction,[14] which together will help to clarify how the machine worked.

> The thread of writing moves now to a useful device of highest ingenuity, passed down to us by ancestors. With it we are able to know, while sitting in a carriage or sailing in the sea, how many miles we have traveled. This happens as follows. The wheels that will be in the carriage are to have a diameter of four feet[15] so that, when a point is marked in the wheel and the wheel begins to progress revolving from this point, touching the road ground, it revolves to the point where it began, after having completed an exact amount of distance of 12 and half feet.[16]

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[12] For epigraphical evidence, see for example *CIL* IX 5943, 5950.

[13] T. Howe, "Commentary and illustrations", in *Vitruvius, Ten Books on Architecture.* Translation by Ingrid D. Rowland, Cambridge 1999, 135–317, 296.

[14] See fig. 1.

[15] After the word *quaternum* all the manuscripts have either *et sextantes* or *et sextantis* or *et sextante*, which has traditionally been deleted since the edition made by C. Perrault in the late 17th century. This will be discussed further below in the chapter "*Pedum quaternum*" *and* "*pedes XII s" in the manuscripts*.

[16] Vitr. 10,9,1: *Transfertur nunc cogitatio scripturae ad rationem non inutilem sed summa sollertia a maioribus traditam, qua in via raeda sedentes vel mari navigantes scire possimus quot milia numero itineris fecerimus. Hoc autem erit sic. Rotae quae erunt in raeda sint latae per medium diametrum pedum quaternum [et sextantes], ut, cum finitum locum habeat in*

Having these prepared in this way a cylinder is to be inserted firmly to the inner part of the hub of the wheel, equipped with one tooth projecting outside from its perimeter.[17] To the body of the carriage above is to be fixed firmly a receptacle containing a revolving cylinder that is placed perpendicularly and fastened to a small axle. To the perimeter of this cylinder are to be shaped four hundred symmetrically distributed teeth that fit the tooth of the lower cylinder. Furthermore to the side of the upper cylinder is to be fixed another tooth projecting further[18] outside the teeth.[19]

Above this is to be located a horizontal one, toothed in the same manner and enclosed in another receptacle so that the teeth match up with the tooth that was fixed to the side of the second cylinder. In this (horizontal cylinder) are to be as many holes as it is possible to travel miles with the carriage on one day's journey. More or less doesn't impede anything. In all these holes are to be located round pebbles and inside this cylinder's box, or receptacle, is to be a hole with a small channel by which the pebbles that were located in the cylinder, after coming to that spot may fall one by one in to the carriage's body and to a bronze container, which has been placed below.[20]

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se rota ab eoque incipiat progrediens in solo viae facere versationem, perveniendo ad eam finitionem a qua coeperit versari certum modum spatii habeat peractum pedes XII s.

[17] See fig. 1.

[18] See fig. 1.

[19] Vitr. 10,9,2: *His ita praeparatis, tunc in rotae modiolo ad partem interiorem tympanum stabiliter includatur habens extra frontem suae rotundationis extantem denticulum unum. Insuper autem ad capsum raedae loculamentum firmiter figatur habens tympanum versatile in cultro conlocatum et in axiculo conclusum, in cuius tympani fronte denticuli perficiantur aequaliter divisi numero quadringenti convenientes denticulo tympani inferioris. Praeterea superiori tympano ad latus figatur alter denticulus prominens extra dentes.*

[20] Vitr. 10,9,3: *Super autem planum eadem ratione dentatum inclusum in alterum loculamentum conlocetur, convenientibus dentibus denticulo qui in secundi tympani latere fuerit fixus, in eoque tympano foramina fiant, quantum diurni itineris miliariorum numero cum raeda possit exire. Minus plusve rem nihil inpedit. Et in his foraminibus omnibus calculi rotundi conlocentur, inque eius tympani theca, sive id loculamentum est, fiat foramen unum habens canaliculum, qua calculi, qui in eo tympano inpositi fuerint, cum ad eum locum venerint, in raedae capsum et vas aeneum quod erit suppositum singuli cadere possint.*

Thus, when the wheel progresses and makes the lowest cylinder and its tooth move, it forces with every rotation the teeth of the upper cylinder to pass by. This leads to, that when the lower has rotated 400 times, the upper cylinder revolves once and the tooth that is fixed to its side makes forth one tooth of the horizontal cylinder. When thus after 400 rotations of the lower cylinder the upper rotates once, it makes a distance of 5000 feet, that is one thousand *passus*. The sound of a falling pebble tells that a mile has been traveled and the number of the pebbles collected from below indicates the sum of the milestones of the day's journey.[21]

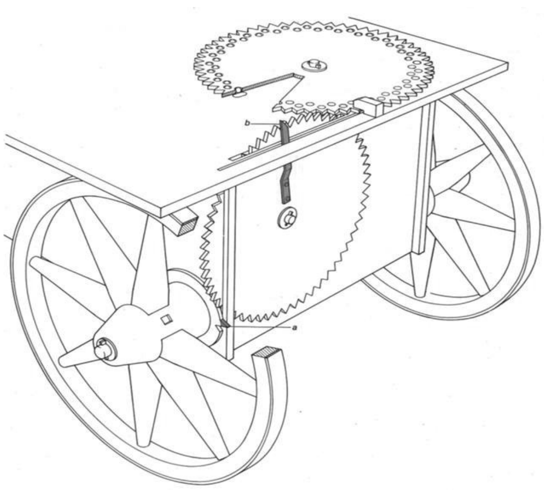

Figure 1. The Vitruvian hodometer as reconstructed by A. Sleeswyk. The cylinder with one tooth is marked with the letter a. The large vertical gear shows fewer than 400 teeth for the sake of clarity. The "tooth projecting outside the teeth" is marked with the letter b. The holes containing the pebbles are in the uppermost gear. Figure from Sleeswyk 1981 (n. 9 above), 166.

The description is somewhat complicated to follow but the basic idea is clear: the measuring is based on the gears connected with the wheel that touches the ground. If the mathematics is in order, the hodometer provides precise linear

[21] Vitr. 10,9,4: *Ita cum rota progrediens secum agat tympanum imum et denticulum eius singulis versationibus tympani superioris denticulos inpulsu cogat praeterire, efficiet ut, cum CCCC imum versatum fuerit, superius tympanum semel circumagatur et denticulus qui est ad latus eius fixus unum denticulum tympani plani producat. Cum ergo CCCC versationibus imi tympani semel superius versabitur, progressus efficiet spatia pedum milia quinque, id est passus mille. Ex eo quot calculi deciderint sonando singula milia exisse monebunt. Numerus vero calculorum ex imo collectus summa diurni <itineris> miliariorum numerum indicabit.*

measures with minor effort. This is a clear advantage of the device when compared with other measuring equipment Romans had: *pertica/decempeda* ("ten-feet"), a ten-feet long rod was used to make linear measurements, but its use on longer distances is not probable.[22] In building roads Romans used also a *groma*, an instrument, which made it possible to plot straight lines and 90-degree angles of established lines – a kind of an ancient total station. However, the *groma* was not used to measure the mileage of a road.[23] One option for measuring longer distances in addition to the hodometer were the βηματισταί, professional "pace-counters" such as Baeton and Diognetus, referred to in Pliny's description of Alexander's conquests as *itinerum eius mensores*, who could provide remarkably accurate measures.[24]

The function of the hodometer and the Vitruvian error

The proper function of the hodometer is dependent on a tolerably accurate value of π, fixed by Archimedes to $3\ 10/71 < \pi < 3\ 1/7$.[25] This is important, because due to the functioning principles of the device even a minute error has drastic consequences for the result: on a mile's journey the error is multiplied 400 times. Nevertheless, regarding the dimensions provided by Vitruvius in his description there's a slight inaccuracy: he tells that the diameter of the wheels in the carriage of the hodometer should be four feet and the perimeter 12 ½ feet.[26] With the equivalent of π known to us we get[27] $C = 2\pi 2 \rightarrow C = 4\pi \rightarrow C \approx 12.566$.[28] Thus, if

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22 Stone 1928 (n. 6 above), 218; the practicality and velocity of measuring with the hodometer is confirmed also by Heron of Alexandria (Her. *dioptr.* 34.).

23 C. Wikander 2008 (n. 5 above), 767–8; Lewis 2001 (n. 10 above), 120–33.

24 Plin. *nat*. 6,25.

25 In decimals (the four decimal place): $3.1408 < \pi < 3.1429$. Archim. *circ.* 3; Ö. Wikander, "Gadgets and scientific instruments", in *The Oxford Handbook of Engineering and Technology in the Classical World*, Oxford 2008, 785–99, 795–6. See also T. Heath, *A History of Greek Mathematics,* vol. II: *From Aristarchus to Diophantus*, New York 1981, 50–6. It might be that Archimedes made an even closer approximation of the value. See T. Heath, *A History of Greek Mathematics,* vol. I: *From Thales to Euclid*, New York 1981, 232–4.

26 Vitr. 10,9,1.

27 From the familiar formula $C = 2\pi r$, where *C* stands for circumference and *r* for radius.

28 With Archimedes's estimation of the value of π the perimeter of a circle with a 4 ft. diameter would measure between 12.563 and 12.571 ft.

the wheels were constructed with a diameter of exactly four feet, the mile measured by a Vitruvian hodometer would become c. 26.55 feet too long.[29] If, on the other hand, the wheels were constructed with a perimeter of exactly 12.5 feet, the diameter would have to be[30] $12.5 = 2\pi r \rightarrow \pi r = 6.25 \rightarrow r \approx 1.989\ (*2) \approx 3.979$ feet. This is remarkably close to 3 47/48 (the fractions based on a denominator of twelve or one of its multiples were relatively easy to express for Romans, instead other than twelve-based fractions were expressed by adding small twelve-based fractions until a good approximation was reached – see the chapter *Roman fractions* below for more) .[31]

On the other hand we could formulate an equation with the parameters provided by Vitruvius inserted in the formula $C = 2\pi r$: $12.5 = 4\pi \rightarrow \pi = 3.125$ (3 1/8 in fraction). This would point out to a fascinating conclusion: Vitruvius was not aware of the value of π! Considering the prominence of Vitruvius as an engineer this seems a bit problematic, even though J. Coulton has shown that in the Greek architecture of the 6th–2nd centuries BC there was a notable tendency to approximations and thus mathematical errors.[32] Also, if Vitruvius is truly reading an account written originally by Archimedes, as Sleeswyk argues, this is hard to accept, as Archimedes's estimate of the value of π was quite accurate and at least not 3 1/8 (see note 25 above). However, the manuscripts show no hesitation with the word for 'four' (*quaternum*).[33] This reveals that the erroneous mathematics was already a part of the archetype.

The manuscript tradition

All the remaining manuscripts can be divided into two families, both of which seem to derive from a 7th century manuscript (marked with x in the figure below) written in Anglo-Saxon script.[34]

[29] $400(4\pi) \approx 5026.55$. One mile is 5000 feet. See for example Vitr. 10,9,4 above.

[30] Again, $C = 2\pi r$.

[31] Maher, W. – Makowski, J. 2001. "Literary evidence for Roman arithmetic with fractions" in *CP* 96 (2001) 376–99, 379.

[32] J. Coulton, "Towards understanding Greek temple design: general considerations", *ABSA* 70 (1975) 59–99.

[33] *Quaternum*, although at first sight seems a singular accusative, is often used as a plural genitive (i.e. with a long last vowel). For other instances of the use, see for example Liv. 6,22.

[34] V. Rose in V. Rose – H. Müller-Strübing (eds.), *Vitruvii de Architectura libri decem*, Leipzig

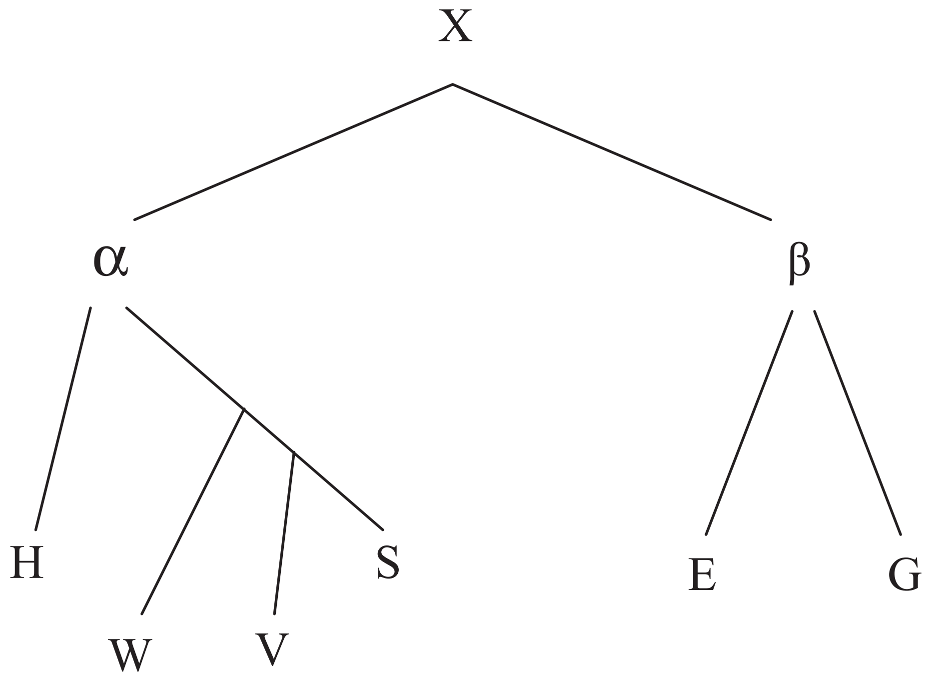

Figure 2. The manuscript tradition of Vitruvius's work.[35]

The family α consists of four independent witnesses. The oldest and most prominent is the *Harley 2767* (H) from c. AD 800, now deposited in the British Library. It remained long as the only witness of the family, until in 1879 the *Bibliothèque et Archives Municipales MS 17* (S) was found in Sélestat (France), where it still is deposited. The other two, *Reg. lat. 2079* (W) (from the 12th century) and *Reg. lat. 1328* (V) (from the 15th century), are in the Vatican. The family β consists of *Gud. Lat. 132* (E) and *Gud. Lat. 69* (G), both deposited in the Herzog August Bibliothek in Wolfenbüttel, former written in the mid/late ninth century and the latter in the 11th century.[36]

Thanks to digital technology it is possible now to consult half of these independent witnesses online: the manuscripts *S*, *E* & *G* can be found digitized on the internet. In other words the whole family β is available to public. When it comes to the representatives of the family α, the situation is somewhat harder, because the only independent witness found online is *S*. The manuscripts *W* and *V*,

1867, vi, (*http://books.google.es/books?id=E6M9AAAAcAAJ&printsec=frontcover&hl=fi&source=gbs_ge_summary_r&cad=0#v=onepage&q&f=false*).

35 The figure is based on the study made by L. Reynolds in L. Reynolds (ed.), *Texts and transmission: a survey of the Latin classics*, Oxford 1983, 440.

36 Reynolds 1983 (n. 35 above), 440–2.

in the Biblioteca Apostolica Vaticana, are not digitized. *H* is digitized only partly by the British Library. However, of the several descendants of *H,*[37] the early (9th-century) *Paris. lat. 10277* can be found online.

***Pedum quaternum* and *pedes XII s* in the manuscripts**

The testimony of the witnesses from the family α consulted for this article is wholly dependent on the *H*, where the loci in question are written *pedum quaternú & sextantes* and *pedes · XII · S ·*. The manuscript *S* is of no use here, because there is a lacuna in it between 10,6,1 *tigno* and 10,10,4 *Crassitudo I*.[38] As regards the manuscripts *W* and *V*, I have not had the possibility to consult them. The Budé edition of Vitruvius[39] anyhow shows that the 15th century V is the only one with the required genitive *sextantis*. At the same time V has dropped the half (*S*) from the correct *pedes · XII · S ·*. The manuscript *W* has both *pedum quaternum et sextantes* and *pedes XII S*.

The two witnesses of the family β show the loci as follows: in the manuscript *G* there is *pedu quaternu et sextante* (with an *s* added afterwards after *sextante*) and *pedes · XII · S ·*.[40] In the manuscript *E* one reads *pedum quaternum & sextante* (with the final *s* of *sextantes* erased, but visible). In *E* we also find *certum modum spatii habeat porrectum pedes · XV · S ·* (the figure XV easily explainable with the misinterpretation II -> \/ -> V, often witnessed in paleography as well as in epigraphy).[41]

Considering the required length of the perimeter of the wheel, 12.5 ft., the manuscript tradition is unanimous enough and the two exceptions can be explained with minor effort. But as regards the length of the diameter, the study of the manuscripts shows that in none of them we see the word *quaternum* alone: they all have something pointing to a fraction after it. In only one of them (*V*) we encounter the required genitive form *sextantis*. Instead we find *sextante* (*G* & *E*) and *sextantes* (*H* & *W*). This shows that the locus is corrupt and the original concept of the passage is lost. The first editor to focus his attention on the locus

37 Reynolds 1983 (n. 35 above), 441.

38 *http://www.ville-selestat.fr/bh/index.php?page=affiche_ouvrage&type=flash&id=326*.

39 Caillebat 1986 (n. 3 above).

40 *http://diglib.hab.de/mss/69-gud-lat/start.htm?image=00166*.

41 *http://diglib.hab.de/mss/132-gud-lat/start.htm?image=00094*.

was C. Perrault in 1684, who noticed that *et sextantis* must be deleted to get the mathematics in order.[42] Since Perrault the tendency among editors has been to treat the *et sextantis/sextante/sextantes* as an error. The reason for this is clear: the problems with congruence refer to hesitation, and in addition, accepting the figure 4 1/6 in the formula $12.5 = 2\pi r$ would mean that the actual value of π for Vitruvius would have been 3 – not exactly the estimate to produce perfect proportions with! On behalf of mathematics it seems quite obvious that Perrault was right with his correction.

Pondering the problematics, A. Choisy suggested in the beginning of the 20th century that following the word *quaternūm* in the archetype there possibly was a group of dots to which the copyists attributed a numeral signification.[43] Choisy's suggestion has not gained much attention, but it is quite interesting regarding that in Latin the fractions were often marked with dots and other diacritics.

Roman fractions

The Roman way of marking fractions was a bit more complicated than ours. Although their number system was a base ten system, their fractional system was twelve-based. The system was unitary with all the basic fractions having a nominator one and a denominator twelve or one of its multiples. These basic fractions were then combined in order to arrive to a close approximation.[44] The Roman convention for marking the value of the diameter[45] that gives the perimeter of 12.5 feet[46] would be expressed 3+11/12+1/24+1/48. The Romans did in any case not notate this in fractions as we do; instead the subparts of the unit were each marked with their own sign, that is, with an independent logograph (as all numbers are). The Roman way of marking the fraction 3 47/48 would

[42] C. Perrault, *Les dix livres d'architecture de Vitruve*, Paris 1684 (*http://architectura.cesr.univ-tours.fr/Traite/Images/B250566101_11604Index.asp*).

[43] A. Choisy, *Vitruve*, III: *Texte et traduction*, livres VII–X, Paris 1909 (*https://archive.org/stream/dearchitecturali03vitruoft#page/208/mode/2up*).

[44] Maher & Makowski 2001 (n. 31 above), 379.

[45] i.e. 3 47/48. With the Archimedean value of π ($3\ 10/71 < \pi < 3\ 1/7$) we get 12.498–12.501 for the perimeter, if the diameter measures 3 47/48.

[46] Rounded from the four decimal place 12.5009.

be III S = = – ℒ ɔ.[47] Why didn't Vitruvius tell this? He had the terminology. Moreover, the operation would have been a rather simple one the denominator being a multiple of twelve. It could naturally be hypothesized that Vitruvius isn't being at his most accurate with the numbers here, rounding the complex III S = = – ℒ ɔ to IV.[48] This is however quite improbable, because Vitruvius doesn't tend to be too rough with figures, as for example in the chapter 10,10 (i.e. right after the chapter that contains the description of the hodometer) where fractions, or better, each subpart is represented minutely. As Pottage notes, the context is also such that Vitruvius might be expected to be as accurate as possible.[49]

It could also be that the error has been made somewhere between Vitruvius's death and the compilation in the Late Antiquity of the archetype, from which the remaining manuscripts derive. This is also highly likely, considering the vulnerability of logographs for change. Taking for example the multiplication tables of Victorius of Aquitaine from the end of the 5th century, the signs for the figures *deunx*, *semuncia* and *sicilicus* are expressed there as ʄʄʄ, ℒ and ʔ respectively.[50] The convention to mark fractions[51] with dots[52] seems to be prevalent as shown for example by early Roman coins (see fig. 4) and witnessed also in the

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[47] 11/12 = *deunx* (S = = –), 1/24 = *semuncia* (∟, ϵ, ⦚ or ℒ) , 1/48 = *sicilicus* (ɔ). (*OLD* s.v. *deunx*; *semuncia*; *sicilicus*; A. Bouché-Leclerq, *Manuel des institutions romaines*, Paris 1886, (*https://archive.org/stream/manueldesinstitu00bouc#page/ii/mode/2up*), 569; Lugli 1957 (n. 2 above), 189–90.) I have chosen ℒ for the sign of *semuncia* in this paper, because it appears in the majority of examples presented. However, see fig. 4 for an example of the sign ⦚ for *semuncia* in an early Roman coin.

[48] Or *quaternum,* 'four, four each, a set of four of anything' (*OLD* s.v. *quaterni.*). Caillebat and Fleury see the figures used by Vitruvius as deliberate choices of simplification (Caillebat 1986 [n. 3 above], 190; Fleury 1993 [n. 8 above], 208.). It is also possible that the approximation is derived directly from the Greek source used by Vitruvius – a view which gains support from the studies of J. Coulton on the frequency of approximations in the Greek architectural context of the 6th – 2nd centuries BC (Coulton 1975 [n. 32 above], 79–83; 98) and for example still in Heron (the 1st century AD) the use of approximations in calculations results in several errors (Coulton 1975, 82; Her. *de mens.* 28,1).

[49] Pottage 1968 (n. 6 above), 192.

[50] The Unicode characters chosen are the ones that resemble the most the characters in the manuscript *Oxford, St. John's College MS 17* (*http://digital.library.mcgill.ca/ms-17/folio.php?p=57v*).

[51] Or better: subparts.

[52] To be more precise, dots were used to mark the subparts 1/12 – 5/12 and 7/12 – 11/12.

manuscripts containing the treatise of Vitruvius.[53] In the *Harley 2767* the manner how the fractions are expressed varies even within the same chapter, as in Vitr. 10,10,4, where the fraction 9/12 is expressed with dots in one occasion and with S :-[54] in another. The latter is also an example of how combinations of lines and dots are used to denote a fraction. The lines are prevalent in the late 2nd century *Assis distributio* of L. Volusius Maecianus, where the signs for *deunx, semuncia* and *sicilicus* relevant for this paper are S = - =, Ɛ and ͻ respectively.[55] In an inscription in the Roman Colosseum, datable probably to the year 82, the signs of *deunx*, *semuncia* and *sicilicus* appear also as S = - =, Ɛ and ͻ, although the sign of *sicilicus* appears to be more elongated, resembling the letter rā' (ر) in Arabic.[56] It is not far-fetched to assume that such diversity in notation may easily have led to confusion and corruption of the original meaning.[57] If the notation originally or at some point was III S::. Ɛ ͻ, how did it then change to *quaternum* or *quaternum et sextantis* or *quaternum et sextantes*?[58]

Pedum quaternum

Even though all the first generation manuscripts have, as seen, some version of *pedum quaternum et sextantes*, the bare *pedum quaternum* is the one accepted by the modern scientific editions. Considering the mathematics involved, it is also the most plausible one of the three available options, because it is only 1/48

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53 Vitruvius also tends to mark fractions occasionally with letters as for example FZ (= 2/3) in 10,10,4.

54 The *S* stands for *semis*.

55 Maecian. *assis distributio* 1,14; 27, 29.

56 *CIL* VI 2059; 32363; J. & A. Gordon, *Contributions to the palaeography of Latin inscriptions*, Los Angeles 1957, 171.

57 One question is how the notation of fractions changed during centuries and whether there was a uniform standard at all. The scarce evidence presented in this paper seems to point out that a change of notation had occurred when coming to the Late Antiquity. On the other hand the examples from Vitruvius and Volusius Maecianus as well as in early Roman coins (see fig. 4) and in the inscription of Colosseum point to a uniform system in use earlier.

58 I haven't taken the option *et sextante* under examination: it is just erroneous with no story behind it. In addition, the manuscripts containing it show hesitation towards it (see chapter "*Pedum quaternum*" *and* "*pedes XII s" in the manuscripts*).

from the desired figure.[59] Mathematics was also the reason that made C. Perrault, the 17th-century editor of Vitruvius's work and the architect of the Louvre,[60] to make his correction. Krohn treats the words *et sextantes* as an interpolation from 10,9,5, where Vitruvius is describing a hodometer suitable for vessels.[61] This might well be the explanatory factor for the misconception concerning the extra "sixth" seen in the manuscripts. But it still leaves us with a Vitruvian value of π of 3 1/8, which, as seen, when applied to a hodometer, produces a mile with 26.55 ft. in excess. How could, then, the required figure for the diameter (i.e. the one that produces a perimeter of exactly 12 ½ ft.), *pedum trium deuncis semunciae sicilici* (III S::. Ɛ ɔ), have turned to *pedum quaternum*?

Let's suppose that instead of numeral, the notation originally or at some point before the making of the 7th century archetype was numeric. As for the figures II and V (see chapter '*Pedum quaternum'* and '*pedes XII s' in the manuscripts*), also figures III and IV get easily mixed with each other: there's only one extra 'I' involved. So, the figure III transforms to figure IV in the same way: III -> /\/-> IV. How to deal then with the remaining fractions S::. Ɛ ɔ? How could they have disappeared in order to leave us with the bare *quaternum*/IV? One option is that the figure was expressed with the subtractive principle which is witnessed in some occasions to have been used also with figures involving fractions. For example the figures 89 ½ and 79 ½ have been represented in some inscriptions with symbols SXC and SXXC respectively.[62] Applying the subtractive method to our figure III S::. Ɛ ɔ (3 47/48) gives thus ɔIV (~"1/48 to 4"). This is however to be left at the level of speculation since the evidence on the use of the subtractive method with Roman fractions is scarce. In addition this doesn't explain how the subtracted ɔ got lost, but the loss of such an infrequent and easily misinterpreted sign is comprehensible. The symbol of *sicilicus* might easily have been interpreted for example as a comma, like the one preceding and following the figures in Vitr. 10,9,1 in the manuscripts (e.g. *pedes · XII · S ·* in the

59 Compare also with the frequency of approximations in the ancient Greek architecture (see Coulton 1975 [n. 32 above]).

60 L. Caillebat, "Éléments d'interprétation et problèmes de réception du Corpus vitruvien sur la mécanique", *Humanitas* 45 (1993) 137–54, 147 (*https://digitalis-dsp.sib.uc.pt/jspui/bitstream/10316.2/7264/1/Art_7_-_Problemes_de_reception_du_corpus_vitruvien.pdf*).

61 Krohn 1912 (n. 3 above), 242–3.

62 D. E. Smith, *History of mathematics* II: *special topics of elementary mathematics*, Boston 1925, 60 (*https://archive.org/details/historyofmathema031897mbp*). Smith doesn't anyhow specify these inscriptions.

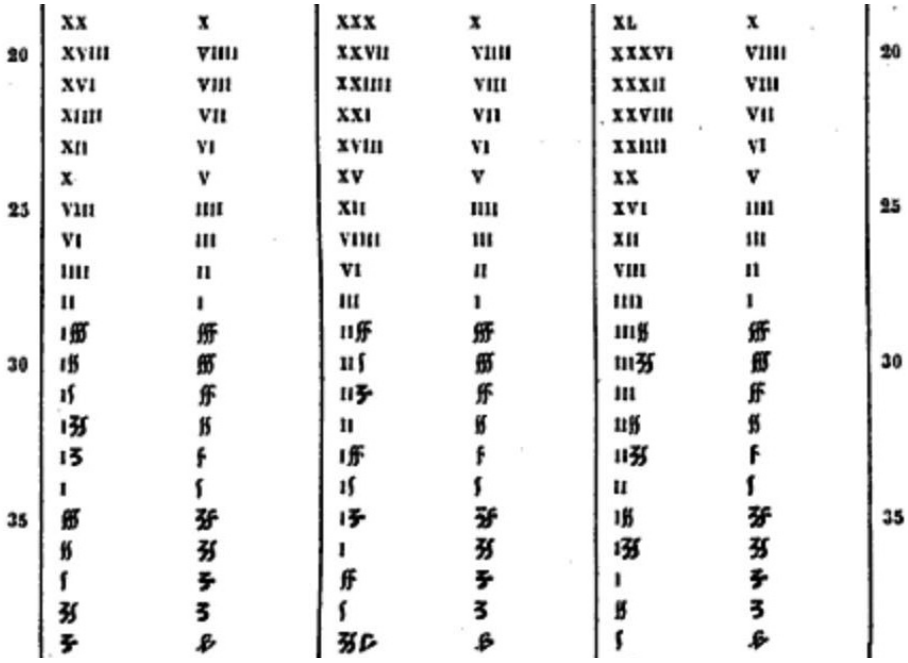

Figure 3. Excerpt from the multiplication tables of Victorius of Aquitaine showing part of the two, three and four times tables. Figure from G. Friedlein, "Victorii calculus ex Codice Vaticano editus", Bullettino della bibliografia e della storia delle scienze matematiche e fisiche 4 (1871) 443–63, 447.

Harley 2767). The variation and changes in notation, as testified for example by the multiplication tables of Victorius of Aquitaine and the Colosseum inscription are naturally also cut out for the loss of the original meaning. This goes also with the whole sequence of fractions S::. £ ɔ, which is reflected in the grammatical confusion that defines the locus in the manuscripts. It is easy to understand that a sequence of symbols, which possibly had no meaning for the copyists got lost during the centuries between Vitruvius's death and the compilation of the Anglo-Saxon archetype in the 7th century. However, the strength behind the option *pedum quaternum* are the manuscripts. The form is grammatical and it appears in all the manuscripts and the grammatically incorrect *et sextantes* that follows it can easily be explained as an interpolation. But what is the story behind 4 1/6, the other grammatically correct form?

Pedum quaternum et sextantis

The line of thought in Vitr. 10,9,1 suggests that if Vitruvius on one hand was not quite aware of the exact value of π, he on the other hand knew that the perimeter of the wheel of the measuring device had to measure exactly 12.5 ft. (hence the choice of words *ut … certum modum spatii* in 10,9,1) in order to give a mile of 5000 ft. The confusion on the length of the diameter and the certainty on the result of the multiplication (i.e. 12.5 ft.) might point out to the use of a multiplication table, which again familiarizes us with the multiplication tables of Victorius of Aquitaine.

Looking at the column of the three times table and supposing that Vitruvius knew that the value of π was a bit over three, the automatic parameters to get the exact result 12.5 are first IIII in order to get XII and then Ʒ (=1/6) in order to get S (=1/2). The multiplication table seems thus to give an automatic answer for the dilemma and turns the blame around to Vitruvius. Following this line of thought, the Vitruvian value of π truly seems to be 3 (*XII S* divided by *IIII Ʒ*). This is also the view supported by the grammar, because the forms preceding the fraction, i.e. *pedum quaternum*, indicate that a genitive is wanted. It is anyhow missing from all the first generation manuscripts except for the rather late (15th-century) *V*. Even though a methodological explanation of how Vitruvius might have arrived to the figure 4 1/6 is offered by the use of multiplication tables, the fact that it appears only in one manuscript might point out to that it is a correction made by a copyist, because the prevalent *et sextantes* is so evidently incorrect. Considering the prominence of Vitruvius as an architect and an engineer apparent in the pages of his treatise I find it also quite unlikely that the value of π for him would have been three. Vitruvius was also clearly aware of the achievements of Archimedes and in addition,[63] if the origins of the hodometer are Archimedean, as Sleeswyk suggests, it is odd that the value of π used in his treatise would originally have been something else than the estimate presented in Archim. *circ.* 3. Notwithstanding, accepting one of the options *pedum quaternum* or *pedum quaternum et sextantis* leaves a chance for this. What might then be the reason behind the prevalent and grammatically incorrect form *pedum quaternum et sextantes*?

Pedum quaternum* and lots of *sextantes

As stated before, the option *pedum quaternum et sextantes* is clearly the least plausible of the three because of the erroneous congruence. This applies also to the mathematical aspect on the question: to say "the diameter is four ft. and sixths" is an utterly imprecise expression. It is also very unlikely that Vitruvius would ever have written *sextantes,* because all the Roman fractions had their spe-

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63 See for example Vitr. 8,5,3.

cific names: *sextans* was one sixth, but two sixths was called quite logically *triens* and three sixths, then, formed a *semis* etc.[64] There is thus hardly any chance that *pedum quaternum et sextantes* was the original form written by Vitruvius. Perrault's deletion of it, based on mathematical necessity, and Krohn's interpretation of it as an interpolation from Vitr. 10,9,5 gets thus support from Latin mathematical terminology.

At this point the observation made by A. Choisy is a step forward. His suggestion was that in the original manuscript following the word *quaternum* there probably was a group of dots to which the copyists attributed a numeric value.[65] I believe Choisy refers to the fact that the subparts of the unit were often symbolized with dots: a *sextans* with two dots, a *triens* with four dots etc. This convention is seen for example in the manuscripts studied for this paper as well as in early Roman coins (see fig. 4).[66] Even other subparts correspondent to 1/12–11/12 are occasionally marked with a group of dots.[67]

Figure 4. Triens (BMC Italy p. 48, no. 8), sextans (BMC Italy p. 49, no. 14) and semuncia (BMC Italy p. 49, no. 21) from 280–276 BC (Crawford 1974 [n. 66 below], 134.). The figures are from the Catalogue of Roman Republican Coins in the British Museum (https://www.britishmuseum.org/research/publications/online_research_catalogues/rrc/roman_republican_coins.aspx.), © Trustees of the British Museum.

64 Smith 1925 (n. 62 above), 209; Maecian. *assis distributio* 1,2; 4; 21.

65 A. Choisy, *Vitruve,* III: *Texte et traduction*, livres VII–X, Paris 1909, (*https://archive.org/stream/dearchitecturali03vitruoft#page/208/mode/2up*).

66 See for example M. Crawford, *Roman Republican coinage* I, Cambridge 1974, 6; W. Metcalf (ed.), *The Oxford Handbook of Greek and Roman coinage*, New York 2012, 302. On the other hand also the letter Z is used for the sign of *sextans* (Lugli 1957 [n. 2 above], 190).

67 See for example the text correspondent to Vitr. 10,10 in the manuscripts *H, E* and *G.*

The option *pedum quaternum et sextantes* gains thus its validity from Choisy's observation. For grammatical, mathematical and terminological reasons its representation must originally have been numeric, the exact form of which anyhow remains obscure. It is to be said however, that a possible transformation from the required III S ::. Ꮭ ɔ (or even III :::::. Ꮭ ɔ) to IV :::: (Ꮭ ɔ) to *quaternum et sextantes* becomes more comprehensible following this line of thought. It is also again easy to see how prone the original locus was to corruption. In fact, for all the reasons presented in this paper, the form *pedum quaternum et sextantes* that at first glance seemed the least plausible one hides behind its ungrammaticality a logical explanation of the destiny of the passage. I find it quite likely that the original notation used by Vitruvius was numeric, but my educated guess is that the final word hasn't been said yet.

Materialization of the immaterial?

If the matter concerning the passage containing the Vitruvian error is so far to be left undecided, is there then something concrete to rely on at this point? The answer to the question is: limestone, and more precisely the milestones whose locations on the ancient roadside were presumably measured by the Roman surveyors with a hodometer. If a hodometer based on erroneous mathematics ever was built and used in Roman road building, the practical consequence would have been a road where milestones are not where they are supposed to be but depending on the distance of a milestone from the starting point of the measurement and on the scale of the mathematical error, misplaced by a distance from few meters up to kilometers. On the other hand, the actual hodometers used by Roman engineers were probably built with the knowledge of the effect the error would have had on measuring and tested before the actual use: the practice of trial and error would presumably have helped in building a correctly functioning machine. It is hard to imagine that a society that among other its architectural achievements built aqueducts relying on millimeter-sharp inclinations would have mismeasured its roads.[68]

Surprisingly, there are several Roman roads, on which the standard measure for a mile, ~1478.5m, does not hold good. The reason for this might naturally be a fluctuating standard or an incomplete present archaeological knowledge of

[68] Naturally a discrepancy in the length of a road would not have had such drastic consequences as one in the length of a planned aqueduct line.

the road lines in question, but also that an ancient measuring device that produced systematic error was used in their building. Roads on which this kind of anomaly is said to manifest itself are to my knowledge Via Appia, Via Laurentina, Via Salaria and Via Tiburtina.[69] On the other hand, on most of them the miles seem to be too short, contrary to the error produced by the parameters expressed by Vitruvius. According to A.-J. Letronne the mile measure on Via Appia for example was only 1471.23m, verifiable by the distance between the 42^{nd} and the 46^{th} milestones.[70]

If thus the Vitruvian value of π really was 3.125[71] and the hodometers used in Roman road building were actually built using the Vitruvian parameters, the consequences for measuring Roman roads would have been significant. As far as I know these possible practical consequences of the Vitruvian error for Roman road building have not been studied before. Using 3.125 for the value of π, the error would thus have led to every mile measured by the hodometer being c. 26.55 Roman feet too long. If we take the case of the ancient Via Salaria as an example, the measuring error of 0.066 ft. (1.95 cm) produced in this way per one rotation of the wheel of the hodometer would multiply to ~3670 ft.[72] on the whole road line.[73]

The best method to study this is to reconstruct the routes of the ancient road lines in question using e.g. the gates of the Servian wall and *in situ* -found milestones or other such fixed sites as points of reference, measure the reconstructed road lines and, if the result appears to differ from the standard mile measure, study the possible cause for this. This kind of a study has recently been done on the ancient road line of Via Salaria ending up in the conclusion that the reason for the view according to which the miles on the road are shorter than the standard, was based on an incomplete archaeological knowledge of the ex-

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69 See M. Capanna, "Il culto di Anna Perenna al I miglio", in A. Carandini – M.T. D'Alessio – H. Di Giuseppe (eds.), *La Fattoria e la villa dell'Auditorium nel quartiere Flaminio di Roma*, Roma 2006, 65–70; A.-J. Letronne, *Recherches critiques, historiques et géographiques sur les fragments d'Héron d'Alexandrie ou du système métrique égyptien* (*http://books.google.fi/books/about/Recherches_critiques_historiques_et_g%C3%A9o.html?id=xhjPAAAAMAAJ&redir_esc=y*), Paris 1851, 10.

70 A.-J. Letronne 1851 (n. 69 above), 10.

71 Or Pottage's suggestion, 3, which makes the practical consequences naturally even worse.

72 = c. 1085 meters.

73 The road line of ancient Via Salaria was c. 139 miles long. See, for example, R. Talbot (ed.), *Barrington Atlas of the Greek and Roman World*, Princeton 2000.

act route of the road line.[74] Such a study would be interesting to execute on the other roads mentioned above, even though the explanation for the anomalies in the mile measures witnessed on them is probably the same. It is anyhow intriguing to hypothesize that a hodometer built with Vitruvian parameters might be the reason behind some of the disturbances. Be that as it may, on a less specific scale this reveals that due to the functioning principles of the hodometer, the actual use of the device for measuring longer distances caused significant problems to the accuracy of the measuring.

Conclusions

The passage in Vitr. 10,9,1 containing the specifications of the parameters with which a hodometer was to be built is clearly corrupt. The original form of the text cannot be ascertained, but the examination of the possible options seems to indicate that originally the notation in the locus was numeric and the fault carried to our days by the manuscripts is due to the mathematical difficulty of the passage and the variation in the notation of fractions. Vitruvius did possess the correct terminology as well as the knowledge to provide his description with the correct parameters and can also be thought to have used them when writing his treatise. Even so, the anomalies witnessed in the mile measures on certain Roman roads leave the possibility that a measuring device that produced systematic error was used in building them. The next step in studying the Vitruvian hodometer could thus be to examine whether the mathematical error in the text was, so to speak, a small drop for one pebble that cumulated to a giant leap with every mile the Vitruvian hodometer traveled. In addition, to understand better the difficulties involved in Vitruvius's description of the hodometer, other mentions of the device in ancient literature would have to be studied, first and foremost the hodometer Heron of Alexandria presents in his treatise *Dioptra*.[75]

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[74] P. Hyppönen, *Salaria via usque ad lapidem XVIII: a reconstruction of the ancient road line between Porta Collina and the 18th milestone of the road*, Oulu 2014 (*http://jultika.oulu.fi/Record/nbnfioulu-201404241310*).

[75] Her. *dioptr.* 34.