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# Theaitetos and Theodoros

HOLGER THESLEFF

The Athenian Theaitetos seems to occupy a secure position in the history of Greek mathematics ever since Eva Sachs, a pupil of Wilamowitz, established it in the beginning of this century.<sup>1</sup> I shall argue here that scepticism regarding his achievements, and a reconsideration of his function in Plato's dialogue, are warranted. The problem of Theaitetos, as I see it, offers very typical examples of the crystallizing of old hypotheses into quasi-facts later used for building new hypotheses, a process all too common in classical scholarship.<sup>2</sup>

Theaitetos is now generally thought to have lived ca. 414 - 369 B.C. The evidence is precarious apart from what can be deduced from Plato's *Theaitetos*. In Eudemos'

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<sup>1</sup> Eva Sachs, *De Theaiteto Atheniensi mathematico*, Diss. Berlin 1914; independently, with similar conclusions, H. Vogt, *Bibliotheca Math.* III:10 (1909/10) 97-155; 14 (1913/14) 9-29; endorsed by Th. Heath, R.S. Brumbaugh, B.L. van der Waerden, and practically everybody who has written on Plato's *Theaitetos* since then. See also the comprehensive RE articles on 'Theaitetos' and 'Theodoros' by K. von Fritz, *V A* (1934) 1351 ff., 1811 ff., S. Heller's conspectus in *Sudhoffs Archiv* 51 (1967) 55 ff., and the recent discussion of Academic mathematics by K. Gaiser in the new *Ueberweg* (1983) and F. Lasserre, *De Léodamas de Thasos à Philippe d'Oponte, témoignages et fragments*, *La Scuola di Platone* 2, Napoli 1987.

<sup>2</sup> I have ventilated this set of problems before, notably in my *Studies in Platonic Chronology* (*Comm.Hum.Litt.* 70), Helsinki 1982, 152-57 (cf. *Phronesis* 34 [1989] 18 n. 67), without being able to shake the consensus about Theaitetos.

list of geometricians he is mentioned together with Leodamas of Thasos, a very shadowy figure, and Archytas of Tarentum, as belonging to the same generation as Plato.<sup>3</sup> According to a confused piece of information in Hesychios and Suda (see below), he had been teaching in Herakleia, and Herakleia on the Pontos was the home city of Plato's pupil Herakleides (born not earlier than ca. 400 B.C.). And various sources attribute to him discoveries concerning irrational numbers and regular solids, which are reflected in Euclid, Books X and XIII, and consequently he is sometimes thought to have written essential parts of these Euclidean texts.

Though the dialogue situation of *Theaetetus* is very probably fictitious, we have no reason whatever to doubt that the presentation of the young Theaitetos approximates to historical truth. A nonsensical distortion of facts known to the readers (as, say, in *Menexenus*) would have been pointless here; but the reappearance of Theaitetos in the *Sophist* and *Statesman* has, of course, no pretensions to historicity. We may take it for granted that Theaitetos was a youngster of less than 17 years in 399 B.C.<sup>4</sup>

The fixing of the death of Theaitetos in 369 B.C. is mainly based upon the following four considerations:

- (a) The *Theaetetus*, at least in its present form and including its present prologue, is a fairly late dialogue.
- (b) The writing of the prologue was occasioned by Theaitetos' death.
- (c) The battle at Corinth referred to must be that of 369 B.C., not 394 B.C. as Zeller and others have suggested.
- (d) Theaitetos' achievements in mathematics, as reflected in Euclid and elsewhere, must have taken a long time to accomplish.

If, however, (d) does not apply, as I shall argue below, it is more natural to interpret (a) – (c) differently.

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<sup>3</sup> Proklos, In Eucl. *Elem.* I, Prol., II p. 64 ff. Friedl., Eudemos fr. 133 W; Lasserre 1987 argues that the list derives from Philip of Opus, not Eudemos.

<sup>4</sup> This is the dramatic date of *Theaetetus* (142c, 210d). The implications of μαιράκιον, beardlessness (168e), etc. are discussed by Sachs 1914:25 f., Lasserre 1987:462. If Theaitetos in reality was very much younger than this it is, apart from other difficulties, reasonable to ask why Plato takes so much trouble to explain the circumstances of Socrates meeting Theaitetos. Unlike *Parmenides* and *Timaeus*, the setting of the *Theaetetus* was within checking reach of contemporary readers; for 'Aristotle', see below, n. 7.

(a) It is true that *Theaetetus* as we have it cannot be an early dialogue. Clearly it links up with the 'critical' dialogues *Parmenides*, the *Sophist*, and the *Statesman*.<sup>5</sup> But very probably it has been revised and re-written from an earlier draft of the *Charmides* type.<sup>6</sup> And even if this were not the case, and we have the text as it was originally composed, Plato is much more likely to have introduced, as Socrates' partner, a long-since dead friend whom he saw from an idealizing perspective (indeed very much like Charmides), than a scholar from his own Academic environment whom many readers would know well. The only obvious counter-argument would be 'Aristotle' in *Parmenides*, but he appears in a 5th century disguise and is not really individualized at all.<sup>7</sup>

(b) A close reading of the prologue and the subsequent presentation of Theaitetos (to 148b) does not suggest to me that the writing of the dialogue was occasioned by the death of Theaitetos. Plato may have had other reasons for introducing him (below, p. 156). But if we assume as a possibility that one reason was his recent death, a date as late as the 360s would seem rather odd after all. A fact not often observed, which makes me suspicious from the start, is the remarkable vitality of Socrates' old friends, Eukleides and Terpsion (note 142a, 143ab): in 369 Eukleides may have been well over 80.<sup>8</sup> And then there is nothing to indicate that Plato thought of Theaitetos as ever having reached the age of 45 or more. He reached manhood, to be sure (ἄνδρα 142b, ἡλικίαν [military age!] 142d), but ἐλλόγιμον in the

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<sup>5</sup> This is a consensus of post-Zellerian scholarship, which I am fully prepared to accept; cf. Thesleff 1982.

<sup>6</sup> I argued this in 1982:152 ff.; cf. 1989:18. H. Tarrant (in a paper known to me from a draft) has added more arguments.

<sup>7</sup> A play with masks is part of the game in Platonic dialogues; cf. the following note and the references in Thesleff 1982. For 'Socrates J:r', see note 23. In fact the *Theaetetus* reflects the beginning of the curious 'split' of Socrates in some later dialogues (including the *Hippias Maior*).

<sup>8</sup> Obviously Plato avoids introducing living persons into his dialogues (Thesleff 1982:32, 154 ff.). Eukleides perhaps was still active about 370 B.C. (ibid. 155), though one may wonder about the long walks implied in the opening scene of *Theaetetus*. He is said to have made Socrates' acquaintance before the Peloponnesian War (Gell. 7,10); at any rate he is likely to have been much older than Plato. In *Parmenides* 127b the 65-year-old Parmenides is described as εὖ μάλα ἤδη...πρεσβύτης, and in *Epistle VII* 338c Plato considers himself a γέρων at that age. The Athenian of the *Laws* stands (and walks) closer to Speusippos than to Plato.

*vaticinium* of Socrates (142d) playfully alludes to his interest in ἄλογα, I believe (cf. again p. 156), and does not as such imply an advanced age.

(c) Eva Sachs<sup>9</sup> made an effort to prove that the battle at Corinth (142a, τὴν μάχην 142b) where Theaitetos was mortally wounded, was a notorious one, and that it occurred in 369 when the Athenians were allied with the Spartans against Thebes. She made it plain, no doubt, that one of the battles in the Isthmian war of 369 was a more important event than earlier critics had thought, and that 45-year-old intellectuals could have taken part in this campaign. She notes that Xenophon (*Hell.* VI 5.49) describes the Athenians' enthusiasm and decision to assist Sparta πανδημεί. And there are additional circumstances which she does not mention but which may suggest that members of Plato's circle were engaged in these operations: Iphikrates assembled his troops in Akademeia (Xen. *ibid.*, somewhat differently Diod.Sic. XV 68); Chabrias took command (Diod. *ibid.*);<sup>10</sup> and Dionysios of Syracuse supplied auxiliary forces (Xen. VII 1.20,28; Diod. XV 70).

Yet thinking of a battle in the Corinthian war around 390 B.C. seems more natural after all: Sachs sweeps this possibility aside on quite insufficient grounds.<sup>11</sup> We happen to know that there was a detachment of Athenian hoplites under Kallias cooperating with Iphikrates' mercenary peltasts in the famous battle when a Spartan regiment was completely defeated (Xen. *Hell.* IV 5.11-18, cf. Demosth. IV 24; Diod.Sic. XIV 86,91 ff. seems to confuse facts). Whatever Xenophon's πανδημεί may imply for the year 369, Theaitetos is somewhat more likely to be found among Kallias' hoplites. The chronology of the events around 390 has been subject to some dispute; today the Spartan disaster is dated not earlier than 392, and Iphikrates' subsequent operations on the Isthmus (Xen. *Hell.* IV 5.19) are thought to have extended to at least 390.<sup>12</sup> And Plato's τὴν μάχην of course refers to the battle

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<sup>9</sup> Sachs 1914:22 ff.

<sup>10</sup> Chabrias seems to have been a personal acquaintance of Plato's, according to the anecdotes in DL III 20,23 f. Plato's alleged pro-Spartan sympathies should not be overrated: he was taken prisoner by the Spartans in 387 (see now Suppl. Plat. I [below, n. 35] 165 ff.).

<sup>11</sup> Her chief target was the view of Schultess and Zeller that the dialogue was an early work, and Zeller and his contemporaries dated 'proelium illud nobilissimum' in 394 B.C.

<sup>12</sup> The dating of the Spartan defeat in 392 by W. Judeich, *Philologus* 81 (1926) 147 A. 6, may still be too early; cf. G.T. Griffith, *Historia* 1 (1950) 252; S. Accame, *Ricerche intorno alla guerra corinzia* (Collana di studi greci 20), Napoli 1951, 108 ff.

where Theaitetos received his wounds, not to the fact that this battle was particularly famous.

(d) Although many mathematicians have reached their peak of brilliance at an early age – can we really trust a young geometrician of 25 years or less with all the discoveries and activities attributed to Theaitetos by the historians of mathematics? The consensus of modern scholarship would point to a simple "No".

I would insist, however, that this consensus is mistaken. "No other branch of history offers such temptations to conjectural reconstruction as does the history of mathematics."<sup>13</sup> Students of Theaitetos have too readily yielded to such temptations.

Let us consider, first, what Plato tells us in the mathematical passage, *Theaitetos* 147c-148c.

Theodoros had been drawing (ἔγραφε) figures, showing (ἀποφαίνων) that lines whose squares have the area of three or five square foot, are incommensurable with the side of a one foot square; and he had proceeded from case to case until he reached the side of a seventeen square foot square where he "somehow met with complications" (ἐν δὲ ταύτῃ πως ἐνέσχετο).<sup>14</sup> In modern times there has been considerable discussion about what Theodoros was in fact doing, how he 'proved' the irrationality of  $\sqrt{3}$ ...  $\sqrt{17}$  (except for the rational numbers,  $\sqrt{4}$ ,  $\sqrt{9}$ ,  $\sqrt{16}$ ), and why he stopped at  $\sqrt{17}$ .<sup>15</sup> I cannot see why he should have 'proved' anything at all. The easiest way to explain his procedure was suggested by H.J. Anderhub in a

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<sup>13</sup> W. Burkert, *Lore and science in ancient Pythagoreanism*, transl. by E.L. Minar, Cambridge Mass. 1972, 404.

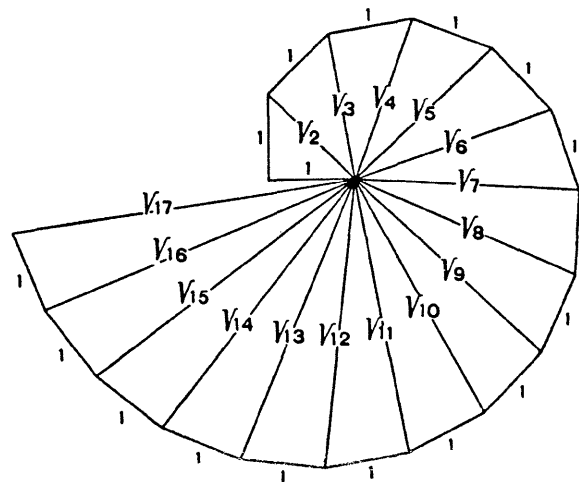
<sup>14</sup> Cf. ἐνέχεσθαι ἀπορίησι Hdt I 190. The participle προαιρούμενος is curious: what did he choose? Should one read προαγόμενος? Cf. below, n. 29.

<sup>15</sup> It is commonly and wrongly assumed that ἔγραφε means 'proved' (note also the imperfect tense). See the references in n. 1 and notably Heath's *History* I 202 ff. and van der Waerden's *Science awakening* (I have used the second German edition, *Erwachende Wissenschaft*, Basel 1966, 235 ff.); add S. Heller's comprehensive discussion in *Centaurus* 5 (1956) 1 ff.; further references in Anderhub (next note), Burkert 1972:463 n. 81 and Malcolm S. Brown, *JHPhilos* 7 (1969) 359 ff.



curious book called 'Joco-Seria' which, as a matter of course, has not been taken seriously by specialists.<sup>16</sup>

Anderhub interpreted the passage approximately as follows: Theodoros must have been well acquainted with the 'theorem of Pythagoras' and with the irrationality of  $\sqrt{2}$  as seen in the relation of the side to the diagonal of a square.<sup>17</sup> Making the diagonal of a one foot square one side of a right-angled triangle, and preserving one foot as the length of the other side, Theodoros was able to 'show' that the hypotenuse of this triangle must have the length of the side of a three square foot square (because  $2 + 1 = 3$ ), and that the new hypotenuse could not be measured in terms of one foot. Remember: the Greeks did not normally operate with fractions. Then he drew the next right-angled triangle, using the former hypotenuse as one side and again a one foot line as the other side. Obviously this  $\sqrt{4}$  foot hypotenuse measured 2 feet. And then he proceeded to draw a spiral-like figure where only  $\sqrt{9}$  and  $\sqrt{16}$  could be seen to be commensurable with one foot.



<sup>16</sup> J.H. Anderhub, *Joco-Seria aus den Papieren eines reisenden Kaufmanns*, Wiesbaden 1941, 161-224; preliminary notes in *Wochenschr.f.klass.Philol.* 1918 (49/50) 598 f. Anderhub rightly insists that  $\gamma\rho\acute{\alpha}\phi\epsilon\iota\nu$  cannot mean 'to prove'. The spiral was also drawn by S. Moraïtes in his *Modern Greek Plato* edition (1913) but he did not see the consequences (Anderhub 222). Heller 1956 adopts a variant of Anderhub's spiral as an illustration, but presumes that Theodoros had given a one hour's lesson on the subject of irrationality.

<sup>17</sup> For the evidence, see now Burkert 1972:428 ff., 462 f.

He stopped at  $\sqrt{17}$  because the  $\sqrt{18}$  triangle would have intruded into his first triangle.<sup>18</sup> I am sure Anderhub was right. There is an additional indication of this, never observed in this connection as far as I know. The only evidence we have of Theodoros' mathematical studies which is seemingly independent of this Platonic passage, is a somewhat cryptic statement on spirals in Proklos. Α ἔλιξ, Proklos says, is a mixed line which does not consist of parts, so Theodoros the mathematician wrongly took it to be a 'κρᾶσις based on lines'.<sup>19</sup> Modern scholars do not seem to have noticed the connection with *Theaitetos*. Without knowing it, Anderhub drew the relevant figure illustrating what Proklos meant. Proklos probably had access to an old tradition about the historical Theodoros having studied triangle-based spirals and Plato having referred to such figures orally and in the dialogue. The Anonymous Commentator on *Theaitetos*, as usual, is not so well informed.<sup>20</sup>

I also find it important to note that throughout the dialogue Theodoros is depicted as an adherent of φαivόμενα, and indeed of Protagoras, who is known to have opposed theoretical geometry.<sup>21</sup> We should definitely not expect any 'proofs' from Theodoros. And shall I add that I am not a believer in the legend of Plato receiving instruction from him in Cyrene?<sup>22</sup>

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<sup>18</sup> The sum of the inner angles of the  $\sqrt{2}$  -  $\sqrt{18}$  triangles would amount to 364.783 degrees, the  $\sqrt{17}$  one reaching 351.150 (and certainly somewhat further, if drawn in sand). I am indebted to Henrik Segercrantz for these calculations.

<sup>19</sup> Proklos, In Eucl. *Elem.* I, p. 117.25-118.8 Friedl., discussing the nature of curves. I understand κρᾶσις ἐπὶ τῶν γραμμῶν to mean a mixture made 'on the basis of' or 'out of' straight lines.

<sup>20</sup> Anonymer Kommentar zu Platons *Theaitetos* (Pap. 9782), unter Mitwirkung von J.L. Heiberg bearb. von H. Diels und W. Schubart, Berliner Klassikertexte II, Berlin 1905, Col. 25 ff., p. 18 ff. Various unreliable guesses are offered at Col. 34 ff., discussed and rejected by Anderhub 1941:183 f. Obviously Platonists in the 2nd c. A.D.(?) were bewildered by the passage.

<sup>21</sup> Theodoros is old and intellectually lazy, though interested in 'appearances' (e.g. 143e, 144bc, 147d, 162ab, 168e, 177c, 180b; 162e is ironical in view of 165a), and he is called upon to defend the tenets of his 'friend', Protagoras (161b, 162a, 171c, 179a). – For Protagoras, see DK 80 B 7, Arist. *Met.* B 998a, cf. Plat. *Prot.* 318de. The outburst of 'Protagoras' in *Theaitetos* 162e about the need of proofs is certainly ironical from Plato's perspective.

<sup>22</sup> DL III 6, cf. II 103 and Thesleff 1982:28.

Plato's *Theaitetos*, however, is a more theoretically and philosophically-minded person.

Since an infinite number of roots appeared (ἐφαίνοντο) to exist – the spiral could be made to grow *ad infinitum* –, he and the Younger Socrates (who probably stands for Plato)<sup>23</sup> looked for a common term for all irrational roots versus rational ones.<sup>24</sup> They divided all numbers into two classes,<sup>25</sup> square numbers (τετράγωνον, ἰσόπλευρον) and 'oblong' numbers (προμήκη, ἕτερομήκη), and returning to geometry, 'defined' (ὠρισάμεθα) the lines corresponding to square numbers as μήκος, and the lines corresponding to oblong numbers as δυνάμεις, because they are not arithmetically, but by their geometrical 'potency', commensurable with the lines of the former class. "And similarly with the solids", *Theaitetos* adds.

So Plato says, simply, that *Theaitetos* and his friend defined geometrical commensurability by means of a new generalizing classification of number, i.e. (as we would say) by introducing 'roots'. The old classification into odd/even was substituted by a more sophisticated one. Presumably, the Pythagoreans had operated with the notion of square and oblong 'gnomon' numbers long before the 390s,<sup>26</sup> but we have no reason to doubt that *Theaitetos* had a share in generalizing the concept of ἕτερομήκης ἀριθμός. Plato at any rate found the idea suggestive of his own metaphysical category of θάτερον versus ταυτόν. I find it practically certain that the play with ἴσον and ἕτερον in *Theaitetos* alludes to the two Platonic principles of the Same and the Different (later linked up with Ἐν and Δύαξ), which constitute measure and knowability on various levels.<sup>27</sup>

<sup>23</sup> See the article by Tuija Jatakari in this journal, p. 29 ff.

<sup>24</sup> This is clearly the implication of 147d8-e1 where ταύτας refers to τρίποδος, πεντέποδος and ἑπτακαιδεκάποδος.

<sup>25</sup> Note the application of the method of διαίρεσις, cf. συλλαβεῖν d9.

<sup>26</sup> See in general Burkert 1972:427 ff., and for the terminology, Lasserre 1987:466 ff. with references.

<sup>27</sup> For ἴσον / ἕτερον cf. 143e, 144d ff., 148ab, 155a, 158c ff., 181c ff., 185b ff., 189a ff., 203e, 208d; μέγα / μικρόν 152d, 155a, 172bc; ἕν / δύο, πολλά, ἄπειρον 146d ff., 154cd, 156a, 185b ff.; κίνησις, δύναμις, ποιεῖν 147d ff., 152e, 156a ff., 181b ff. (ποιότης! 182ab), 185c ff., 197c; *Theaitetos* the γεννικός (γενναῖος) somehow representing 'generation' of numbers and λόγοι 144d, 146d, 149a ff., 156cd, 158e ff. Plato's world is a κρᾶσις of unequal opposites (cf. 152d and Theodoros' spiral); his 'two-level model' (Thesleff 1989:24 f.) is presented in the digression 172c-177c. – The *locus classicus* for the metaphysics of the Same versus the Different is *Timaeus* 35a ff., cf. the μέγιστα γένη *Sophist* 254cd. I shall not enter into the question of oral teaching, but note the abundant references to γράφειν 143a-c, as if

Theodoros and Theaitetos had been 'measuring' the sides of their triangles; finding a common μέτρον is a central topic in the dialogue.<sup>28</sup> The methods of geometrical measuring cannot have been very refined in those days, but Plato's contemporaries are likely to have used an approximative method sometimes called 'reciprocal subtraction'.<sup>29</sup>

Then we have the interesting statements of Eudemos (in an Arabic text of Pappos, overlooked by Wehrli), which are sometimes thought to represent a tradition independent of Plato. There it is said in connection with a reference to the dialogue that Theaitetos "divided the most generally known irrational lines according to the different [i.e. geometric, arithmetic, and harmonic] means", perhaps using the terms μέση (medial), ἐκ δυοῖν ὀνομάτοιιν (binomial surd) and ἀποτομή (subtractive binomial surd); and that he "assumed two lines commensurable in square and proved that if he took between them a line in ratio according to geometric proportion (the geometric mean), then the line named the medial was produced, but if he took (the line) according to harmonic proportion (the harmonic mean), then the apotome was produced".<sup>30</sup>

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Plato wanted to remind his readers that the μεταξύ τῶν λόγων (c1) are indeed δυνάμεις in their own way (147e9). For an attentive interpretation of *Theaitetos*, where an attempt is made to give the mathematical passages their proper philosophical bearing, see Brown 1969 (above, n. 15) who, however, shares the conventional view of Theaitetos' achievements in geometry.

<sup>28</sup> Cornford's commentary (1935), useful in its time, is of little help in these matters; see notably P. Friedländer, *Platon III*<sup>2</sup>, Berlin 1960, 151, and Brown 1969. For μετρητική, cf. *Protagoras* 356e-357b (with metaphysical allusions similar to *Theaitetos*!), *Statesman* 283c-287b, also *Gorgias* 508a ff.

<sup>29</sup> Cf. Eucl. X 2 ff. and see e.g. Heath I 206 ff., R.S. Brumbaugh, *Plato's mathematical imagination* (Indiana Univ. Publ., Humanities Ser. 29), Bloomington 1954, 54 f.; Heller 1956:23 ff.; G. Junge, *Class. & Med.* 19 (1958) 42-44; Burkert 1972:459; Brown 1969:363-365; Lasserre 1987:447 ff., 476 ff.; each making a somewhat different approach. An illustrative example is given by van der Waerden 1966:208. In Arist. *Top.* VIII 158b33 ff. ἀντανάσεις probably means the same as ἀνθυφάσεις in Euclid. The notions of ἔλλειψις (ἐνδεία) / ὑπεροχή (ὑπερβολή) belonged to geometry in Plato's days; cf. *Prot.* 357a, *Men.* 87a, *Rep.* VIII 546c, and the methods for 'squaring' the circle from Hippokrates of Chios onwards, and the 'Golden Section' (see esp. Burkert 1972:452 f.); cf. also the 'Divided Line', *Rep.* VI 509d ff. with its metaphysical διαίρεσεις. Possibly the use of the verb προαιρείσθαι in *Theaet.* 147d7 has something to do with all this, but cf. above, n. 14.

<sup>30</sup> The Commentary of Pappus on Book X of Euclid's Elements, Arabic text & translation by W. Thomson with introductory remarks, notes [etc.] by G. Junge and W. Thomson (Harvard Semitic Series VIII), Cambridge Mass. 1930, p. 63 ff., 72 ff., 138 ff., and Junge's comments p. 15-17; frgs D 3-4 Lasserre 1987 (with comments, p. 467 ff.). For the terms, cf. Eucl. *Elem.*

It is doubtful what 'proving' means in the latter quotation even if the translation from Arabic is literally correct. Proportionals of the 'arithmetic' and 'geometric' type were easily obtained from Theodoros' triangles, and for the *apotome* one might think of the traditional construction of the 'Golden Section' by means of 'cut-offs' from a right-angled triangle though in fact the geometrical constructions needed for illustrating a harmonic mean ( $2\sqrt{2}$  to  $1 + \sqrt{2}$ ) by ἀποτομαί of three lines,  $(a - b) : (b - c) = a : c$ , are not very sophisticated.

Apparently it was known in the Academy that Theaitetos used to classify various combinations of surds, i.e. δυνάμει commensurable lines, in relation to the three means. But assuming this does not mean accepting that he systematized the doctrine of surds as we have it in Euclid.

In the dialogue, Theaitetos' function is to act as an intelligent discussion partner with Socrates (cf. the slave-boy in *Meno*), well versed in geometry and ἄλογα, and prepared to use λόγοι as well, potentially an ideal philosopher. But as far as I can see, Plato or Eudemos give no further hints about his achievements in the study of irrationals.<sup>31</sup>

Now Euclid's Book X, which contains the theory of incommensurability and surds, may indeed somehow represent the essentials of what the historical Theaitetos thought in this matter. Ancient sources seem to take this for granted, and it seems to fit in with other pieces of evidence which can be gathered from the historians of mathematics from Eudemos onwards.<sup>32</sup> But to infer that Theaitetos 'wrote' Euclid X, or at least formulated the main part of its propositions and proofs,

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X 21 ff., 47 ff., 73 ff.; cf. also Ps.-Arist. *De lineis insec.* 968b13 ff., which may reflect an earlier tradition. For the Pythagorean or at least pre-Platonic origins of the three 'means' as represented in *Timaeus* (31cd, 36a, cf. *Epin.* 991a), see Burkert 1972:440-42 and M. Brown, *Phronesis* 20 (1975) 173 ff. Von Fritz 1934:1354 ff. and some later critics (van der Waerden 1966:275 ff. among them, also Lasserre l.c. in spite of his generally cautious interpretation) have made too much of this notice in Pappos.

<sup>31</sup> For λόγοι, cf. συλλαβαί 202b ff. (also συλλαβεῖν 147d), and ἄρρητα 152c, 155e, 156b (with ἄλογα, 202b ff.); cf. above, n. 27. Elsewhere in Plato no manifest allusions occur to Theaitetos' studies: see the critical remarks of Lasserre 1987:487 ff. (who, however, accepts *Laws* VII 820c and *Epin.* 990d as 'fragments').

<sup>32</sup> After Sachs the evidence was recorded by von Fritz 1934:1353 ff. (who did not know Thomson's Pappos, above, n. 30). Cf. also the rather negative evidence of the Testimonia collected by Stamatis in his new B.T. edition of Euclid, Book X (1972).

is a modern idea.<sup>33</sup> I can see no reason at all for accepting this view. The Academy had fostered many prominent mathematicians before Euclid's times.<sup>34</sup> Supposing that Theaitetos died as a young man, surely Plato, the 'architect' of Academic geometry,<sup>35</sup> would have been able to transmit his dead friend's visions of irrationality and commensurability to younger generations who were capable of elaborating the theories and giving them a fixed written form. Perhaps, too, Plato was the only transmitter of the tradition about Theodoros' ἔλιξις.

Theaitetos' other speciality is said to have been the construction of the five regular solids. The evidence was discussed in detail by Sachs,<sup>36</sup> who argued a point later doubted by very few, namely that Theaitetos made his discoveries mainly after Plato had written the *Republic*, where (VII 528a ff.) Socrates remarks on the deplorable state of stereometry, and before the *Timaeus* where the theory of the solids is implied (31b-34a, 53c-55c, cf. *Epin.* 990d). Euclid's Book XIII would largely derive from Theaitetos' work on the solids. Again, I think, the moderns have gone far too far.

The brief reference to stereometry in *Theaitetos* (148b) certainly points to Theaitetos' activities in this field though, as such, it only implies that Theaitetos and his friend 'saw' that the same rule of δυνάμει commensurability must apply to the relation of the edges to the volume of cubes, pyramids, etc.<sup>37</sup> The rest of the

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<sup>33</sup> Sachs 1914:11-13, 41-42. It is reflected very clearly in van der Waerden (e.g. 1966:271-91) with his tendency to dogmatic conclusions. Even the generally cautious von Fritz insists that Theaitetos offered Euclidean proofs (e.g. 1934:1358). Lasserre 1987:464 is wisely sceptical about the possibility of reconstructing Theaitetos' formulations.

<sup>34</sup> Menaichmos, Deinostratos, Athenaios, Hermostimos, Theudios (who actually published a book of 'Elements'), and Eudoxos, to mention just a few; see now Lasserre 1987. Eudoxos' relation to Theaitetos is a matter of conjecture. The sweeping statements of Proklos, In Eucl. *Elem.* I, p. 67 Friedl. 'Ερμότιμος... τὰ ὑπ' Εὐδόξου προηυπορημένα καὶ Θεαιτήτου προήγαγεν ἐπὶ πλέον (probably from Eudemos), p. 68 ...Εὐκλείδης...πολλὰ μὲν τῶν Εὐδόξου συντάξας πολλὰ δὲ τῶν Θεαιτήτου τελεωσάμενος (probably his own addition), have very little relevance.

<sup>35</sup> The new Dikaiarchos text published by K. Gaiser in Supplementum Platonicum I, Stuttgart 1988, comments on the development of mathematics and μετρολογία in the Academy, ἀρχιτεκτονοῦντος... τοῦ Πλάτωνος (Col. 1 Y.4 ff., p. 152) and emphasizing the role of Eudoxos in developing the ὀρισμοί (cf. *Theait.* 148a8 and Gaiser p. 348). Plato, of course, was not himself very active as a mathematician.

<sup>36</sup> Eva Sachs, Die fünf platonischen Körper (Philologische Untersuchungen 24), Berlin 1917, esp. 146 ff.; cf. Sachs 1914:40.

<sup>37</sup> von Fritz 1934:1360 seems to admit this.

ancient sources referring to Theaitetos' studies of the solids do not suggest more than that he was able to construct and explain the cube, the regular tetrahedron, the octahedron, the icosahedron, and the dodecahedron.<sup>38</sup>

If no proofs or systematized theories are required, this is not so very remarkable. The theory of the cube (6 squares) and the tetrahedron (4 equilateral triangles) had been well known for a long time, the dodecahedron (12 regular pentagons) had been an object of wonder among the Pythagoreans and before,<sup>39</sup> and the octahedron (8 equilateral triangles) was easily, the icosahedron (20 equilateral triangles) possibly, derived from the tetrahedron. It is reasonable, however, to infer that Theaitetos applied his knowledge of reciprocal subtraction and ἀποτομαί to the construction of the icosahedron and the dodecahedron and, hence, to the regular pentagon.<sup>40</sup> But as far as I can see, there is no evidence of his producing the system of regular solids as we have it in Euclid XIII.

Plato's *Republic* took shape gradually, and I find it quite plausible that Glaukon's and Socrates' complaints about stereometry in Book VII reflect the state of affairs in the late 370s.<sup>41</sup> But to force Theaitetos' alleged discoveries of the 'five Platonic bodies' in between that date and 369 B.C., is simply to overinterpret a series of hypotheses. And surely we should expect Plato, who was not afraid of anachronisms, to have made more than a casual reference to τὰ στερεά in *Theaetetus* if his friend had made such remarkable progress just before the dialogue was written.

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<sup>38</sup> The evidence collected by Sachs is discussed with critical cautiousness by von Fritz 1934:1363 ff. and Lasserre 1987:492 ff. (who emphasizes the intermediary role of Hermotimos). The most explicit piece of information is a Scholium on Euclid, Book XIII, where it is stated that the octahedron and the icosahedron are Θεαιτήτου.

<sup>39</sup> Burkert 1972:460.

<sup>40</sup> See von Fritz 1934:1369-71; Junge 1958 (above, n. 29). The 'Golden Section' was used in antiquity for constructing the pentagon. The Anonymous Commentator, Col. 41 ff. (p. 28 Diels-Schubart), in explaining *Theaet.* 148b, starts from arithmetic (which is probably correct) but proceeds by guesses and hardly supplies historically reliable information.

<sup>41</sup> *Rep.* VII 528b-d: Glaukon remarks that "this subject (i.e., of the third dimension) does not appear to have been investigated yet", and Socrates gives some reasons why this is so, one being that "the investigators need a director", and "as things are now, seekers in this field would be too arrogant to submit to this guidance" (transl. P. Shorey). This reproach surely would not apply to the promising young geometrician of *Theaetetus*. For the date of the *Republic*, cf. Thesleff 1982:138, 185.

In my view, it can be rather safely concluded that Theaitetos only laid the foundations for stereometry by trying to generalize the rules for square roots to cubic roots and by studying the properties of the regular solids. After the 380s, Eudoxos and some others actually built the system eventually laid out in *Timaeus*, *Epinomis*, in various Aristotelian passages, and in Euclid. And so Theaitetos the stereometrician will have to take a similar step backwards in history as Theaitetos the irrationalist. I am not, however, questioning his brilliance in relation to his contemporaries.

Finally, there is the odd notice in Hesychios and Suda about Theaitetos having taught in Herakleia.<sup>42</sup> A mistake, similar to the emerging of 'Theaitetos of Rhegion',<sup>43</sup> is quite possible. Still, Theaitetos may have visited Herakleia in the 390s; he was a friend of Eukleides of Megara, and Herakleia was a Megarian colony. The awkward fact that Theaitetos is not mentioned in the Philodemic list of members of Plato's Academy traditionally known as the 'Academicorum Index', has sometimes been taken to indicate that he was working abroad and not in Athens.<sup>44</sup> My explanation of why he does not appear in the list is different.

I am inclined to think that Theaitetos lived ca. 415-390 B.C. and that the explicitness of his discoveries has been exaggerated by modern interpreters. Plato's reasons for introducing him and Theodoros in *Theaitetos* may be looked for along the general lines suggested by Malcolm Brown.<sup>45</sup>

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<sup>42</sup> Suda ends by having two Theaitetoses, the Athenian who was an ἀστρολόγος etc., and a pupil of Socrates and who taught in Herakleia, and the Heracleote, who was a pupil of Plato.

<sup>43</sup> Iambl. *VP* 172; here Theokles is meant, cf. *ibid.* 130.

<sup>44</sup> Cf. Lasserre 1987:434. For the Academicorum Index, see now *Suppl. Plat.* (above, n. 35), Col. 5.32 ff., and Gaiser's comments p. 443 ff. (also p. 15 f., 90, on Lasserre's solutions). Nor does Dikaiarchos mention Theaitetos.

<sup>45</sup> In the article referred to above, n. 15. See also above, n. 27.