CULTIVATING FLEXIBILITY:

A NEW APPROACH TO TEACHING AND LEARNING MATHEMATICS AT THE UNIVERSITY LEVEL

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DEFINING FLEXIBILITY

When it comes to mathematics, many of us remember formulas and specific methods etched into our minds in school. Imagine, if instead of one right way to solve a problem, there were multiple approaches – each uniquely suited to a specific mathematical task. This skill, known as mathematical flexibility or simply flexibility, has gained increased attention from education and research. Flexibility is about adapting and selecting the best strategies to solve a problem effectively (Star & Rittle-Johnson, 2007; Xu et al., 2017).

In primary- and upper-secondary education mathematics it is often viewed as following a set of rigid steps. However, in the world of higher education, things get a little trickier. University mathematics presents students with more complex problems that require not only knowing procedures and concepts but also being able to apply them in different situations. In this environment, flexibility is crucial. Teaching students how to tackle a problem flexibly helps them to become better problem-solvers and more creative thinkers.

Consider a simple example of equation solving. You are given a linear equation

$$\frac{6x-9}{3}+\frac{8x-12}{4}=3,$$

with multiple fractions.

The generic strategy taught for solving this problem involves expanding the fractions to a common denominator before adding them together. This method is long and inefficient; students must handle larger numbers, and in the end simplify the fraction. In such a manner the students are more prone to make careless errors. A more efficient approach, known as the situational strategy, involves identifying a common factor and simplifying the fraction accordingly.

Generic strategy	Situational strategy
$\frac{4(6x-9)}{12}+\frac{3(8x-12)}{12}=3$	$\frac{3(2x-3)}{3}+\frac{4(2x-3)}{4}=3$
$\frac{4(6x-9)+3(8x-12)}{12}=3$	$2x-3+2x-3=3$
$4(6x-9)+3(8x-12)=36$	$4x-6=3$
$24x-36+24x-36=36$	$4x=9$
$48x=108, x=\frac{108}{48}=\frac{9}{4}$	$x=\frac{9}{4}$

UNDERSTANDING THE IMPORTANCE OF RESEARCHING FLEXIBILITY

There are concerns about students' ability to apply learned methods and the lack of depth in their procedural and conceptual knowledge. The overall level of mathematical skills students possesses before entering university is also for worry. Studying flexibility helps students to develop multiple problem-solving strategies and foster a deeper understanding of mathematical concepts. This approach promotes critical thinking, which is valuable not only in mathematics but in various aspects of life. Students with strong flexibility skills are better equipped to adapt to new challenges, and flexible teaching methods enhance engagement, making learning more enjoyable and meaningful.

How can students and teachers then cultivate flexible thinking? The studies suggest creating a classroom environment where students are encouraged to explore multiple strategies (Star et al., 2012) and compare alternative solution methods (Star & Rittle-Johnson, 2007). Research indicates that comparing different strategies promotes flexible learning more effectively than studying each method individually (Durkin et al., 2017). Additionally, opportunities for collaboration among students and the introduction of non-routine problems can further enhance flexibility. While previous research demonstrates that educators can actively influence the development of students' flexibility, further investigation is needed to gain a deeper understanding of this area

MATHEMATICAL FLEXIBILITY IN HIGHER EDUCATION: CURRENT RESEARCH

At the university level, research on mathematical flexibility has been relatively limited, mostly focusing on arithmetics and calculus. In our research related to my dissertation, we first examined flexibility in early university mathematics, focusing specifically on solving algebraic tasks such as linear equations, quadratic equations, and systems of equations. We also investigated the relationship between accuracy, flexibility, and exam performance. The results demonstrated high levels of both accuracy and flexibility. Interestingly, although accuracy and flexibility were correlated, flexibility was a more significant predictor of exam scores than accuracy (Ernvall-Hytönen, et al., 2022).

The second article examines the relationship between flexibility in solving linear- and quadratic equations and mathematics achievement. It explores university students' strategy choices, the spontaneity of these choices, and flexibility across different degree programs. The study also compares the flexibility between university students and high-school students. The studies indicated that higher flexibility is associated with improved accuracy in problem-solving and greater success in academic performance (Ernvall-Hytönen, et al., 2025).

In conclusion, fostering mathematical flexibility in university students is a crucial step toward equipping them with the skills needed for complex problem-solving and critical thinking in higher education. While research on flexibility at the university level is still in its early stages, the findings thus far emphasize the importance of flexibility in not only improving mathematical skills but also in enhancing overall academic success. As we continue to explore how flexibility can be cultivated in university mathematics, it is essential for both students and educators to embrace a mindset that values multiple approaches and creative problem-solving strategies. By doing so, we can create a learning environment where students are better prepared to face the challenges of both their academic studies and real-world situations.

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