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RECOGNISING ARTICULATED REASONING IN STUDENTS' ARGUMENTATIVE TALK IN MATHEMATICS LESSONS

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ABSTRACT

Previous studies have reported difficulties in analysing students' argumentation. Especially the warrant, which connects data with claims, is found to be problematic to recognise. We examine how specific criteria for different forms of reasoning could decrease this difficulty. Instead of identifying warrants, we recognise whether students' argumentation contains explicitly stated reasoning. We have developed an analysis method for recognising reasoning in argumentative talk. We collected video data from two mathematics lessons where students had to state claims about a mathematical situation and build convincing justification for their claims. The focus of analysis is in how the students articulated why the posed claim is true. Three cases illustrate how criteria for different forms of reasoning help in recognising whether the students articulate their reasoning.

INTRODUCTION

Argumentation is one of the 21st century skills and it is not important only in STEM (science, technology, engineering, mathematics) subjects but has wide applicability in different fields (Erduran, Simon & Osborne, 2004). In this study, we consider argumentation from the discussion perspective and take into account only voiced arguments. This means that we consider argumentation as a particular type of discussion containing a claim and statements that are intended to support the claim.

Previous studies have also examined argumentation from discussion perspective. Some studies have focused on collective argumentation and have explored instances where both students and teachers pose mathematical claims and provide evidence to support them (e.g., Conner, Singletary, Smith, Wagner & Francisco, 2014). Science education research has focused on studying argumentation as a discussion in which students make claims, defend them and criticise others' arguments (e.g., Berland & McNeill, 2010). Argumentation has been studied also as a verbal process where the students examine the validity of a conclusion, model or prediction in the light of evidence and theory (Duschl & Osborne, 2002).

There are numerous studies in the field of mathematics and science education on argumentation and analysing the structure of arguments. Many of them have used modified Toulmin's (1958/2003) Argumentation Pattern (TAP) as the analytic framework to examine the structure of argument (e.g., Whitenack & Knipping, 2002; Krummheuer, 2007; Conner & et al., 2014). The main components in Toulmin's model are claim, data and warrant. Toulmin (1958/2003) defines claims as statements whose validity is to be established and data as support for the claim. Warrants are statements that connect data with claims. These elements form the core of argument (Krummheuer, 1995).

There are some difficulties in using Toulmin's (1958/2003) model as an analytic framework. The model is restricted to relatively short arguments and the components possess ambiguities according to Kelly, Druker and Chen (1998). A particular utterance may be considered as a claim in one context, but the same utterance may serve as a warrant in another (Conner et al., 2014, p. 406). It is difficult to differentiate between data and warrant because they relate to each other (Krummheuer, 1995). Many earlier studies have modified Toulmin's model and tried to diminish the ambiguity of the model in different ways (e.g., Kelly, Druker & Chen, 1998; Erduran et al., 2004). Still several studies report about the difficulties in recognising the warrants (e.g., Erduran et al., 2004). Some researchers have defined in advance the content of the desired warrant in a given situation and then looked if corresponding utterances are found in the argumentation (e.g., Jimenez-Aleixandre, Rodriguez & Duschl, 2000). Also McNeill, Lizotte, Krajcik and Marx (2006) have defined item-specific coding criteria for recognising warrants. Erduran et al. (2004), instead, have examined words, such as 'so' or 'because', which could indicate giving warrant. Another problem in recognising warrants is that warrants are not often explicitly stated but researchers have to interpret them from the context (e.g. Conner et al., 2014).

There still seems to be a need for a more unambiguous way to analyse students' arguments. The aim of this study is to develop an analytic tool for recognising elements of argumentation. TAP has been a starting point for our analysis, but we have modified it. Instead of trying to recognise warrants, we examine whether an argument contains explicitly stated reasoning, which we call *articulated reasoning*. The articulated reasoning means stating the reasons why a certain conclusion can be made. Articulated reasoning includes warrant, but we do not need to recognise which part in students' utterances is data and which part warrant. Instead of defining articulated reasoning separately for different topics, we use specific criteria for several different kinds of reasoning identified in previous research (e.g., Reid & Knipping, 2010). Thus, we set the following research question: How can explicitly articulated reasoning be recognized using criteria that are specific to several forms of reasoning that are not content spesific?

METHODS

This study concerns the pilot phase of a two-year longitudinal research project on how students develop their argumentation when using argumentation tasks regularly in mathematics and physics. During the pilot phase, we designed some argumentation tasks, studied how these were implemented in mathematics and physics classes and developed our analysis methods.

Data collection

Data in the pilot phase was collected in one school class in Central Finland. The mathematics teacher of the class had taught several years mathematics in lower secondary school. We video- and audio-recorded four lower secondary mathematics lessons with a handheld video camera, which followed the teacher. In every lesson, students were first working in groups of 2–4 students, and the work in the small groups was recorded with wide-angle GoPro-cameras. The group work was followed by a whole class discussion orchestrated by the teacher.

In this paper, we introduce more closely two eighth-grade mathematics lessons, which we already have analysed in depth. The tasks for both lessons were designed by our research group to correspond the mathematical topic that the class was studying. Tasks were related to everyday life context and designed to stimulate the natural need to state a claim about the mathematical situation. In addition, students were asked to give convincing justifications for the claims.

Data analysis

In this study, we focus on the most essential elements in the structure of argument: *claim*, *describing support* and *articulated reasoning*. This compares to a core of an argument (Krummheuer, 1995). A *claim* is defined as a statement, which is supported, for instance, with a fact, observation or calculation or an explanation for why the claim can be concluded. A claim may also be a statement whose validity is explicitly questioned or challenged. *Describing support* means presenting facts, statements, figures or calculation to support the claim. *Articulated reasoning* means explicitly articulating why a claim can be concluded from what is known. Because we examine argumentation from the discussion perspective, we are interested in only orally verbalised reasons why the claim can be concluded. It is important to differentiate between reasoning and articulated reasoning. Reasoning is not happening only when students are speaking, but instead of any kind of reasoning, we have chosen to observe articulated reasoning. The articulated reasoning or described support for the claim do not have to be scientifically correct or complete.

To form an argument, in minimum the students have to pose a claim and justify it by describing support or articulating reasoning. In addition, the argument may contain other components, such as a competing claim and components, which specify different situations or conditions for the argument. In other words, these components describe details of the argument but are not necessary to form an argument. However, in this paper we focus on recognising articulated reasoning. Compared to the elements of TAP, reasoning can be seen as a combination of a warrant and data. Unlike in the Toulmin model, we do not have to recognise which statements are data and which are warrants. We merely have to recognise whether the argument includes articulated reasoning or not. To help to recognise this, we developed more specific criteria for different forms of reasoning to define conditions for articulated reasoning (Table 1).

Table 1. Forms of reasoning and criteria for articulated reasoning

Form of reasoning	Criteria for articulated reasoning
Deductive reasoning	
1. Reasoning by counterexample	Students explain why a posed counterexample refutes the claim. Giving a counterexample without an explanation is only describing support.
2. Existence proof by example	Students explain why a certain example demonstrates that the claim is true.
3. Pleading to a known fact or theory	Students explain why a known fact or theory can be applied to the case in which the claim is posed.
4. Chain of logical steps	Students explain the logical steps that follow from each other and lead to the claim.
5. Reasoning by exhaustion	Students list systematically all possible cases and explain why their list contains all the possible cases.
6. Indirect reasoning by contradiction	Students explain why a counterclaim would lead to something impossible.
Non-deductive reasoning	
7. Inductive reasoning	Students give individual cases in which the claim holds true and explain why these cases demonstrate the claim.

The forms of reasoning are composed from known justification types (e.g., Reid & Knipping, 2010; Stylianides, 2007; Smith & Henderson, 1959). Most of the deductive forms of reasoning have a corresponding mathematical proof strategy such as indirect proof. However, the same forms of reasoning are often used in everyday argumentation too. The chosen forms are the ones we have found to be useful at this point of the study. In addition to these, we are prepared to add more categories (e.g., analogic and abductive reasoning) if needed.

Video data was transcribed and analysed using Atlas.ti-software. At first, we recognised the claims from the transcript and identified the parts where students were describing support for the claim. Then, we identified the form of reasoning. For example, if students were using individual cases to support a general claim, it indicated the inductive form of reasoning. After identifying the form of reasoning, we carefully compared the students' utterances with the specific criteria of this form of reasoning to decide whether the argument included articulated reasoning or not. In some cases, students' reasoning may have the features of two forms of reasoning. In these cases both applicable forms of reasoning and their criteria for articulated reasoning would be observed carefully. In this pilot study, a consensus about articulated reasoning within four researchers was reached by discussing different options.

RESULTS

We have analysed students' argumentative talk in small group discussions by identifying the form of reasoning and examining whether students' talk fulfils the criteria of articulated reasoning or not.

In the following, we elaborate on two instances of students' articulated reasoning and one situation where a justification does not fulfil the requirements of articulated reasoning (Pleading to a known fact or theory).

Chain of logical steps

In one of the lessons, the students were thinking about the relation between a person's length and the length of the person's shadow. A student group was trying to justify why the length of a person divided by the length of her/his shadow is always the same when the Sun is shining from a certain angle (Figure 1).

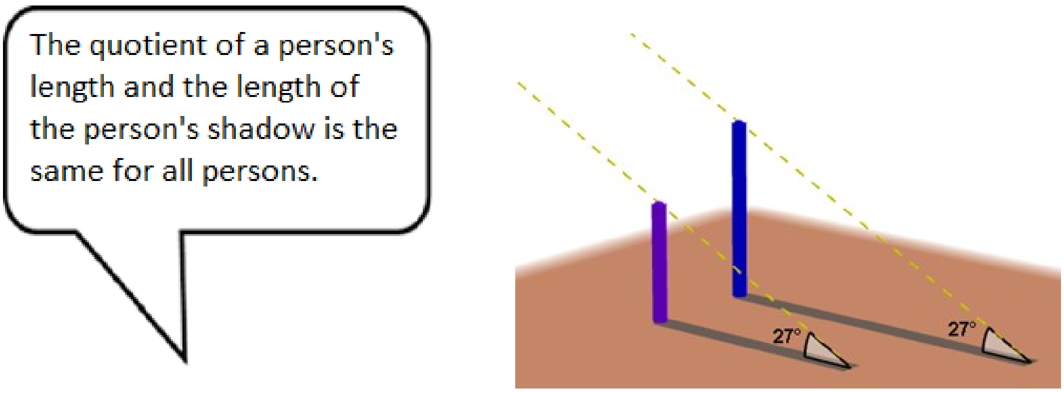

Figure 1. Two persons with their shadows when the Sun is shining at a 27-degree angle.

As part of their argumentation, they concluded that the two triangles shown in Figure 1 are similar:

Bruno: …the sum of triangles.

Sally: The sum [of the angles] is 180.

Bruno: That's right. Minus 27, minus 9… this must be 63 (points the angle on one triangle) Pen! We know now that these are similar.

Sally: Yeah.

Bruno: Because the angles are the same.

In the above episode, the claim is that the two triangles are similar. The students started from the knowledge of the sum of the angles of triangles, which is 180 degrees. By using this information, they calculated the missing value of one angle and found out that both triangles have same sized angles. After that, the similarity of the triangles followed from the knowledge that both triangles have the same sized angles. The students' explicitly explained the chain of logical steps, which leads to the conclusion that the triangles are similar. This meets the criteria for articulated reasoning in the form of reasoning "Chain of logical steps".

Note that this is not a complete justification for the original claim (the main claim), but a step towards it. The knowledge of similarity of the triangles could be used to justify that person's length divided by the length of the shadow is the same for all persons, or the leg of the triangle divided by the other leg of the triangle is the same for all triangles. However, the students did not reach this level during the lesson.

Inductive reasoning

In the following episode, another group was discussing about the same task as in the previous example. They claimed that the proportion of a person's length to the length of the shadow is the same for all the persons as shown in Figure 1. Susan had measured the lengths from the picture she had drawn and explained to the others what she had done:

Walter: We changed our opinion to Joan. (The quotient of person's length and the length of the shadow is the same for all the persons).

Susan: …these are in the same proportion, this side divides this, so it's approximately zero point fifty-three or fifty-two, because I didn't measure it precisely. After that, this was divided by this, it was also something about zero point fifty-two. It's approximately the same, because I didn't measure so precisely, as I said. So it is the same, so it is Joan.

Susan got almost the same result when she calculated the quotient of the sides of two triangles she had measured. The triangles represented two persons with their shadows. In the next phase of the lesson, students had a whole class discussion. Susan described her work to the whole class:

Susan: Well, we drew like two right-angled triangles, which had one 27-degree angle, but they were different size, in the same proportion, in principle. Then we divided it, them… Like the length of the person with the length of the person's shadow and got more or less the same answer from both pictures. I don't know how to explain…

Susan had drawn two triangles and measured the sides of the triangles. She explained which sides she measured, which division she calculated and which result were the same in the both triangles. Thus, she explained why the general claim is true in the case of these two triangles. This reached the criteria for articulated reasoning in the form of reasoning "Inductive reasoning". Of course, measuring and using inductive reasoning is not a scientifically acceptable way to justify the claim in mathematics, but from the point of view of the structure of an argument, this is not relevant.

Pleading to a known fact or theory

In another lesson, the teacher gave students two suggestions for a solution to a problem (Figure 2) about probability and the students had to defend one option and criticise the other. In the task, two persons had flipped a coin three times and Mary had won two times and Ben once. Their game ends when one or the other has won three flips. The task included two solution for the probability of Ben winning the game.

Solution A

Game can continue as follows:

4th flip	*5th flip*	*Prize goes to...*
Ben wins	Ben wins	Ben
Ben wins	Mary wins	Mary
Mary wins	-	Mary

Ben's probability to win is $\frac{1}{3}$

Solution B

Game can continue as follows:

4th flip	*5th flip*	*Prize goes to...*
Ben wins	Ben wins	Ben
Ben wins	Mary wins	Mary
Mary wins	(Ben would win)	Mary
Mary wins	(Mary would win)	Mary

Ben's probability to win is $\frac{1}{4}$

Figure 2. Solution suggestions to a probability task under discussion

In the following episode, a student group was discussing solution B and trying to defend it as the right one:

William: The only option that Ben wins this game is that he wins both flips.

David: Yeah, well, it's one out of two. But is it, because then it is…

William: No, because isn't it, that, what we have just studied what happens in series

David: Yeah, it has…

William: Isn't it so that the chance is one out of two times one out of two? Is it one out of four then? One out of four. Isn't it so?

David: Well, you can tell your theory. I don't know if it is like that.

In this discussion, William referred to something they have learned before (the probability of a series of independent events) and the situation has indications for the form of reasoning "Pleading to a known fact or theory". However, no one explains how the theory presented in the conversation is connected with the situation in the task or why the theory can be applied to this case. To meet the criteria of articulated reasoning, students would have needed to explain, for example, that on every flip Ben's probability to win is ½ and he has to win next the two flips in a row to win the whole game and, therefore, the chances for Ben to win the prize can be calculated by multiplying the probabilities of these two independent events. The argument given by the students is interpreted to include describing support, but not articulated reasoning.

DISCUSSION

As the results indicate, we were able to identify articulated reasoning in students' talk using the developed analysis method. Thus, the specific criteria for the forms of reasoning introduced in this paper may be used as a tool for recognising students' articulated reasoning. The introduced analytic tool is based on Toulmin's model (1958/2003), but we have modified it to simplify the analysis. In Toulmin's model (1958/2003) an argument consists of a claim, data and warrant. In our model, in addition to a claim, we have to recognise whether students' conversation includes articulating reasoning or only describing support. Articulated reasoning incorporates warrant, but we do not need to recognise it separately, which makes analysis easier to perform. For example, Krummheuer (1995) has brought up the difficulty of making the distinction between data and warrant. When using our analysis method, we do not have to be concerned with that problem. Articulated reasoning might include some data whose relation to the claim is explicitly stated, but we do not need to separate which utterance, or a piece of utterance, is warrant and which is data. In addition, in a spoken language, it might be difficult to interpret a single element from a single utterance. Instead, the whole conversation is easier to interpret.

There are several earlier studies on analysing students' argumentation and many of them have reported on the complexity of identifying the different components of argument, especially the warrant as it is defined in Toulmin's model (1958/2003). For example, Erduran et al. (2004) have used indication words (such as 'because') for resolving ambiguity, but some warrants may be unrecognised due to the unorganised way of talk that students use. McNeill et al. (2006) have approached the problem of recognising warrants from the task-specific point of view and defined the coding criteria for each item. This of course makes recognising warrants easy but demands work in designing the criteria separately for every respective task. The benefit of the system introduced in this paper is that it is more universal and can be applied to many kinds of tasks without using further task-specific definitions or indication words.

Classroom talk can be very vague even though a student might have a clear idea of how to solve the problem. Part of the difficulties stem from the nature of talk: it may not seem necessary to voice everything in the natural way of talking or discussing. Although our focus is on students' talk and voiced thinking, part of the discussion consists of, for example, facial expressions or gestures. Some of the content in a discussion is not necessarily said out loud, since a conversationalist may know and assume others to know what it is about.

Student discussions do not typically proceed in a straightforward manner and this makes reasoning difficult to recognise. Student groups can change the subject very often. Articulated reasoning might not be stated in one turn of speech or in one conversational unity. We have seen that students are often too timid to voice their ideas and thinking if they do not feel they have convincing grounds for them. At the same time, students might find very vague ideas and explanations convincing. Sometimes it seems like the aim for the students is not so much to understand the posed problem or other students' thinking as it is to give any answer to the question.

Students may give a single support, such as a calculation or a fact instead of articulating reasoning. In addition, a student might articulate an incomplete reasoning, which does not fully justify the claim, but it can be a step towards it or a part of the satisfying explanation. Sometimes the line between describing support and articulating reasoning is vague, because students' thinking might include some sort of logical deduction, but it does not meet the requirements of articulated reasoning. Articulated reasoning, as we define it, needs to make the reasons, why the conclusion can be made, explicit.

In the end, we believe that the argumentation skills can be practised, and thus making the students' thinking visible becomes more fluent and organised. Students need to be encouraged to voice their thinking even if it is incomplete, and thus help each other to build reasoning and take their thinking to the next level together.

The forms of reasoning and the defined criteria for articulated reasoning helped us to be consistent in our analysis. However, we have come across many problems in coding students' discussion, so consensus in recognising reasoning cannot be considered self-evident. In applying this analysis method, negotiations between researchers and comparing the interpretations are very important.

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