

Assaying U.S. Schooling: More Money or More Performance

Arthur Melmed & Jean H.P. Paelinck

ABSTRACT

U.S. schools are failing the nation and the nation's economy. It may be that a greater level of expenditure can be justified and urged upon the nation based on an analysis of schooling effects on economic growth. Alternatively, the nation should aim to provide the highest quality of schooling at the level of expenditure preferred by the taxpayer. The application of a choice model for social expenditure suggests the latter course.

In this case, the nation is left to try to improve the productivity and efficiency of schooling. The historical record reveals many failed approaches taken towards this goal, with one major option as yet untried: changing the traditional mix of capital and labor in the schools.

Within this option, there can yet be many political and educational goals, e.g.: (i) improving the attainment of students in the lower tail of the achievement distribution, which can improve parental satisfaction of the included students and improve the quality of manpower available to certain sectors of the economy; and (ii) improving the attainment of students in the upper tail of the achievement distribution, which can improve the quality of manpower available to high value-added sectors of the economy.

For practical political reasons, the use of any radically different mix of capital and labor in the nation's schools, would and should aim to improve the achievement of all school children, but leaves unanswered the question of equity, and the policy question of how to get "from here to there."

1. INTRODUCTION

The nation's system of formal education is an exceptional achievement, enrolling over 50 million children in elementary and secondary schools, and over twelve million youths and adults in higher education. Yet, U.S. education may be failing the nation. With costs rising faster than inflation, the quality of the schooling product doesn't satisfy society or meet the manpower needs of the nation's advanced economy. The reverse may also be true however; that the nation is failing public education; that educational expenditure is too slight and unbalanced relative to the nation's investment in physical infrastructure and in expenditure for health care.

A greater level of educational expenditure can perhaps be justified and urged upon the nation based on the benefits of schooling for economic growth. Alternatively, the nation can aim to provide the highest quality of schooling possible at the level of expenditure preferred by the taxpayer.

School improvement strategies can emphasize changing instructional practice, marginally or radically. Examples of a radical strategy include a sharp increase, say doubling, of teacher salary, or a sharp change in the classroom mix of capital and labor. In principle, a marginal or radical change in instructional practice should be possible in a regulated or free market environment.

School improvement efforts by federal, State and local governments since Sputnik in 1957 have taken the easier route, politically and socially, of marginal improvement in instructional practice at a marginal increase in annual student cost. Oft-times, fierce debate has raged among office holders, educational experts and economists about whether previous efforts at school improvement have had any effect - 4th grade test scores on international comparisons have

recently improved while 8th grade test scores remain desultory; on whether the nation has persisted long enough and hard enough; on whether tests test the "right" things; and on whether the taxpayer is getting what he's paying for.

We discuss these matters briefly, sufficiently to reveal our biases. Analysis does not and probably can not resolve these conflicting views decisively for three reasons. Parents, social groups and representatives of value-added sectors of the economy often have different goals for the nation's schooling system; measures for student outcomes are not up to the analytic task; and there is besides a problem in causal inference, of sorting out the student's school learning from the student's learning at home and outside of school.

We concern ourselves here primarily with two policy issues. We consider first the contribution of schooling to growth in domestic product. Comparing the relative spending preferences in for education and health of the U.S. and another OECD nation, we conclude, albeit tentatively, that U.S. expenditure for schooling is on the high side. Casting additional doubt on the policy option of increasing quality by means of a substantial increase in educational expenditure is the taxpayers' manifest resistance to higher property taxes, which effectively limits the annual increase in educational expenditure to something close to the rate of inflation.

Many experienced educational observers argue that only by improving the quality of the classroom teacher and/or reducing class size sharply and/or increasing instructional time significantly can improvement in student achievement be realized. As expenses related to the classroom teacher already total to a substantial fraction of the annual student cost of schooling, averaging just under 50% nationally and approaching 80% in some educational jurisdictions, and as U.S. business and industry has long since outbid the taxpayer for the quality of human capital (classroom teacher) that could be wished for in the schools, any improvement in schooling output appears to depend upon a different approach.

Second, we consider an as yet untried, politically and socially demanding approach to improving education for all students; i.e., by employing a different mix of human and physical capital than is presently regnant in the schools.¹ By schools,

we intend public, charter and "voucherized" private schools, and conclude with a note on the need for more data from theoretical and empirical research.

2. POLICY MODEL

In this section, we attempt an initial, rough-and-ready effort at understanding the scale of U.S. schooling expenditure by comparing it with that of another OECD nation, The Netherlands, whose citizens demonstrate manifest satisfaction with their schooling system. Only the principal substantive points underlying the necessary mathematical development are presented below. The interested and mathematically oriented reader can find details of the simulation model in the Appendix.

2.1 *Computation on Social Choice*

We use here a general choice model for social expenditure, which includes consumption, and investing in physical capital, education and health care. In the authors' experience, these are the principal categories generally considered for our purpose. One could of course go into greater depth, and selected exercises down this path are outlined in the Appendix, but we aim primarily for a model that gives essential and fruitful insights into the issue under investigation, and other possible categories like military and space exploration expenditure are not given explicit consideration at this time.

We consider then only the following variables, the initials accounting for the name we give to the model, KECH:

- k : the share of physical capital investment;
- e : the share of educational expenditure;
- c : the share of consumption in domestic product; and,
- h : the share of health care expenditure.

We start with the well-known national growth model, "Harrod-Domar," which relates growth in domestic product to relative level of investment, absolute growth being related to the amount of

Table 1

	U.S.		The Netherlands	
	US \$ in billions	%/100 of domestic product	Hfl in millions	%/100 of domestic product
C,c	5,893.2	.6337	499,666	.6136
K,k	1,861.7	.2002	174,637	.2145
E,e	557.0	.0599	37,051	.0455
H,h	1,198.0	.1288	66,624	.0818

Sources are OECD (2000a, b and c.) and Seskin and Sullivan (2000).

net investment divided by the amount of capital needed to produce a unit of domestic product, or the so-called (net) capital coefficient. Dividing the current volume of domestic product by the capital coefficient relates the growth rate of that product to its share of investment.

We extend this model by considering that investment in physical capital provides only a partial contribution to growth, with education and health care also generating their partial contributions.

In order to rationalize social choices, we employ a preference function, one with constant preference elasticities, meaning that it relates by means of a constant coefficient the relative increase in preference to a relative increase of the choice item considered.

Finally, the specified choice problem is computed over a "complete" future in terms of yearly total consumption, investment in physical capital, and expenditures for education and health care. As future flows of these categories are valued at less than present ones, they are discounted in order to derive their total present value.

The interested reader will find details in the development of this model in the Appendix, as well as selected simulations aimed at illustrating its workings. The next section shows the results for the U.S. and The Netherlands based on recent data.

2.2 Some tentative results

Table 1 (below) presents computed figures for the U.S. and The Netherlands corresponding to the KECH-variables, based mostly on 1999 data.

The reader is forewarned that these computed figures are only indicative at this stage, due to some inconsistency in the input data. For example, the figures for education draw on data for the year 1997, so the relative values for consumption (by both households and governments) are almost certainly underestimated for both countries. The health care figure for The Netherlands is based on data for the year 1998. It should be noted that, wherever possible, the investment figures for education and health care have been imputed to E,e and H,h respectively and subtracted from K,k. That the relative figures do not always add up to one is attributable to the fact that certain minor items (changes in inventories and balance of trade) have not been strictly reproduced in the computations.

In a later stage of this work, the authors intend to prepare an input-output table for the U.S., focusing on the education and health care sectors and aggregating all other sectors so as to ensure consistency of the results presented. (For an analogous exercise in medicine and health, see van der Burg and Paelinck, 1993.)

An industrialized nation with a substantial immigrant population, a fellow member of OECD, The Netherlands was selected for comparison because of general satisfaction by the Dutch public over its schooling product; because it spends less per student than the U.S., just under \$5000 and just over \$7000 respectively at the elementary and secondary school levels, (Education at a Glance, OECD Indicators, 2000, p83, B4;) and because its students consistently outscore U.S. students on the International Mathematics and Science Study (<http://www.nces.gov/TIMSS/>.)

In light of these factors, the discrepancy bet-

ween the relative education figures for the two countries shown in Table 1, 4.5% for The Netherlands as against 6% for the U.S. is striking, given moreover the discrepancies in annual income per head, about \$25,000 vs. \$34,000.

3. IMPROVING U.S. SCHOOLING AT A PRICE THE TAXPAYER WILL PAY

Using the substantial percentage difference in education shares of domestic product between the U.S. and The Netherlands as evidence that U.S. education spending is at least not deficient (from a rational, analytic perspective,) and that motivating the taxpayer to increase educational expenditure can only serve to skew relative shares further, we consider in this section, subject to certain assumptions, options for improving the product of U.S. schooling at current levels of expenditure.

Since Sputnik in 1957, many approaches have been employed aimed mostly at marginal change in instructional practice with little salutary effect. Modest efforts have also been made to introduce into the educational system the "carrot and stick" of a competitive marketing structure.

We review these summarily, and then go on to consider the as yet untried, politically and socially sensitive and demanding approach of radical change in the mix of human and physical capital in the schools.

3.1 *Past efforts at schooling improvement*

For some 40 years or more, the U.S. has unsuccessfully endeavored, by almost every measure, to improve public education. Despite an average annual student expenditure greater than that for almost every other OECD nation, U.S. secondary school students consistently score towards the bottom of the rankings among students of industrialized nations in international comparisons. The marginal approach the nation has employed almost exclusively² over the past 40 years, which aimed to improve the curriculum, increase teacher salaries, emphasize school leadership, and reduce class size, has failed.³

Secondary school achievement scores do not meet expectations, with U.S. students scoring no better than in the middle of the country ran-

kings in international tests. Average annual cost per K-12 student increases annually, often faster than inflation. U.S. industry demands a growing number of visas for individuals trained overseas. College and university science and engineering departments are manned by a faculty "foreign legion".

By spending more, perhaps substantially more, on students in the lower tail of the achievement distribution, the nation might successfully reduce the variance in the distribution at secondary school leaving. This approach would increase average student cost; could be expected to improve satisfaction with schooling by students and by the parents of students directly affected; and could be expected to improve the quality of manpower available to certain sectors of the U.S. economy. From historical experience, this policy choice is unlikely at a level of expenditure that could be expected to achieve the desired outcome.

The U.S. taxpayer has demonstrated a persistent unwillingness to increase the financing of education at a rate much faster than inflation. The so-called exception clause to the U.S. constitution leaves responsibility for public education to State and local governments. The federal government has regularly contributed only some 6% to the financing of public education, and no more than a modest set-aside for slow learners in the early grades has been politically feasible, before other constituencies argue volubly for their "fair" share. (For the same reason, greater spending on students in the upper tail of the achievement distribution isn't feasible either, although this approach could be expected to improve the quality of manpower available to value added sectors of the economy and favorably affect the rate of growth of the U.S. economy.)

Perhaps in reaction, incentive approaches like accountability, charter schools and publicly financed private schools, all aimed at mimicking a competitive marketing structure, now have their vocal advocates. Absent a new input to schooling, however, there seems little reason to think that a market approach, which improves curriculum, increases teacher salaries, emphasizes school leadership, and reduces class size, (in addition to possibly aiming to turn a profit), should succeed, when more than 40 years of trials using this approach in some 85,000 schools has failed. The "invisible hand" is not a guaranteed miracle

worker in any and every circumstance.

In this initial exercise, we do not further consider the potential of "marketized" but only marginally different traditional classroom practice alone to improve the outputs of schooling. We consider instead the as yet untried approach of sharply reallocating the ratio of human to physical capital - at the secondary school level - for which charter and voucher schools, because less heavily systematized and bureaucratized, can potentially serve as exploratory venues.

3.2 *An untried approach*

We take up, under certain assumptions, whether any mix of human and physical capital exists, which can improve schooling at a price the taxpayer is prepared to pay.

Unlike the practice of medicine, which has now entered onto a scientific stage of development, education remains a practitioner-based art, with each empirical study of effectiveness inevitably contested or contradicted by another. In order to proceed with this initial analysis, we depend upon widely accepted observations for certain necessary assumptions, including especially the following:

- Student learning is widely believed to be primarily attributable to the classroom teacher, responsible for "baby sitting" and supervision, for student motivation, and for presenting necessary subject matter knowledge. Adults, who have experienced the teaching classroom, often retain a romanticized memory of it. Belief in the educational primacy of the teacher is also supported by examples in the popular culture, including Socrates in the market place, Aristotle tutoring Alexander, and Mark Hopkins at the other end of a log. But the most important question continues to remain empirically undecided: what is the marginal variation in student learning with marginal variation in the quality of teaching. Alternatively, a large research literature finds that knowledge can be transmitted and learned through a variety of channels, with little loss in effectiveness.
- Teachers can, in principle, redirect and reallocate their attention and effort in favor of students in the lower tail of the achievement distribution, or to the so-called weaker or slower students. This would presumably improve the achievement of these students, in some measure, without much affecting the achievement of students in the upper tail of the distribution. In practice, classroom teachers teach to their intuitive assessment of the mean in the achievement distribution of their students, or the so-called "middle."
- Teachers are ill prepared by pre-service training to teach the so-called "difficult" subjects like upper grade mathematics and science education.
- Students learn at different rates and in different ways. Subjects like foreign language instruction, mechanics, mathematics and history demand different mental processes and conceptual models for successful learning, and require different instructional approaches.
- The cost of teachers in the technologically lagging sector of the economy, schooling, increases with time; and will continue to do so. In the "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," American Economic Review, 57, June 1967, William Baumol, explains, "The logic of the entire analysis can be put rather simply in intuitive terms. If productivity growth per man hour rises cumulatively in one sector relative to its rate of growth elsewhere in the economy, while wages rise commensurately in all areas, then relative costs in the non-progressive sectors must inevitably rise, 'and these costs will rise cumulatively and without limit' [emphasis in original]."
- The cost of information technology, hardware and software, tends to decline over time. Hardware is processed sand. The primary input for software is high-priced labor. The declining cost of both hardware and software ultimately depends upon amortizing development, production and distribution costs over a large number of units. Schooling is characterized by large numbers, e.g., some 50 million students and nearly 2.5 million teachers.
- The U.S. has the capacity to produce electronic learning materials responsive to different curriculum demands and student learning

styles. This is mostly a matter of trial-and-error, just as for productivity software, say, Microsoft Word. And, just as in the case of Microsoft Word, development costs should not be trivialized. But with sharply reduced delivery cost thanks to the Internet, and student users across the country numbering some nine to 12 million per grade level in any three-year period, even a development cost of \$500 million computes to a unit cost of only \$50 per software curriculum per student.

We do not presume to design specific mass schooling arrangements, but this chain of credible assumptions suggests the possibility of reallocating schooling resources in a way that could lead to moving the mean in the student achievement distribution "uphill." To illustrate and in order to lend some concreteness to the discussion, we provide (below) the boundary conditions for one possible example of schooling practice for students in grades seven through twelve. In this example, we ignore desired and important outcomes of schooling like psychomotor and athletic skills, affective and social skills, or the long-term benefits of hot lunch. We focus exclusively on cognitive outcomes and mastery of the curriculum.

- A number of highly qualified,⁴ and therefore high-salaried instructional staff is in attendance. These individuals, "teachers by name", although not necessarily classroom teachers in the usual sense, would be psychologically and socially skilled practitioners with deep subject matter knowledge, able to interpret test results and diagnose student learning problems, and to counsel and motivate effective remedial action by students.
- A number of non-instructional, lower-salaried non-instructional staff is in attendance to supervise students, the larger number of whom are at any one time not under the immediate guidance or supervision of one of the instructional staff.
- Students, motivated and counseled to learn, carry out assignments independently and in small groups using sophisticated electronic curriculums. These groups can include an older or more academically able student, who

has already demonstrated a mastery of the material. When in self-acknowledged difficulty or whenever indicated by test results, students are instructed, advised and counseled individually or in small groups by a member of the instructional staff.

Budgets constrain costs. How does this compute? If we consider reducing the size of the present teaching staff, grades seven through twelve, by two-thirds, then we could in principle double the current average teacher salary in those grades, while leaving a significant surplus. Whatever the circumstance, whether studying independently or in small groups, whether in the corridor or the gym, students require supervision. The surplus could go to pay the wages of non-instructional staff for that purpose. If it were thought necessary or desirable to maintain the same staff-to-student ratio as is presently employed in traditional classroom instruction, the surplus would cover the cost of salary and wages for non-teaching staff at one-half the current average teacher salary.

For example, assuming a current average annual teacher salary of \$40,000, then the highly qualified individual we postulate could be paid an annual salary of \$80,000; and the noninstructional staff member \$20,000.

How might this be expected to work in practice? No doubt it should work differently for fact-based vs. concept-based vs. practice-based junior and senior high school subjects. In general however, students would be expected to learn as much as they could independently or in small groups or assisted by other academically more advanced students, using the highest quality of electronic curriculums the nation is capable of producing.⁵ When in difficulty, students would be instructed, advised and counseled by a highly qualified instructor, who at twice the current annual teacher salary, might successfully be recruited from the ranks of college students with a combined SAT score of 1200 and up, as opposed to 900 and down, as is roughly the case presently, on average.

Academically less able students would be expected to take up more of the time of the instructional staff; and academically more able students, less. Whether the numbers postulated here (or something like them) can be made to work is a matter for empirical resolution, as is

the effect of this approach on the achievement distribution of students. Specifically, would all students learn more?

This approach serves to overcome well-known disadvantages of traditional classroom practice. A short list includes:

- Slow learners hold back fast learners.
- Slow learners can feel pressed by too rapid a pace set by fast learners.
- All classroom students can have their focus and concentration disturbed by distracting, often irrelevant, questions and answers shouted into the classroom environment by other classroom students.
- Days spent by students in the classroom with substitute teacher are most often wasted learning days.
- Electronic curriculums used in school could also be used to extend and reinforce learning outside of school, e.g., at home.
- Instructional staff could be kept updated on findings from research and practice with electronic training materials.

This schematic approach leaves many important questions unanswered. A short list includes:

- The definition of the high-quality teacher as used here is empirically unknown.
- The thrust of this approach is to advance the mean in the achievement distribution "uphill." Even if successful, the effect on the variance in the distribution remains to be learned and adjusted empirically. An increase in variance could be unacceptable politically and socially, even if academically weaker students were found to be learning more than previously.
- We provide no transition strategy for getting "from here to there." Much trial-and-error and small-scale experimentation would be required. During such a trial period with only a modest-sized market, there is little motivation for private sector firms to develop the electronic curriculums and instructional staff training materials required to satisfy the demands of this approach.

4. CONCLUDING REMARKS

We conclude with some remarks on the absence of a rational scheme for deciding the allocation of funds for schooling suggested by the marked discrepancy found in expenditures between the two nations, the U.S. and The Netherlands, and the possible misallocation of funds for schooling in the U.S.

First, any allocation approach based on the national averaging of large numbers of heterogeneous elements will inevitably be flawed, locally and regionally. In the case of schooling, manpower pools from which qualified applicants for instructional and non-instructional positions are chosen can vary considerably by region; taxing capacity for schooling can vary sharply; and parents can place different values on the various outcomes of schooling. A convincing analysis depends upon the extension of output measures beyond the combination of unsatisfactory proxies and narrowly defined scholastic measures now in common use; and an accounting for the substantial regional differences within the U.S.

Second, politics and cultural attitudes can successfully prevent empirical trials of improvement approaches suggested by rational analysis, for example, the attitude in the U.S. that what's worth trying with a sample of the school population should be good enough to be used with the entire school population, immediately and simultaneously. Less bureaucratized charter, publicly voucherized and independent schools can play a work-around role for this attitude.

Third, there can exist a curve of rising growth in domestic product as the productivity and efficiency of schooling is improved, and as students in the upper tail of the distribution are given improved opportunity and resources to increase their learning and achievement, and to move on more quickly and more ably to higher education and work. There can also exist a curve of rising satisfaction among students in the lower tail of the achievement distribution as they learn more and advance in life, and realize a higher standard of living due to national economic growth and perhaps to the application of a measure of distributive justice in the nation's tax code. Whether the improvement approach described here and others like it are economically and socially optimal from a policy perspective can only be derived from an explicit choice model like the one

illustrated here. Whether they are politically and socially acceptable is another matter.

ENDNOTES

¹In this exploratory paper, we give only slight consideration to approaches that depend exclusively upon changes in management and organization for reasons we discuss in the text.

²Almost exclusively, but not altogether. Efforts at management and organizational restructuring include experiments with performance contracting and education vouchers. Community control and the establishment of alternative schools inside and outside the formally recognized system represent other efforts aimed at making education more responsive to particular client groups.

³There is dispute over this point. While finely tuned measures for specific cognitive skills like basic reading and arithmetic achievement exist, adequate assessment techniques for measuring attainment of complex academic skills and affective and social development outcomes of schooling are lacking. Proponents for higher educational expenditure and traditional classroom practice can point to some success in 4th grade international test comparisons, where the U.S. has made it to the center of the pack. On the other hand, conclusions stemming from such surveys as Coleman's Equality of Educational Opportunity study and its various analyses have been foreshadowed in the educational literature since the 1920s, generally showing that spending more money, adding graduate courses and degrees to teacher training, or reducing studentteacher ratios produce statistically insignificant differences in student achievement. And there can be little doubt about the failure of upper grade instruction, where subject matter knowledge by teachers counts for more. In the 8th grade international test comparisons, U.S. students score at the bottom of the pack of industrialized nations.

⁴True, we know little of what the training requirements for such an individual might be or whether the nation's institutions for pre-service training of teachers could master the necessary effort and discipline.

⁵We do not address here the issue of whether the market can produce the highest quality electronic curriculums of which the nation is capable without a suitably structured public incentive system, particularly during a transition period when only a small fraction of the nation's schools are prepared to experiment with this approach and the market over which development costs can be amortized is small.

REFERENCES

- Burg, H.G. van der, and Paelinck, J.H.P., 1993, *Structure et simulation d'un modèle d'équilibre général calculable axé sur les activités médicales (MEGICAL)*, in J.-P. Gallet (editor), *Actes des premières journées françaises de métronomie*, A.E.S.V.L. Chinon, pp.85-92.
- O.E.C.D., 2000a, *National Accounts of OECD countries, Detailed Tables, Volume II, 1988-1998*, Paris.
- O.E.C.D., 2000b, *Education at a glance, OECD Database 2000*, CD-ROM, Paris.
- O.E.C.D., 2000c, *OECD Health Data 2000, A Comparative Analysis of 29 Countries*, CD-ROM, Paris.
- Seskin, E.P. and Sullivan, D.F., 2000, *Annual Revision of the National Income and Production Accounts, Survey of Current Business*, January 2000, pp.6-33.

A.1. KECH Model

Using the extended Harrod-Domar model, growth of product y is governed by the expression:

$$y_t = y_0 \exp[(\alpha k + \beta e + \gamma h)t], \quad (1)$$

the rate of growth being governed by the term between parentheses, and this expression in being turn made up of the contributions of physical capital investment, and investments in education and in health care.

Taking the logarithm of the Cobb-Douglas function discounted over an infinite horizon at rate δ yields the preference function:

$$\varphi = \ln y_0 + \int_0^{\infty} (\rho c + \sigma e + \tau h)(\alpha k + \beta e + \gamma h)t \exp(-\delta t) dt, \quad (2)$$

The preference elasticities are ρ (for consumption), σ (for education) and τ (for health care), expressing in this manner the relative preferences for each of those three categories of social choice:

- c : the share of consumption in domestic product;
- e : the share of educational expenditure; and,
- h : the share of health care expenditure; and

Without going into the details of the mathematical reasoning to follow, we only enumerate the different steps. Integrating (2) and then deriving with respect to k , e and h , taking into account that :

$$c + k + e + h = 1 \quad (3)$$

(the shares of consumption and the three types of investment add up to 100%), one obtains the following system of equations :

$$(2\alpha\rho)k + (\alpha\rho - \alpha\sigma + \beta\rho)e + (\alpha\rho - \alpha\tau + \gamma\rho)h \geq \alpha\rho \quad (4)$$

$$(\alpha\rho - \alpha\sigma + \beta\rho)k + 2\beta(\rho - \sigma)e + [(\beta(\rho - \tau) + \gamma(\rho - \sigma))]h \geq \beta\rho \quad (5)$$

$$(\alpha\rho - \alpha\tau + \gamma\rho)k + [\gamma(\rho - \sigma) + \beta(\rho - \tau)]e + 2\gamma(\rho - \tau)h \geq \gamma\rho \quad (6)$$

Given the complexity of this system of inequalities – inequalities resulting from the conditions c , k , e , $h \geq 0$ – and the difficulty of realizing an analytical (i.e. explicit) solution for α , β and γ , the model has been simulated.

A.2. *Simulation values*

The following parameter values have provisionally been chosen to highlight the workings of the model:

α : .5
 β : .2
 γ : .1
 ρ : .5
 σ : .1
 τ : .4

The result of solving the model is : $c=.27$, $k=.44$, $e=0$, $h=.29$. In this case, the share of growth expenditure allocated to education is shown to be 0, due to assumptions on the average efficiency of educational expenditure promoting growth (β) and to the relative low preference for education (σ).

If with respect to its initial parameter value, .2, β is increased to .4, all variables take the value .25; and continuing, if with respect to its initial parameter value of 0.1, σ is increased to 0.3, the result becomes : $c=.12$, $k=.33$, $e=.28$, $h=.27$.

In fact, these values, chosen for illustrative purposes, are far removed from the observed statistical ones; and, in order to bring the application of the model closer to reality, recent figures for the U.S. have been substituted in Table 1, Section 2.2 of the text.

A.3. *Some caveats and conclusions*

The following points will receive specific attention in the next stage of this work:

- a. One should try to obtain econometric (i.e. measured) estimates of α , β and γ ; to the knowledge of the authors, such values have not yet been computed;
- b. One should be aware of the importance of the role (implicit or explicit) the social choice parameters ρ , σ and τ play in allocating the available resources to their possible uses; and,
- c. One can observe that the results depend on the *joint* impact of technical - α , β , γ - and social choice - ρ , σ , τ - parameters;
- d. One should consider that technical possibilities and social preferences influence each other; and this calls for a study of the inter-linkages between parameters α , β , γ , ρ , σ , and τ .

From the figures in Table 1, and once the coefficients α , β and γ of equation (1) have been computed, the preference coefficients in that equation can also be computed from equations (4), (5) and (6) (Appendix). As stated above, these coefficients together determine the optimal allocation of domestic product among the four uses identified there, i.e., consumption, investment in physical capital and expenditures for education and health care.

A.4. Possible extensions to the model

The first extension to be envisaged is to distinguish in the preference function between technical education, e_1 , and general and "humanistic" education, e_2 . Again, for the sake of initial simplicity, no distinction is made between "levels" of education (primary, high school level, university).

The second possible extension is to render the educational efficiency parameter in the growth function, β , some "production function" of e_1 and k_1 , the latter being the share devoted to "technical training capital" (e.g., computers, business methods, and so on), e_1 being now also considered as investment in "technical teaching capital":

$$\beta = \beta(e_1, k_1) \quad (7)$$

and in the same vein the preference coefficient for general and humanistic education, now σ_2 , could be made a function of e_2 and k_2 , both this teaching and technical capital being devoted to the type of education mentioned; this idea ties up with parents' and students' satisfactions mentioned earlier.