On Medical Activity

An Essay in Theoretical Medicometrics

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ABSTRACT

As an interdisciplinary field of study, "medicometrics" relates health problems to other disciplines, and in particular to economics and econometrics.

The present study, a theoretical one - for one wants a firms theoretlcal foundation for interdisciplinary studies -, starts with the derivation of a macroeconomic production function, incorporating explicitly medical activities, and medical and other externalities; an optimal level and share of medical activities are derived. Relative prices of medical and other activities are computed, and in those terms – and in terms of health insurance – consumer behaviour is studied, introducing to a trust capital approach, to be dynamised later on.

Dynamics is first introduced via a generalised Harrod-Domar model, and medical activity is optimised over an infinite horizon; as health care is a much discussed issue, a dynamic model on opinion building and diffusion is presented.

A last point taken up is one of the spatial dimensions of the problem; a multiregional generalisation of some previous results is derived.

Conclusions refer to further research topics.

KEY WORDS : medicometrics; model; health; theory.

1. INTRODUCTION

Since the early eighties, studies in *"regional medicometrics"* (for a definition, see [PAE 84]) have been regularly undertaken and published.

It is well known that throughout the world, health care expenses represent an important fraction of national products (see {CAS 90] and [CAS 92]); judgments about the efficiency, possibly the optimality of such outlays, can only be based on a model integrating medical activities in a larger framework of overall economic activities.

Section 2 hereafter presents such a model; it allows of computing a one-period optimum for medical activities, and, as an interesting by-product, their relative price level compared to other productive activities; health insurance effects are considered, and a trust capital approach is investigated. Section 3 then goes on integrating this model in a longer term perspective by optimising over an infinite horizon, dynamising the trust capital model, and presenting the possible dynamics of health care opinion diffusion. Section 4 introduces the spatial element.

Conclusions and references follow.

2. STATICS

2.1. Basic model

Start from the following equations for a closed economy:

$$q \equiv s + r \qquad (1)$$

q being national product, , *s* "medical product", and *r* the product of other economic activities; input-output relations could be introduced but would not fundamentally change the reasoning.

$$q = p^*\pi \qquad (2)$$

Here p^* is (active) population, π average productivity (per person or worker); exogenous magnitudes are starred.

$$\pi = \sigma^*s - \tau^*s^2 + \omega^* - \rho^*r - \xi^*s \qquad (3)$$

Equation (3) pictures how medical activities contribute to maintain (and over time to increase; Cimetidine is only one example) the productivity level (with a decreasing marginal intensity), ω^* representing the other productivity factors ; the last terms stand for the average negative influence of the other activities ("negative externalities", possibly of a "medical nature": road and workplace accidents, certain excessive consumptions,...) and also for negative medical externalities (medical errors, medicinal diseases,...); of course, in the long run investment and medical activities will increase productivity, but here we stick to a static one period model (for its dynamic extension, see section 3).

Substituting (3) into (2), and using (1), one obtains the following relation between *q* and *s*:

$$q = (1+ p^*\rho^*)^{-1}p^*[\omega^* + (\sigma^* + \rho^* - \xi^*)s - \tau^*s^2] \quad (4)$$

Deriving with respect to *s*, equating to zero and neglecting the *1* in the denominator, supposed to be negligible compared to its twin term, one obtains:

$$s^o = (2\tau^*)^{-1}(\sigma^* + \rho^* - \xi^*) \stackrel{\Delta}{=} (2\tau^*)^{-1}\kappa^* \quad (5)$$

"medical production" being a direct function of its marginal contribution to productivity and of the coefficient of the general negative externalities, and further negatively related to the medical externalities, and inversely to the parameter measuring the decrease in its marginal contribution; this result is entirely logical.

The ratio s^o/q^o can be computed as:

$$s^o/q^o = \rho^*[2\omega^*\tau^*\kappa^{*-1} + 1/2(\kappa^*)]^{-1} \quad (6)$$

which allows of concluding that an increase in τ^* and ω^* have a decreasing effect on that ratio, while an increase in σ^* and ρ^* lead to its increase (decrease as far as ξ^* is concerned), at least for $\kappa^* < 4\omega^*\tau^*$; these results again are logical.

One can also think of maximising a weighted sum of *r* (coefficient α^*) and *s* ("health for health"; coefficient $1-\alpha^*$); the result is that in (5) ρ^* has to be replaced by $[1 - (2\alpha^*-1)/\alpha^*]\rho^*$, but the results in comparative statics remain entirely valid. If then $\alpha^* \to 0$, one will in (5) have a dominance of ρ^*, reducing (6) to the term $[2\omega^*\tau^* + ½]^{-1}$; for $\alpha^*= ½$, solution (6) prevails (logical again, as *s* and *r* are equally valued), and for $\alpha^*=1$ the term ρ^* disappears from the denominator of (6).

2.2. *Pricing*

Pricing according to marginal productivity would lead to zero pricing of medical activities [see equation (5) and the rationale for its derivation], so an appropriate demand equation should be introduced next to the national income identity at (relative) current prices (the price of total production being set equal to one):

$$q \equiv p_s s + p_r r \quad (7)$$

The demand equation will be derived from a Minkowski-type utility function [constant elasticity of substitution : $(1-\theta)^{-1}$]:

$$u = (\lambda\, s^\theta + \mu\, r^\theta)^{1/\theta} \quad (8)$$

leading to a relative price function :

$$p_s/p_r = \lambda/\mu\, (r/s)^{1-\theta} \quad (9)$$

implying $\theta<1$. For an optimal s^o/q^o-ratio, noted v^o, this leads to:

$$p_s = [(1-v^o)^{-1}(\lambda/\mu)(1-v^o/v^o)^{1-\theta}][1+(\lambda/\mu)(v^o/1-v^o)^\theta]^{-1} \quad (10)$$

for v^o having values according to the results of section 2.1. It can be shown that the higher the ratio λ/μ (preference ratio for *s* and *r*) and the lower v^o, the higher p_s.

2.3. *Health insurance effects*

As health insurance is a much debated issue, the following developments are presented.

Suppose a representative consumer to have a utility function like (8), but with a quadratic term for *s*:

$$u = [\lambda(s - ½\,\gamma\, s^2)^\theta + \mu\, r^\theta)^{1/\theta} \quad (11)$$

In case of direct consumer pricing for *s*, the traditional neoclassical condition $u'_s/u'_r = p_s/p_r$ would hold; if however a lump-sum medical insurance is paid (say *m*, possibly legally imposed), the consumer would prefer to maximise his medical consumption up to $s = \gamma^{-1}$, and to consume $(y-m)/p_r$ of *r*, where *y-m* is his residual income.

This could start a dynamic process along the following lines; define $\gamma^{-1}=\delta$, then a possible path could be generated by:

$$\delta_\tau - \delta_{\tau-1} = \zeta\, m_{\tau-1} + \delta* \quad (12a)$$

$$m_\tau - m_{\tau-1} = \eta\, \delta_{\tau-1} + m* \quad (12b)$$

Computing the eigenvalues β from the transition matrix:

(13)

one finds $\beta = 1 \pm\sqrt{\zeta\eta}$, giving an asymptotic positive growth rate for *m* of $\sqrt{\zeta\eta}$, hence the search for controlling mechanisms accompanied by declarations of patients' rights. Formally a system like (12a)-(12b) is controllable (see [KAA 98]), but the point here is to give that control a specific content.

2.4. *A trust capital approach*

A way of looking at the latter problem might be along the lines of trust capital ([HAR 97]).

Define:

x,y: activity levels of each of two agents;

z: total activity of the two agents;

and consider the following equations :

$$x + y \equiv z \quad (14)$$

an identity,

$$x = a\,y \quad (15)$$

a (potential) input relation between *x* and *y* (*x* being a technical input to activity *y*), and:

(16)

with $\alpha + \beta < 1$ (usual diseconomies of scale to guarantee convexity); this is the "trust relation" between the two agents ([HAR 97], p.3 : "trust in human interaction") with "trust-parameter γ.

Substituting (15) into (14) and equating to (16) gives:

$$(1 + a)\, y = \gamma\, (ay)^\alpha y^\beta \quad (17a)$$

(17b)

so:

$$y^{1-\alpha-\beta} = \gamma\, a^\alpha\, (1 + a)^{-1} \quad (18)$$

and :

(19)

It is obvious that in the absence of trust, activity *y* (and thus *x*) does not materialise; on the other hand, activity *y* (and thus *x*, and also total activity *z*) increases with γ, the "degree of mutual trust".

The model can be readily expanded to the multi-agent case.

Equation (15) now turns into:

$$\mathbf{q} = \mathbf{A\,q} + \mathbf{f} \quad (20)$$

the well-known input-output specification with:

q = vector of activity levels;

f = vector of final deliveries.

The following system of equations generalises two-agent equation (16); the *'s represent powers smaller than 1, with sums also smaller than 1 (see the remark above on decreasing returns). Matrix Γ and diagonal matrix $\hat{\gamma}$ generalise scalar coefficient γ.

$$\mathbf{q} = \hat{\mathbf{q}}^* \Gamma\, \hat{\mathbf{q}}^* + \hat{\mathbf{q}}^* \hat{\gamma}\, \mathbf{f}^* \quad (21)$$

Write now (21) as:

$$\mathbf{i} = \mathbf{q}\, \mathbf{q}^{*-1} \Gamma\, \hat{\mathbf{q}}^{*-1} \mathbf{q} + \hat{\mathbf{q}}\, \hat{\mathbf{q}}^{*-1} \gamma\, \mathbf{f}^{*-1} \mathbf{f} \quad (22)$$

i being the unit column vector; substituting from (20), there comes:

$$\mathbf{i} = [\, \Gamma^* (\mathbf{I} - \mathbf{A})^{-1} + \gamma^*]\, \mathbf{f} \quad (23)$$

Γ^* and γ^* being the respective *-corrected matrices; so:

$$\mathbf{f} = [\, \Gamma^* (\mathbf{I} - \mathbf{A})^{-1} + \gamma^*]^{-1}\, \mathbf{i} \quad (24)$$

and:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1} [\, \Gamma^* (\mathbf{I} - \mathbf{A})^{-1} + \gamma\,]^{-1}\, \mathbf{I} \quad (25)$$

The decreasing returns effect being taken up as said, larger Γ- and γ-values can be proven to lead to higher production and final demand levels.

The dynamics of this approach is taken up in section 3.2.

3. DYNAMICS

3.1. Infinite horizon model

Suppose now the objective function to be maximised, and referred to in section 2.1, to be the following:

$$\varphi = q_o(v^o) \int_o^{\infty} \exp[\chi v^o + \varepsilon (1-v^o) - \phi]\, t\, dt \qquad (26)$$

which leads to:

$$\ln \varphi = \ln q_o(v^o) - \ln[\phi - \varepsilon - (\chi - \varepsilon)v^o] \qquad (27)$$

for the expression between brackets in the exponential of (26) being negative. That term between brackets is the discounted (rate ϕ) constant growth rate of production, a generalisation of the Harrod-Domar model (in which only the savings rate and the capital coefficient appear).

Consider the second term of the right hand side of (27); as $0<\phi<1$, the expression between brackets must also have the same bounds, so the logarithm is negative. If $\chi-\varepsilon>0$, $v^o=1$ maximises the negative logarithmic expression, and vice-versa for $\chi-\varepsilon<0$ for which $v^o=0$ is a maximiser. If now $0<v^o<1$ is a maximiser of the first term on the right hand side of (27) (see section 2.1), an upward or downward correction should be applied, according to the two cases distinguished above; the correction finally depends on the relative growth efficiencies of medical and other activities in the total production process.

3.2. Trust capital dynamics

The model presented will again be macro-economic; its first equation:

$$q = a\, q + f \qquad (28)$$

is a macroeconomic version of (20);

$$q = \gamma\, q^{2\alpha} + \gamma^*\, f^{\alpha} q^{\alpha} \qquad (29)$$

is a macro-version of (21);

$$\dot{q} \overset{\Delta}{=} \partial q/\partial t = \sigma\, \kappa^{-1} f \qquad (30)$$

is an absolute growth equation, in which s is the savings rate, and k the gross marginal capital coefficient;

(31)

this equation stating the dependence of the savings ratio on trust.

To simplify the solution, the "trust-elasticities" have all been fixed at $0 < \alpha < .5$, but the more general specification of (21) could also be applied here.

Solving (28) and (29) gives:

$$q = (1-a)\, [\gamma\, (1-a)^{1-2\alpha} + \gamma^*(1-a)^{1-\alpha}]^{1/(1-2\alpha)} \qquad (32)$$

and combining (28) with (30)-(31) gives :

(33)

hence:

$$\hat{r} = \dot{q}/q = \kappa^{-1}\delta(1-a)^{1-\alpha}[*]^{-1} \qquad (34)$$

where the expression between square brackets is that of (32). So growth depends immediately on the savings-investment "trust-parameter" δ.

3.3. Opinion diffusion dynamics

Health care policies being intensively discussed, it has appeared interesting to develop an opinion diffusion model; a fuzzy probability approach is pursued here, first in a linear, then in a non-linear (Lotka-Volterra type) setting.

Let $x_{i,t}$ be the opinion of agent i at time t ; $x_{i,t} \in [-1,+1]$; a value $+1$ means a strong positive opinion, a value -1 a strong negative opinion, a value 0 indifference towards the problem posed ("no opinion").

Let the dynamics of agent i be ruled by the following equation:

(35)

where p_{ij} is the probabilty of i meeting j (it could be time-dependent in a more complex model; one will come back to that), and a_{ij} the degree of influence of j on i ($a_{ij} \in [0,1]$); $p_{ii}=1$, and possibly $a_{ii} \neq 1$(individual i might revise his opinion). Generalising to n agents, and using vector-matrix notation, there comes:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \overset{\Delta}{\mathbf{B}}\mathbf{x}_{t-1} = \mathbf{C}\mathbf{x}_{t-1} \tag{36}$$

which leads to

(37)

where $\mathbf{x}_0^*$ is the vector of (given) initial opinions.

The evolution of (37) depends on the vector of eigenvalues of $\mathbf{C}$, λ; if $-\mathbf{i}<\lambda<\mathbf{i}$, then the process will converge to $\mathbf{0}$, an "amorphous" situation; some eigenvalues could have the value 1, and together with the corresponging elements of $\mathbf{L}$, the matrix of eigenvectors of $\mathbf{C}$, they will determine the asymptotic state of the system; as values of $\mathbf{x}_t$ have to remain bounded, a diagonal scaling matrix, $\hat{\mathbf{s}}$, could premultiply $\mathbf{C}$.

An indicator of the stability of opinions might be $s=n^{-1}n_u$, where n_u is the number of unit roots of $\mathbf{C}$; indeed, if opinions are perfectly stable and uninfluenceable, then $\mathbf{C} \equiv \mathbf{I}$, which has only unit roots, so $s=1$; values of $s<1$ denote a certain degree of "volatility" of opinions. If some eigenvalues are complex, opinions might "fluctuate".

As said before, probabilities (maybe influence coefficients) could be time dependent, so each $\mathbf{C}$ and $\mathbf{s}$ should be given an index t, and the final state computed as:

(38)

Whatever the specification [(37) or (38)], one could compute an "average" opinion as [e.g. from (38)]:

$$\mu(\mathbf{x}_t) = n^{-1}\,[\mathbf{i}'\mathbf{x}_0^* + \mathbf{i}'(\Pi_t \mathbf{s}_t \mathbf{C}_t - \mathbf{I})\mathbf{x}_0^*] = \mu_0 + \Delta_{0,t}\mu \tag{39}$$

the expression for the change in m being possibly zero, leading to stability of average opinions.

Let now $\mathbf{y}$ be defined as:

$$\mathbf{0} < \mathbf{y} \overset{\Delta}{=} \tfrac{1}{2}\,(\mathbf{x} + \mathbf{i}) < \mathbf{i} \tag{40}$$

and its dynamic process as:

$$\dot{\mathbf{y}} = \hat{\mathbf{B}}\mathbf{y}\,(\mathbf{i} - \mathbf{y}) \tag{41}$$

which is a vector-matrix generalisation of the well-known logistic (scalar) curve, and as such a special case of a Lotka-Volterra process.

As the elements of $\mathbf{B}$ are constrained (see the linear model above), "shattering" around $\mathbf{0}$ or $\mathbf{i}$ will probably be absent; furthermore, no $\hat{\mathbf{s}}$-correction is then necessary as $\mathbf{y}$ is constrained by (40).

An interesting point is that $\mathbf{0}$ and $\mathbf{i}$ are not the only (asymptotic) singular points; indeed, the possibility of:

$$\hat{\mathbf{B}}\,(\mathbf{y}\text{-}\mathbf{y}\mathbf{y}) = \mathbf{0} \tag{42}$$

exists, even as a multiplicity, so stable intermediate opinion pattems can exist.

4. SPATIAL DIMENSION

Let us write the production function for region i as :

$$q_i = q_i\,(s_i,r_i) \tag{43}$$

and the function for the reference space as :

$$q = q[q_1(s_1,r_1),\ldots,q_I(s_I,r_I)] \tag{44}$$

A static preference function can now be specified as:

$$\varphi = \omega q + \Sigma_i \omega_i q_i \tag{45}$$

with :

$$\omega + \Sigma_i \omega_i = 1 \tag{46}$$

Static optimality conditions w.r.t medical activities can then be written, i, as :

$$\varphi_i' = \omega\, q_i'\, q_{isi}' + \omega_i\, q_{isi}' \tag{47a}$$

$$= (\omega\, q_i' + \omega_i)\, q_{isi}' \tag{47b}$$

which takes into account general reference area welfare and local welfare in terms of incomes; an extension along the lines of section 2.1, in fine, is immediate.

5. CONCLUSIONS

It has been shown that an analytical reflexion on the role of a specific activity – in casu, the medical one – allows of clarifying its relative importance in a global production-cum-consumption process.

The following step is to try and identify the parameters of the process; some headway has been made in previous studies ([BUR 93] and [BUR 98]).

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