Varhaiskasvatuksen Tiedelehti
Journal of Early Childhood Education Research
Volume 9, Issue 2, 2020, 420–436

VERTAISARVIOITU
KOLLEGIALT GRANSKAD
PEER-REVIEWED
www.tsv.fi/tunnus

An investigation of Swedish pre-service preschool teachers' knowledge about the negative numbers

Timo Tossavainen[a], Maria Edholm[b], Ewa-Charlotte Faarinen[b] & Maria Lundkvist[b]

[a] *Lulea University of Technology, corresponding author, e-mail: timo.tossavainen@ltu.se*
[b] *Lulea University of Technology*

ABSTRACT: In this paper, we examine the Swedish pre-service preschool teachers' knowledge about the negative numbers, which is one of the central content areas in their compulsory mathematics course. Our results show that almost a half of the pre-service preschool teachers can give a reasonable definition for the negative numbers, but only a few of them set the negative numbers in relation to the other common number sets. The participants' performance in a set of mathematical tasks related to the negative numbers follows the qualitative variance in their definitions of the negative numbers, yet this relation is not completely direct; there were also such participants who were not able to give a definition but succeeded well in the tasks. Those who gave a vague definition performed weaker than the others. Although the participants' knowledge about the negative numbers is, perhaps, not satisfactory with every respect, it does not significantly differ from that of a control group that consists of (non-mathematician) university teachers from their study programme.

Keywords: *mathematical concepts, numbers, pre-service preschool teacher*

Introduction

In Sweden, the discussion about what teaching in preschool is or should be has been going on for years. The debate has become even more actual after the revisions of the national curriculum in the 2010s which have been interpreted to imply a shift from play-based

 Peer-review under responsibility of the editorial board of the journal ISSN 2323-7414; ISSN-L 2323-7414 online

education to more teaching oriented activities. For example, the 2018 revision (Skolverket, 2018) says that preschool mathematics education must give a child a possibility to use mathematics for exploring and describing his/her surrounding world and solving everyday problems. This includes, among other things, that every child must be given prerequisites to develop understanding about space, time, different shapes, the fundamental properties of sets, patterns, amount, order, number, measurement, and change, and further, an ability to reason mathematically on these notions (Skolverket, p. 14). These goals can be considered very demanding, yet the interpretation of the extent to which they should be covered in preschool is left completely to teachers. No concrete examples are given and this is one of the main reasons for the continuous debate.

Not surprisingly, the teaching of mathematics to young children has been discussed in several research articles in Sweden and elsewhere. Palmér and Björklund (2016) have surveyed them and found a large diversity of aims within the context of preschool mathematics; there are similar finding also from Finland (Björklund, 2015). Swedish pre-service preschool teachers' views of the means and goals of mathematics education in preschool are as well varying. Tossavainen, Johansson, Faarinen, Klisinska, and Tossavainen (2018) and Johansson, Tossavainen, Faarinen, and Tossavainen (2020) noted that the Swedish student teachers want to include quite large content areas in preschool mathematics education. Further, students have a rather positive conception of using digital technology, but they also emphasise that learning mathematics should be fun.

In addition to the indefinite description of the content of preschool mathematics education in the national curriculum, there are also other reasons for the variety of opinions in Sweden. One element in the public discussion seems to be a relatively low common trust in the preschool teachers' mathematical abilities to conduct the teaching of mathematics successfully (cf. Ekenryd, 2016, March). The suspicious beliefs may be rooted in a low appreciation of a teacher's profession in Sweden in general. Further, when measured in terms of study credits, preschool teachers' mathematical education usually covers only a half (15 ECTS) of that of primary teachers (30 ECTS). And in spite of the demanding goals in the national curriculum, in most universities, their mathematics course focuses less on formal mathematics than courses in other teacher programmes.

The aim of the present paper is to provide some evidence-based facts to contribute to this debate in the Nordic context. Even though some differences exist between the preschool teacher education programmes in the Nordic countries, the challenges related to mathematics education are more common than diversified (cf. Einarsdottir, 2013). For example, all Nordic countries have put an emphasis on strengthening the theoretical foundation of education in general and moving it to a higher educational level (ibid., p.

307). Hence, we believe that the findings from a group of Swedish pre-service preschool teachers are interesting and useful also to the readership in Finland and other Nordic countries as they shed some light on the quality of these students' mathematical content knowledge (Shulman, 1987).

Students' mathematical content knowledge can be studied in several ways. For example, by surveying the students' performance simultaneously in several mathematical knowledge areas as is done in the PISA surveys (OECD, 2020), or by focusing on the examination of understanding about a more restricted topic such as a single concept or an activity (e.g., Tossavainen, Suomalainen, & Mäkäläinen, 2017). The former approach aims at giving an overview of how well students perform in general in mathematics, the latter at a more sophisticated view of the variation in students' knowledge about a learning object. In addition to that, the latter approach also provides valuable information about students' mathematical thinking and abilities to apply and generalise their mathematical knowledge.

Our study is of the latter type. We focus on pre-service preschool teachers' knowledge about the negative numbers, i.e., the real numbers that are less than zero. These include also the negative integers and the negative rational numbers. We motivate this choice by the above discussion about the Swedish national curriculum for preschool. The negative numbers play an essential role in many of the mentioned content areas. For instance, they are needed for discussions about temperature. And even though one can speak of order or the decreased amount to children without explicitly mentioning any negative numbers, the national curriculum and its most recent revision clearly indicate that the negative numbers should be included in every Swedish preschool teacher's mathematical content knowledge (Skolverket, 2018). The negative numbers constitute also an essential part of the compulsory mathematics course in the preschool teacher programmes in the Swedish universities.

We formulate our research questions in terms of the theory of concept images and concept definitions (Tall & Vinner, 1981). This theory depicts a mathematical concept, on the one hand, as an individual's total cognitive structure that is associated with the concept (image) and, on the other hand, more explicitly as a verbal expression that is used to specify the concept (definition). More detailed introduction will be provided in the Theoretical framework.

1. What kind of concept definitions do pre-service preschool teachers have of the negative numbers?

2. How are these definitions related to pre-service preschool teachers' performance on the negative numbers?

3. How does pre-service preschool teachers' knowledge about the negative numbers relate to their non-mathematician educators' knowledge about these numbers?

Quite obviously, we can answer the last question only in a suggestive way. We contrast the participating pre-service preschool teachers' responses to our questionnaire with the responses given by a randomly selected group of their non-mathematician teacher educators.

Theoretical framework

Shulman's (1987) classical article discusses the sources of the knowledge base for teaching. It distinguishes between particular kinds of content knowledge and pedagogical strategies. A central idea behind the notion of pedagogical content knowledge is that content and pedagogy must come together in order to provide for effective teaching. More precisely, since content and pedagogy are interrelated, powerful teaching is a result of the recognition of this interrelation (Segall, 2004). An obvious prerequisite for that is that a teacher must have a good understanding about both content and relevant pedagogical approaches.

A teacher's content knowledge is knowledge about the content of the subject(s) to be taught. In mathematics, this knowledge can be divided into components in many ways. The most common approaches acknowledge, at least, knowledge about mathematical concepts and knowledge about mathematical procedures, i.e., how to use concepts and their properties to solve mathematical problems (Hill, Rowan, & Ball, 2005). A slightly different division is given by Hiebert and Lefevre (1986): They defined the *conceptual knowledge* of mathematics as knowledge that is rich in relationships, whereas the *procedural knowledge* of mathematics covers, e.g., rules and procedures for solving mathematical problems and familiarity with how to use mathematical symbols.

Tall's and Vinner's (1981) theory of concept definitions and concept images describes two approaches to understanding about mathematical concepts. First, a mathematical concept has always a formal *definition* that can be expressed verbally. These definitions are to be found in textbooks etc., but an individual can also have a private definition, his/her own verbalization of a mathematical notion. Second, an individual always constructs also a mental model, an *image* of a mathematical concept. This cognitive structure consists of examples of tasks, memories, and interpretations of situations where the individual has used the concept. As the image is cumulative and contains elements from very different kinds of situations, it does not need (or even cannot) be coherent. In new situations, where the applying of the concept is assumed, only a part of the image is provoked to an individual's mind. Therefore, it is possible that an image can contain contradictory elements and, especially, it may conflict with the definition.

In this study, we especially investigate pre-service preschool teachers' concept definitions but examine also their images of the negative numbers.

Review of literature

It seems that there are no previous studies that directly or primarily investigate the Nordic (pre-service) preschool teachers' conceptual knowledge about the negative numbers. Kilhamn's (2011) dissertation is a longitudinal study on how Swedish students' understanding about the negative numbers evolves in the transition from primary to lower secondary school. This study shows a large variance in students' concept images of the negative numbers. When it comes to the present study, Kilhamn's (2011) thesis is most relevant to us in explaining the challenges that compulsory school students meet when studying the negative numbers since, most probably, the challenges are the same also for all learners including pre-service preschool teachers.

According to Kilhamn (2011), the negative numbers cannot be understood as long as the mathematical truths must be proved physically or in a concrete manner. Some properties of the negative numbers are the result of the integration of numbers into an algebraic structure and, therefore, they need to be proved by mathematical reasoning. If a learner uses only a metaphorical reasoning, it is more likely that he/she results in an incorrect answer. For example, when Kilhamn (2009) gave 99 student teachers the task $(-3) - (-8)$, thirty percent of them could not solve it. It turned out that all those students who used a metaphorical reasoning such as "it's minus 3 degrees and then it gets 8 degrees cooler" came up with an incorrect answer. Consequently, Kilhamn (2011, 1) warns of risks in using too informal language in teaching about the negative numbers: "*Although metaphors initially help students to make sense of negative numbers, extended and inconsistent metaphors can create confusion. This suggests that the goal to give metaphorical meaning to specific tasks with negative numbers can be counteractive to the transition from intuitive to formal mathematics.*"

Similar results have been reported outside the Nordic context, too. For example, Chrysostomou and Mousoulides (2010) studied pre-service primary teachers' knowledge about the negative numbers in Cyprus. They noticed that students build on process-based explanations such as rules and memorization and lack conceptual understandings about the negative numbers. They also concluded that students' limited knowledge about the negative numbers "*created even more difficulties on their pedagogical content knowledge and prevented them from being able to realize what was actually needed for successfully teaching the negative numbers* (Chrysostomou & Mousoulides, 2010, 272)". According to Widjaja, Stacey and Steinle (2011), there are, at least, two different sources for incorrect perception of negative (decimal) numbers. First, some students' concept image of these numbers bases on the use of separate rays for positive and negative numbers, but they arrange them so that they have the same orientation. Misconceptions occur when a student joins parts of these rays in a point whose placement varies depending on the given task. Second, some students construct the image of the negative number line by merging translated positive intervals $[0, 1], [1, 2], \ldots$ as puzzle bits on left-hand side of zero. This leads to the correct order of the intervals but a wrong order of numbers within the intervals.

As a summary, one can say that students' misconceptions of the negative numbers are not rare (cf. Altiparmak & Özdoğan, 2010, 31). This reflects a more universal problem with learning mathematical concepts that is noticed at all levels of mathematics education. For example, Tossavainen, Attorps, and Väisänen (2011) conducted a study in Sweden, Finland, and South Africa that focused on pre-service mathematics subject teachers' understanding about the concept of equation. This concept should be familiar to all mathematics students. Nonetheless, only a half of the participants were able to give a correct definition for this notion, and most participants had several misconceptions of equations in their concept images.

Method

The data for the current study were collected using a questionnaire, without giving any notice about the study in advance, during one of the first lectures on a compulsory mathematics course in the preschool teacher education programme at one Swedish university. The course is run during their second year at university. The total number of students in the course is 31 and all 23 students who were present that day participated on the voluntary basis after hearing a short introduction to the purpose of the study. The data can thus be considered to be representative even though the number of the participants is rather low. To guarantee that the students felt safe and that their responses are treated anonymously, all responses where folded and handed in through a closed box.

To answer the last research question, randomly selected group of ten non-mathematician teacher educators replied to the same questionnaire as the students. They teach, for example, courses in early childhood education, general and special pedagogy, and English. Their age varies between 30 and 62, so, they represent well the staff of the teacher education programme where the participating students study.

The methodological design of this study was inspired by a quite recent investigation by Tossavainen, Suomalainen, and Mäkäläinen (2017) who explored pre-service primary teachers' knowledge of the area concept. Similarly as theirs, our questionnaire surveys a respondent's concept definition and then his/her concept image with a set of specially designed tasks related to the negative numbers. The design of the tasks is based on the APOS theory (Asiala et al., 1996; Dubinsky & McDonald, 2001) which describes the learning of conceptual knowledge in mathematics as a process with four consecutive stages or levels. At the level of *Actions*, a learner can perform a simple action on the negative numbers, e.g., add or subtract two such numbers. At the level of *Process*, a learner can manage simple processes on the negative numbers, e.g., to order a given small set of negative numbers. At the level of *Object*, a learner can understand the negative numbers as a coherent and independent object that have certain properties. Further, he/she can acknowledge that there are more general properties and operations related to the negative numbers. An example of such properties is that the addition of negative numbers gives always a negative number, but the multiplication and division of negative numbers do not have a similar property. At the level of *Scheme*, a learner can set the negative numbers in a more general mathematical framework and sees relationships between the negative numbers and other independent mathematical objects. An example of this is that a learner masters the diverse usage of the minus sign both in the set of the negative numbers and in other number sets. Consequently, the APOS theory also induces a taxonomy for evaluating the challenges related to mathematical tasks within a given content area. The tasks included in our questionnaire are intended to measure the respondent's knowledge about the negative numbers both at the level of a novice and an advanced learner.

Originally, the questionnaire was designed by one author and then tested and revised by all authors. It consists of an introduction and a set of tasks divided into two sections. The first one contains four calculation exercises surveying the arithmetic skills related to the negative numbers. This is followed by a section of four verbal tasks (see Appendix). In order not to influence any respondent's definition of negative numbers, the calculation tasks were designed so that only negative integers appear in them.

The tasks were examined by two authors and scored as follows. In the calculation tasks, the scale is 0 = *"An empty or seriously incorrect response"*, 1 = *"A response with minor mistakes"*, and 2 = *"A correct response"*. In the first verbal task, one point was given for each relevant usage of the minus sign, which are subtraction, taking an opposite number, and multiplication by -1. In three remaining tasks, the scale is 0 = *"An empty or irrelevant response"*, 1 = *"A response with some mistakes"*, and 3 = *"A correct response"*. The higher maximum score in these tasks is motivated by the fact that they are more challenging or require more careful explaining of the solution than the calculation tasks. Therefore, the maximum scores between these sections are weighted 60% and 40%, respectively. The maximum of the sum scores in the whole test is hence 20.

In order to examine the respondents' concept definitions and images of the negative numbers, we conducted a content analysis of the responses for the definition question and four verbal tasks (see Appendix, questions 1, 3–6). Krippendorff (2013) mentions three different traditions in the methodology of content analysis. We build on the one that assumes that the participants' written responses truly express their actual conceptions of the negative numbers, and that their concept images of the negative numbers are reflected in their responses to the tasks in the questionnaire. Two researchers participated in the analysis, and the final categorisation of the definitions was reached after several cycles of analyses and discussions.

Further, we analysed our data using some standard quantitative methods such as Spearman correlation analysis and Student's t-test for independent samples. The use of correlation analysis raises a question of the sample size. It can be calculated that, for N = 23, the absolute value of the correlation coefficients should be 0.41 or more in order to that the correlation were significant at the level $p < 0.05$, and 0.35 or more if the significance level is set to $p < 0.10$. Therefore, our data should be sufficiently large to show it if the effect size of a covariance between any two variables is medium or larger than that. Our data are rather small for using t-tests but, technically, there are no reasons why the mean differences could not be analysed in this way. The biggest problem with small data is that the mean difference between two groups has to be quite large with respect to the standard deviation in order that a t-test can show a sufficient support for the rejection of the null hypothesis. Therefore, it is now more meaningful to evaluate the significance of mean differences by using the effect size (Cohen's *d*). We do so when we study the relationship between the quality of participants' definitions and their task performance.

Results

We begin by reporting from the participants' concept definitions of the negative numbers. The content analysis of the definitions resulted in finding four categories whereof one is reserved for the empty answers. Table 1 below gives the name and the description of each category, an example of a definition belonging to the category, and the distribution of the participants into the categories.

In Table 1, the fourth category represents definitions that can be accepted as mathematically correct and sufficiently precise. In our data, nine out of 23 participants are associated to this category, i.e., approximately 40% pre-service preschool teachers succeeded in giving a reasonable definition.

TABLE 1 Pre-service preschool teachers' categories of concept definitions (N = 23)

NAME OF CATEGORY	*DESCRIPTION*	*EXAMPLE OF DEFINITION*	*N*
1. Empty answer	No answer at all	-	6
2. Irrelevant	Definitions that fail to discuss the negative numbers in a meaningful way	*"A number with a bad attitude"*	4
3. Allegoric	Definitions that depict the negative numbers using allegories or concrete examples (cold temperature, debt, etc.)	*"Negative numbers are such as those who show the minus temperatures."*	4
4. Below zero	Definitions that express that the negative numbers are numbers less than zero.	*"Negative numbers are all those numbers that are below 0. In other words, those that start with –"*	9

At the level of individual definitions, we discover a richer and more nuanced variation of conceptions of the negative numbers than four categories alone can depict. For example, one participant belonging to the fourth category, emphasised also the operational aspect of the negative numbers, i.e., a negative number always reduces the summa in the addition: *"Negative number is a number that subtracts from another integer. Something that subtracts from something else".*

The above citation also reveals something about the respondent's concept image: the use of word 'integer' instead of 'number' suggests that the respondent associates the negative numbers with the set of negative integers although the beginning of the sentence and second sentence do not make this restriction. In general, there were only a few indications of setting the negative numbers in relation to other number sets in the responses. Here are two of the rare cases which can be interpreted to do so: *"Something that is not a positive number"*, i.e., a definition based on using the positive numbers which indeed are the inverses of the negative numbers, and *"Positive [numbers] are all those that are over 1, negative [numbers] are all those below"*. Interestingly, the latter definition harvests all positive numbers from the interval from zero to one into the set of negative numbers, which is not correct. This respondent's provoked image of numbers seems now to consist only of integers and she divides them into positive and negative subsets.

Both of these alleged properties (the subtractive operator and the inverse of a positive numbers) seem to be more common in the participants' concept images than in their definitions. For example, in Task 4 (see Appendix), some respondents motivated that −2 is larger than −3, for example, stating that *"−2 is larger because it is closer to a positive number"*, and in Task 5, for example, claiming that the sum of two negative numbers is always negative *"Yes, because it gives even more – [=the minus sign]"* or *"Yes. Because [then] one does not add anything".*

On the other hand, the participants' answers in the tasks also reveal that their concept images contain incorrect rules and conceptions of the negative numbers, e.g., *"negative + negative = positive"* and *"The sum of two negative numbers is always a positive number because two negative numbers cancel one another"*. Further, there are conflicts between the participants' definitions and images of the negative numbers. For example, one respondent defined the negative numbers by stating *"A negative number is a number below zero"* and, in Task 2, she indicated also that the negative numbers have a subtracting property. Nevertheless, in Task 5, she claimed that *"In certain cases [of addition] two negative numbers becomes positive"*.

The four categories in Table 1 have been organised qualitatively on an ordinal scale. The first category stands for the missing definitions and the other three categories for "the failed/underdeveloped definitions", "the referentially correct definitions", and the correct definitions, respectively. Hence, it is relevant to ask whether there is a covariance relation between the category scale and the participants' sum scores from the tasks. A Spearman correlation analysis ($\rho = 0.29,\ p = 0.17 > 0.05$) reveals that the risk in making such a conclusion is too high (because $p > 0.05$), yet one might like to answer the question positively with some reservations (because $\rho = 0.29$).

To get a more thorough view of the relationship between the quality of the participants' definitions and their performance on the calculations with negative numbers, we present the distribution of the students' sum scores in Figure 1 and then, in Table 2, summarise the descriptive statistics of their performance across the category groups. As the figure shows, altogether twelve students got seven or more points, and the remaining eleven students got six or less points. The extreme values of the students' sum scores are one and eleven points. There is only one student for each of these values. As the possible maximum of the sum scores is twenty points, the students' performance on the negative numbers cannot be considered especially high, given that the designed tasks should be solvable for any person with a good command of secondary school mathematics.

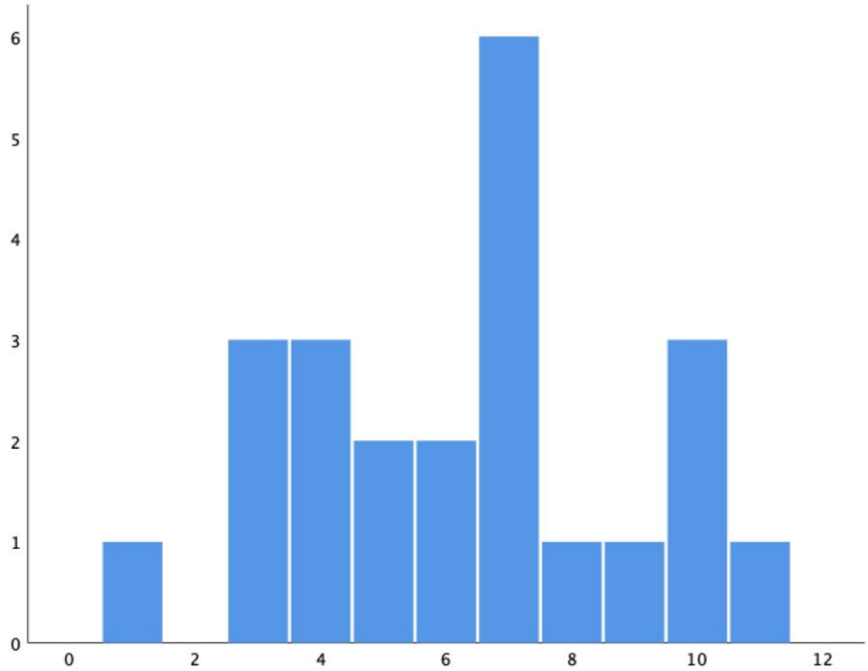

FIGURE 1 The distribution of students' sum scores

TABLE 2 Pre-service preschool teachers' task performance across the categories

NAME OF CATEGORY	*MEAN SUM SCORES*	*STD. DEVIATION*	*MIN–MAX*
1. Empty answer	6.33	3.20	3–11
2. Irrelevant	3.75	2.50	1–7
3. Allegoric	6.00	0.82	5–7
4. Below zero	7.44	2.67	3–10

In Table 2, the mean sum scores are highest for the group consisting of those participants who were able to define the negative numbers correctly. Indeed, their mean is almost the double compared to the mean of the group 'Irrelevant'. The effect size for this mean difference is actually very large ($d = 1.43$). Further, the effect size for the mean difference between the third and fourth category groups is also medium, almost large ($d = 0.73$). Here we have used the limit values of the effect sizes given by Sawilowsky (2009).

Concerning the second research question from the perspective of the effect sizes, there is a significant relation between the quality of the participants' definitions and their task performance, but the relationship is not direct. In Table 2, we see from the minimum and maximum scores that the task performances in the first and fourth category group remind one another, although these should be the most distant with respect to ability to define the negative numbers. Similarly, the second and third category group are closer to one another than to other two groups. We elaborate this finding in more detail in the Discussion section.

To answer our third research question, we first summarise the task performance of the control group and then report from the analysis of the mean differences between the pre-service preschool teachers and their educators.

TABLE 3 Teacher educators' distribution of definitions and task performance (N = 10)

NAME OF CATEGORY	*N*	*MEAN SUM SCORES*	*STD. DEVIATION*	*MIN–MAX*
1. Empty answer	3	6.33	6.11	1–13
2. Irrelevant	1	8.00	-	8–8
3. Allegoric	1	9.00	-	9–9
4. Below zero	5	7.80	3.77	2–12

If we compare the teacher educators' distribution of concept definitions to that of their students, a significant difference is that only one teacher educator falls into the second and the third category. Their mode is the same as the mode of the pre-service preschool teachers, i.e., 'Below zero'. The number of empty answers is approximately as common as for the pre-service preschool teachers. Hence, one can summarise the findings in Table 3 by saying that the teacher educators' concept definitions are qualitatively more polarized than those of their students, but the relative proportion of correct definitions is quite the same for the teacher educators, when the effect of random sampling is taken into account.

TABLE 4 Task performance of pre-service preschool teachers and their educators (N = 33)

GROUP	*MEAN SUM SCORES*	*STD. DEVIATION*	*MIN–MAX*
1. Pre-service preschool teachers	6.26	2.67	1–11
2. Their educators	7.50	3.92	1–13

Table 4 may seem to speak for a slightly better performance of the teacher educators since their mean scores are higher than those of their students. However, the mean difference is not statistically significant ($t(31) = -1.06,\ p > 0.05$). This is mainly due to a large variation of the sum scores within both groups. The effect size is now $d = 0.37$ which is small (yet not non-existing). Consequently, the mean difference is merely insignificant in relation to the standard deviations and the group sizes. To sum up, by Tables 3 and 4 one cannot claim that one or another group had significantly better knowledge about the negative numbers.

Discussion and conclusions

This study draws on the responses from a quite small group of Swedish pre-service preschool teachers, yet it succeeds to reveal quite large variation of conceptions of the negative numbers. Almost half of the participants were able to give a correct definition of this number set, yet only a few of them set the negative numbers in relation to other common number sets.

We summarised the participants' definitions into four categories, but as several quotes showed, the variety of their concept images is more versatile. Moreover, there are some conflicts between the concept definitions and images. One indication of that is contained in our answer to the second research question: a respondent did not mention the subtracting property of the negative numbers in her definition, but used it as a main argument in one task, and yet argued against this property in another task.

A significant but not especially surprising finding related to Table 2 is that those pre-service preschool teachers who were able to give a correct definition of the negative numbers, also performed better than others in the given set of tasks, related to the negative numbers. A somewhat unexpected result is that those participants who did not give a definition at all, nevertheless, performed almost as well as the best group. This outcome can be explained, at least, in two different ways. A practical explanation is that there are people who refuse to give any definition if they feel unsure about the correctness of their definition. A more theoretical, but empirically verified, explanation is given by our theoretical framework; concept images and concept definitions can be independent from one another (Tall & Vinner, 1981; Tossavainen, Attorp, & Väisänen, 2011). An individual may have met a formal definition of a concept but, thereafter, as he/she has started to apply the concept, all his/her activities have based on his/her image of the concept and the formal definition has faded away. Moreover, if a learner is expected to solve two different tasks related to the same concept, the tasks may provoke such elements or aspects of the concept image into working memory, that are contradictory to one another. Indeed, this was the case with a respondent to whom we referred to in the previous paragraph. Further, Tossavainen, Haukkanen, and Pesonen (2013) verified the same phenomenon in a study, which concerned pre-service mathematics subject teachers' knowledge about monotone functions. In that study, the same task was formulated in four different ways, and the participants' performance between the tasks varied significantly in spite of the fact, that each task could have been solved by referring to a certain central property of the concept.

Table 2 shows also that putting emphasis on the construction of thorough knowledge about mathematical concepts has a positive effect on task performance. This result supports the conclusions made by Kilhamn (2009, 2011) who warns about the risk of learners constructing misdeveloped conceptions of the negative numbers if the discussion of these numbers is based on informal language. In preschool teacher education, a key to finding a balance between playful and more formal discussions about the negative numbers has been suggested, for example, by Jahnke (2016) who emphasises the importance of maintaining mathematical knowledge through practice. A concrete example related to the negative numbers could be the following. When a negative number is met in a learning situation, it can most often be related to its positive inverse. For example, the problem with two missing toys can be solved by getting two new corresponding toys. Similarly, if you lend a coin to your friend, you can expect to have a similar coin back in order to balance your wallet. The difference between lent and received coins can be designated by using the minus and plus signs in front of number indicating the amount of coins. Such discussions maintain children's knowledge about the important property of the negative numbers, i.e., that they and the positive numbers are inverse to one another, and leads in a natural way to further discussion what other properties the negative numbers may have.

Concerning our third research question, the results related to Tables 3 and 4 suggest that the Swedish pre-service preschool teachers do not in any significant way differ from a very relevant reference group, when knowledge about the negative numbers is concerned. Another question, of course, is what is a sufficient mathematical knowledge for any individual in the modern society? The mean scores for both groups were relatively low. Mathematics is applied in our everyday life, more than ever, through the digitalisation of work and a rapidly technologizing environment – including home. Therefore, we predict that the discussion on the aims and goals of mathematics education to young children will continue also in the future.

References

Altiparmak, K., & Özdoğan, E. (2010). A study on the teaching of the concept of negative numbers. *International Journal of Mathematical Education in Science and Technology, 41*(1), 31–47.

Asiala, M., Brown, A., DeVries, D., Dubinsky, E., Mathews, D., & Thomas, K. (1996). A framework for research and curriculum development in undergraduate mathematics education. *Research in Collegiate Mathematics Education II, CBMS Issues in Mathematics Education, 6*, 1–32.

Björklund, C. (2015). Pre-primary school teachers' approaches to mathematics education in Finland. *Journal of Early Childhood Education Research, 4*(2), 69–92.

Chrysostomou, M., & Mousoulides, N. (2010). Pre-service teachers' knowledge of negative numbers. In M. Pinto & T. Kawakaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 265–272). Belo Horizonte, Brazil: PME.

Dubinsky, E., & McDonald, M. A. (2001) 'APOS: A constructivist theory of learning in undergraduate mathematics education research'. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 273–280). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Einarsdottir, J. (2013). Early childhood teacher education in the Nordic countries. *European Early Childhood Education Research Journal, 21*, 307–310.

Ekenryd, S. (2016, March). "Ifrågasätt inte lärarnas kompetens!" [Do not question teachers' competence!]. Skolvärlden (30.3.2016). Retrieved from https://skolvarlden.se/artiklar/ifragasatt-inte-lararnas-kompetens

Hiebert, J., & Lefevre, T. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hieber (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 291-293). Hilsdale, NJ: Erlbaum Associates.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–406.

Jahnke, A. (2016). *Skolans och förskolans matematik: kunskapssyn och praktik*. Lund: Studentlitteratur.

Johansson, M., Tossavainen, T., Faarinen, E.-C., & Tossavainen, A. (2020). Student teachers' definition of the concept "teaching mathematics in preschool". In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 2277–2284). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Kilhamn, C. (2009). Making sense of negative numbers through metaphorical reasoning. In C. Bergsten, B. Grevholm & T. Lingefjärd (Eds.), *Perspectives on mathematical knowledge. Proceedings of MADIF6* (pp. 30–35). Linköping, Sweden: SMDF.

Kilhamn, C. (2011). *Making sense of negative numbers.* Gothenburg: Univ. of Gothenburg.

Krippendorff, K. (2013). *Content Analysis – An Introduction to Its Methodology* (3rd ed.). London: Sage Publications.

OECD (2020). Pisa 2018 Results. Retrieved from https://www.oecd.org/pisa/publications/pisa-2018-results.htm

Palmér, H., & Björklund, C. (2016). Different perspectives on possible–desirable–plausible mathematics learning in preschool. *Nordisk matematikkdidaktikk, 21*(4), 177–191.

Sawilowsky, S. S. (2009). The effect size rules of thumb*. Journal of modern applied statistical methods, 8*(2), Article 26.

Segall, A. (2004). Revisiting pedagogical content knowledge: The pedagogy of content/the content of pedagogy. *Teaching and Teacher Education, 20*(5), 489–504.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–23.

Skolverket. (2018). *Läroplan för förskolan. Lpfö 18*. Stockholm: Skolverket.

Tall, D., & Vinner, S. (1981). Concept image and concept definition with particular reference to limits and continuity. *Educational studies in mathematics, 12*, 151–169.

Tossavainen, T., Attorps, I., & Väisänen, P. (2011). On mathematics students' understanding of the equation concept. *Far East Journal of Mathematical Education, 6*(2), 127–147.

Tossavainen, T., Haukkanen, P., & Pesonen, M. (2013). Different aspects of the monotonicity of a function, *International Journal of Mathematical Education in Science and Technology, 44*(8), 1117–1130.

Tossavainen, T., Suomalainen, H., & Mäkäläinen, T. (2017). Student teachers' concept definitions of area and their understanding about two-dimensionality of area. *International Journal of Mathematical Education in Science and Technology, 48*(4), 520–532.

Tossavainen, T., Johansson, M., Faarinen, E. C., Klisinska, A., & Tossavainen, A. (2018). Swedish primary and preprimary student teachers' views of using digital tools in preprimary mathematics education. *Journal of Technology and Information Education, 10*(2), 16–23.

Widjaja, W., Stacey, K., & Steinle, V. (2011). Locating negative decimals on the number line: Insights into the thinking of pre-service primary teachers. *The Journal of Mathematical Behavior, 30*(1), 80-91.

Appendix – the tasks in the questionnaire

1. Use a couple of minutes by reflecting what the negative numbers are. Thereafter, describe how you would define what the notion of a negative number means.

Section 1

2. Solve the following tasks.

a) $-12 + (-9) =$

b) $-27 - (-32) =$

c) $(-1)^{11} \cdot (-4) =$

d) $\frac{6 \cdot (-5)}{-10} =$

Section 2

3. Which purposes the minus sign can have in mathematics?

4. Is -2 larger or smaller than -3? Motivate your answer.

5. Is the sum of two negative numbers always a negative number. Motivate your answer.

6. Explain what it means to divide a given number by -2.

Remark. In the introduction, a student was encouraged to motivate her/his answers as carefully as possible. Especially, it was urged that a student reflects also on the possible reasons if she/he cannot solve the given task.