Pricing the Pharmaceuticals when the Ability to Pay Differs*

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Abstract

A non-trivial fraction of people cannot afford to buy pharmaceutical products at unregulated market prices. This paper analyses the public insurance of a patent-protected pharmaceutical product in terms of price controls and socially optimal third-degree price discrimination. First, the paper characterizes the Ramsey pricing rule in the case where the producer price has to cover the *R&D* costs of the firm and patients' pharmaceutical expenditures are not covered by health insurance. Subsequently, conditions for a welfare increasing departure from the Ramsey pricing rule are stated in terms of price regulation and health insurance coverage. Unlike the earlier views expressed, the increased consumption of the pharmaceutical is shown to be welfare increasing. In the spirit of the Rawlsian view, a criterion for vertical equity is examined as an optimal means-tested health insurance. In this scheme, the regulator chooses a higher insurance coverage for individuals whose income is below an endogenously determined income threshold. The means-tested insurance scheme improves social welfare but also yields very equal market outcomes.

Keywords: *Pharmaceuticals, price regulation, public health insurance, third-degree price discrimination, equity criterion*
JEL codes: *L1, L5, I18*

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1. Introduction

The ability to pay for pharmaceuticals varies among people. A non-trivial fraction of people cannot afford to buy pharmaceutical products at unregulated market prices. Those products are created through expensive and risky *R&D* programmes committing the pharmaceutical firms to rather high expenditures. Those expenditures should subsequently be covered through prices, which, however, may turn out to be too high to be socially acceptable. In the current paper, a question is raised concerning how to introduce means-tested subsidies to low-income citizens as part of optimal regulation and yet to maintain the incentives for a pharmaceutical company to invest in the *R&D*. Therefore, the conflict between efficiency and equity has to be resolved via optimal pricing.

Apart from the efficiency considerations, policy-makers typically emphasize equitable access to services due to the fact that in many countries, if not in most, low-income people are not able to buy the medication they need. Indeed, the health policy concerning the medical industry often expressed in the official documents states that "the purpose of the medical policy is to provide to citizens high-quality and cost-efficient pharmaceuticals at reasonable prices....". Moreover, the PPRI Report 2018 provides information about currently existing pharmaceutical pricing and reimbursement policies in the 47 PPRI member countries. It turned out that 42 PPRI network member countries have mechanisms in place to set medicine prices at the ex-factory (or sometimes wholesale) price level, mostly targeting reimbursable medicines or prescription-only medicines. 46 PPRI network member countries have at least one reimbursement list for outpatient medicines in place, and in 31 PPRI countries the reimbursement lists relate to both outpatient and inpatient sectors. In addition, hospital pharmaceutical formularies are managed at the level of hospitals in most PPRI countries. At least 43 countries charge co-payments for outpatient reimbursable medicines (frequently percentage copayments, but also a prescription fee and/or a deductible). All these 43 countries apply exemptions from or reductions of co-payments for vulnerable and other defined population groups (Vogler et al., 2019).

In Finland, pharmaceuticals are delivered and financed by different channels although the system is tax-based. Reimbursed drugs are delivered from pharmacies and costs are covered by Social Insurance Institution and patient copayments. Reimbursement categories (40%, 65% and 100%) are based on disease severity. Medication during hospital visits is covered by municipalities and costs are incorporated into the hospital payment. Decision-making bodies and the criteria used in health technology assessment and regulation vary in different channels. Patient income is not a decision-making criterion, but people with extremely low incomes may get pharmaceuticals for free from the Social Insurance Institution's income support.

Previous work based on the efficient price regulation of pharmaceutical products and health insurance has produced a number of important contributions. The basic idea has been cast in terms of the optimal product taxation in a one-person or many-people economy with Ramsey's (1927) idea of equal percentage reductions in (compensated) demands for all commodities (Diamond, 1975). Based on such foundations, Besley (1988) explored the trade-off between risk sharing and the incentives to consume medical care inherent in health insurance. Earlier, Feldstein (1973) had expressed concerns about the welfare cost of excess health insurance induced by the adverse incentives of the consumption of health care. The interaction of pricing and insurance coverage in the pharmaceutical market was addressed by Barros and Martinez-Giralt (2008), who considered the normative allocation of *R&D* costs across different markets served by a pharmaceutical firm. They showed that a higher insurance coverage calls for higher prices not only because of a lower demand elasticity but also due to a larger moral hazard effect in the consumption of the pharmaceuticals. The equilibrium pricing rule appeared to deviate from the standard Ramsey pricing rule: for equal demand elasticities, and given the distortion cost of funds, a country with a higher coverage rate will have higher-priced pharmaceuticals as well.

Gaynor et al. (2000) also focussed on the excessive consumption of medical products caused by insurance, that is, the moral hazard. In a related area, Grassi and Ma (2011; 2012) studied the provision of public supply of health care services but with non-price rationing when the income levels of people are different. When the rationing is based on wealth information (as is the case in the USA), the optimal policy in their analysis rations public services to low-income people, while leaving the high-income people to buy services from the private market. If also the cost is observed, the optimal rationing turns out to be based on cost-effectiveness (as in most European countries and Canada). Baicker, Mullainathan and Schwartzstein (2015) suggest that "behavioural hazard" can make people misuse health care. They suggest that health insurance can do more than just provide financial protection – it can also improve health care efficiency. A comprehensive survey of the literature on pricing pharmaceuticals has been documented by Borges dos Santos et al. (2019).

Abbot and Vernon (2005) have demonstrated how pharmaceutical price controls will significantly diminish the incentives to undertake early-stage *R&D* investment. In the current paper, and in contrast to the existing work in the area, the question is raised of how to introduce means-tested subsidies to low-income citizens as part of the optimal regulation and yet to maintain the incentives of a pharmaceutical company in investing in *R&D*. To fix the ideas of the paper, a market for a pharmaceutical product with one firm having innovated a new product is considered. The firm is the sole producer of the product, say through the patent protection. The cost of innovation is sunk at the time the product is sold on the market, and it causes a decrease in the average cost of producing the pharmaceutical product. Three policies will be analysed: Ramsey pricing without insurance, price regulation with insurance (henceforth, price-insurance policy), and a means-tested price and insurance policy (henceforth, means-tested price-insurance policy). Throughout the analysis, we will allow the patient population to be heterogeneous in terms of the ability to pay (income) and analyse questions related to the access to pharmaceutical treatment.

Initially, equity issues are ignored. As the equality between price and the marginal cost of producing the pharmaceutical does not represent a feasible starting point for the price regulation, the Ramsey pricing rule is a natural candidate to be studied in the absence of health insurance. Next, conditions for a welfare increasing departure from Ramsey pricing in terms of price regulation and optimal insurance coverage are derived, taking the social cost of public funds into account. Our results provide insights in to why both price regulation and social insurance are desirable. Subsequently, the paper also addresses the fact that a non-trivial fraction of patients cannot afford to buy the pharmaceutical product even at regulated and subsidized market prices and poses a question of whether means-tested insurance coverage rates have the potential to improve welfare. We thus arrive at the socially optimal, third-degree out-of-pocket price discrimination. Our analysis complements that of Grassi and Ma (2011; 2012) who analysed efficient non-price rationing schemes. Moreover, while Gaynor et al. (2000) worked with the case of a private health insurance market, the focus in the current paper is instead on the public (or social) health insurance.

When Ramsey pricing is compared with the price-insurance policy, our findings indicate that the moral hazard in terms of increased consumption of pharmaceutical products is welfare increasing. Without the health insurance, the prices would be excessively high as the firm's *R&D* costs have to be recovered. The result that the introduction of health insurance improves social welfare is due to our focus on designing a socially optimal health insurance coverage in a price-regulated pharmaceutical market characterized by increasing returns to scale.

Yet, the second-best equilibrium with public health insurance also has some undesirable properties: low-income people are left without the medication they need. As the optimal means-tested insurance, we explore an equity-based health insurance scheme in the spirit of the Rawlsian view. In this scheme, the regulator chooses a higher insurance coverage for individuals with an income below a threshold (low-income patients) and a lower insurance coverage for individuals above the income threshold (high-income patients). Under this scheme, the income threshold categorising patients into the low-income and high-income groups is determined endogenously.

Our results show that in the Rawlsian world with equity based on maximizing the aggregate consumer surplus and conditional on the improved access to pharmaceutical treatment by means-tested insurance coverages, the consumption of the pharmaceutical and the consumer surpluses are split equally between the low- and high-income patients. In this respect, the optimal means-tested policy yields very equal market outcomes. It is also shown that the optimal means-tested price-insurance policy provides a strictly higher social welfare than the optimal price-insurance policy with no equity concern.

Before presenting the model (Section 2) and its analysis (Sections 3–5), we comment on the potential information problems as follows. First, although the regulator is uninformed about the individual incomes of the patients in Sections 2–4, the income distribution is known. This is all the information needed in the Ramsey problem and in the optimal price-insurance policy analysis. Secondly, in Section 5, the regulator uses the means-tested approach to classify the patients into low-income and high-income groups and the regulator is assumed to have access to income information that is needed to construct optimal policies.

2. Model

We consider a market for a new pharmaceutical product. There is a single monopoly producer holding the patent and selling the product. The demand side of the market consists of patients in need of the pharmaceutical treatment that the firm produces. The current health state of each patient is h_0. The consumption of the pharmaceutical improves patients' health to $h_1 > h_0$. The effectiveness of the pharmaceutical treatment can be measured by the difference $\Delta = h_1 - h_0$.

Patients derive utility from health h and consumption goods x. Each patient has a utility function $u(x, h)$, which is assumed to be a strictly increasing function in both consumption goods and health. In the spirit of Grossman (1972), patients consume the pharmaceutical to produce health. The production function for health is $h = h(j) = h_0 + \Delta j$, where the indicator $j = 1, 0$ describes whether or not a patient consumes the pharmaceutical.

2.1 Ability to pay, willingness to pay and the demand for the pharmaceutical

Patients are heterogeneous in their ability to pay. To capture such heterogeneity formally, we introduce a randomly distributed income variable w, assumed to follow the $U\,[0, 1]$ distribution. The income variable measures disposable income and is adjusted for the patients' tax payments to the government.

We first show how the willingness to pay for the pharmaceutical product, denoted θ, is determined by the patient's ability to pay using the approach developed by Grassi and Ma (2011; 2012). Let the variable p denote the producer price of the pharmaceutical product. The budget constraint of the patient with income w can then be written as $w = x + (1 - r)pj$, where the binary indicator j describes the patient's consumption of the pharmaceutical, the variable r stands for the insurance coverage, and the price of consumption goods is normalized to one.

Assuming a separable utility function, the patient with income w obtains utility

$$u(w - (1 - r)pj) + v(h(j)) \tag{1}$$

from the consumption of the pharmaceutical. The first (second) part of the utility function $u(x)$ ($v(h)$) measures the patient's utility from consumption goods (health).

Using the Cobb-Douglas utility function with constant returns to scale, we show in the Appendix (Result 1) that the relationship between the willingness to pay and income is $\theta(w) = w\eta$, where

$$\eta = 1 - (1 + \frac{\Delta}{h_0})^{\frac{-\alpha}{1-\alpha}}, \tag{2}$$

and the fraction $\frac{\Delta}{h_0}$ measures the relative effectiveness of the pharmaceutical and $0 < \alpha < 1$ is the preference weight that patients give to health. Henceforth, the parameter $\eta > 0$ will be called quality weight as it is determined by the health effects of the pharmaceutical.

We will assume conditions that allow us to adopt the parametrization $\theta(w) = w\eta$, where η is given by (2). As a consequence, a patient with income w obtains consumer surplus

$$CS_w(j) = (w\eta - (1 - r)p)\, j \tag{3}$$

from consumption of the pharmaceutical. The consumer with income w_i is indifferent with regard to consuming or not consuming the pharmaceutical. Therefore, the condition $CS_w(1) = CS_w(0) = 0$ can be solved with respect to the income of the indifferent consumer:

(4) $$w_i = \frac{(1-r)p}{\eta}.$$

Given the producer price and the insurance coverage, the demand for the pharmaceutical is given by the number of buying, high-income patients:

(5) $$q(p,r) = 1 - w_i = 1 - \frac{(1-r)p}{\eta}.$$

The total consumer surplus from the consumption of pharmaceuticals is defined as follows:

(6) $$CS(p,r) = \int_{\frac{p(1-r)}{\eta}}^{1} (w\eta - (1-r)\,p)\,dw.$$

2.2 Producer

The profit of the pharmaceutical firm is

(7) $$\pi(p,r) = (p-c)q(p,r) - F,$$

where $c > 0$ is the marginal cost of production and $F > 0$ is a fixed (sunk) cost from *R&D* activities prior to the launch of the pharmaceutical product. In the following analysis, we assume that the quality weight exceeds the marginal cost of producing the pharmaceutical:

Assumption 1: $c < \eta$.

Assumption 1 guarantees the existence of an active market for the pharmaceutical. If Assumption 1 was not true, there would be no patients whose willingness to pay for the pharmaceutical exceeds the marginal cost of producing the pharmaceutical. This implies that there would be no possibilities for market exchange.

2.3 Regulator

The regulator is benevolent and chooses the producer price and the insurance coverage to maximize social welfare, which is defined as the sum of the consumer surplus and the firm's profit subtracted by the cost of financing health insurance:

(8) $$W = CS + \pi - (1+\lambda)T.$$

In (8), T is the tax revenue raised to finance the health insurance. We assume that each euro collected through taxation to finance the pharmaceutical expenditures costs $(1 + \lambda)$ for society and where $\lambda \geq 0$ measures the marginal cost of public funds. The regulator maximizes social welfare (8) subject to the budget constraint

(9) $$T \geq rpq(p,r) \equiv IE(p,r).$$

The right-hand side of the inequality (9) measures the public health insurance expenditures due to the consumption of the pharmaceutical.

Since the value of the social welfare function (8) decreases as the tax revenue T increases, the regulator is not willing to collect more tax revenue than the amount of the aggregate health insurance expenditure. This implies that the budget constraint (9) must be binding in any solution to the regulator's problem. The social welfare function can then be restated as follows:

(10) $$W = CS(p,r) + \pi(p,r) - (1+\lambda)IE(p,r).$$

The regulator's problem is to choose the price-insurance policy (p, r) which maximizes the value of social welfare (10) subject to the profit constraint

(11) $$\pi(p,r) \geq 0$$

and the constraints defining feasible price-insurance policies: $p \geq 0$ and $0 \leq r \leq 1$.

2.4 Timing

We will examine a strategic game between the regulator and the producer of the pharmaceutical. The sequence of moves in the game is as follows. The regulator first chooses the producer price p and the insurance coverage r, after which the firm either accepts or rejects the regulator's proposal. If the firm accepts the proposal, patients decide whether or not to consume the pharmaceutical and the firm produces the amount of the pharmaceutical demanded by the patients.[1] To concentrate on analysing equity consequences of various price-insurance policies, it is assumed throughout the article that the quality weight, marginal and fixed costs and the marginal cost of public funds are common knowledge.

2.5 First-best solution

An efficient benchmark to the regulator's problem is the first-best price and quantity of the pharmaceutical, which maximize social welfare that is not influenced by health insurance coverage:

(12) $$W_f = CS(p,0) + \pi(p,0).$$

The first-best welfare is achieved by setting the price of the pharmaceutical equal to the marginal cost, that is $p_f = c$. The amount of pharmaceuticals consumed in the first-best solution is $q(c, 0) = (\eta - c)/\eta$, and the corresponding social welfare value is

(13) $$\bar{W}_f = CS(p_f,0) + \pi(p_f,0) = \frac{(\eta - c)^2}{2\eta} - F.$$

It is also understood that the regulator cannot implement the marginal-cost pricing scheme because that would yield the profit $-F$, which the firm is not willing to accept.

3. Ramsey price

Understanding that the marginal-cost pricing cannot be implemented, we first consider the pricing that maximizes welfare and satisfies the firm's profit constraint as the benchmark case. Furthermore, and to leave the analysis of the optimal insurance coverage to the subsequent sections, we assume that the regulator does not subsidize the patients' pharmaceutical expenditures through health insurance, but selects $r = 0$. Under such policy, the consumption of the pharmaceutical has no effect on public health insurance expenditures.

[1] *The regulator acts as a Stackelberg leader relative to the producer and consumers.*

The problem of the regulator can be defined as finding the pharmaceutical price which maximizes social welfare

$$W = CS(p,0) + \pi(p,0) \tag{14}$$

subject to the profit constraint

$$\pi(p,0) \geq 0. \tag{15}$$

The solution of the above problem defines the Ramsey-Boiteux price (e.g. Armstrong and Sappington, 2007). With L denoting the value of the Lagrangian function, the necessary condition for the Ramsey price is defined as follows:

$$\frac{\partial L}{\partial p} = \frac{\partial CS(p,0)}{\partial p} + (1+\mu)\frac{\partial \pi(p,0)}{\partial p}$$

$$= -\left(1-\frac{p}{\eta}\right) + (1+\mu)\left(\left(1-\frac{p}{\eta}\right) - \frac{(p-c)}{\eta}\right) = 0, \tag{16}$$

where μ is a positive-valued Lagrange multiplier of the profit constraint. In addition to the condition (16), the solution of the regulator's problem must satisfy $-\pi(p, 0) \leq 0$, $\mu \geq 0$ and $-\mu\pi(p, 0) = 0$.

Straightforward computation shows that social welfare (14) is decreasing with all pharmaceutical prices higher than the marginal cost[2]. Therefore, the regulator wants to reduce the price of the pharmaceutical until the excess profit of the pharmaceutical firm is exhausted. This implies that the firm must earn zero profit in the solution of the regulator's problem.

The first-order condition (16) can be solved together with the zero-profit condition $\pi(p, 0) = 0$ to obtain[3] the Ramsey price:

$$p_R = \frac{1}{2}\left(\eta + c - \sqrt{(\eta-c)^2 - 4\eta F}\right). \tag{17}$$

For the Ramsey price (17) to be well-defined, we must assume that

$$F < \frac{(\eta-c)^2}{4\eta}. \tag{18}$$

The assumption (18) is essential, because it guarantees that prices exist for which $\pi(p, 0) \geq 0$ and the regulator's strategy set is non-empty. Intuitively, the Ramsey price is sufficiently high so as to allow the firm to break even but it is lower than the monopoly price $(1/2)(\eta + c)$. The Ramsey price is related not only to the marginal or the fixed costs but also to the price elasticity of the demand (e.g. Armstrong and Sappington, 2007).

The solution is characterized by zero profits, which implies that social welfare equals the value of the consumer surplus. Therefore, the social welfare value in the Ramsey solution is

$$W_R = CS(p_R,0) = \frac{1}{8\eta}\left(\eta - c + \sqrt{(\eta-c)^2 - 4\eta F}\right)^2. \tag{19}$$

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[2] *The first derivative of social welfare with respect to price is* $-(p-c)\frac{1}{\eta}$ *and the statement follows from this.*

[3] *The solution of the first-order condition (16) and* $\pi(p, 0) = 0$ *defines the Ramsey price and the value of the Lagrange multiplier. The system of equations has two solutions,* $x_1 = (p_1, \mu_1)$ *and* $x_2 = (p_2, \mu_2)$. *The first (second) solution corresponds to the lower (higher) root of the zero profit condition. The value of social welfare is strictly decreasing at all price levels that exceed the marginal cost. Since the prices in the feasible set (i.e. prices which satisfy the profit constraint) are higher than the marginal cost, the lower root* x_1 *is the solution to the regulator's problem.*

We note from the Ramsey price that even if it eliminates excess profits, it forcefully limits the number of people who are able to buy the pharmaceutical.

4. Second-best efficient price and insurance policy

We next introduce public health insurance and ask whether adding a distortionary policy instrument to the regulator's strategy has the potential to improve social welfare. Intuitively, health insurance improves patients' welfare by lowering the out-of-pocket price that patients pay for the pharmaceutical, but the obvious social cost of health insurance is that it increases health insurance expenditures, which are financed through taxation. To examine whether the social benefits of public health insurance exceed social costs, we first derive the optimal price-insurance policy and thereafter assess its welfare properties.

The regulator's policy problem is to choose the price and insurance coverage (p, r) that maximize social welfare (10) subject to the profit constraint (11) and the feasibility constraints $p \geq 0$ and $0 \leq r \leq 1$. The solution of the regulator's problem is characterized in Proposition 1 below.

One of the features of the optimal price-insurance policy is that $\mu = \lambda$, where μ is the Lagrange multiplier of the profit constraint. To explain the logic of this result, we note that the multiplier μ measures the marginal social benefit of relaxing the firm's profit constraint, while λ is the marginal cost of tax funding. Since part of the firm's revenues are financed through the tax-funded health insurance expenditures $rpq(p, r)$, the regulator can use the price-insurance policy (p, r) to relax the firm's profit constraint. Proof of Proposition 1 in the Appendix shows that the optimal policy must satisfy the condition that the marginal social benefit of relaxing the profit constraint equals the marginal cost of tax funding.

Proposition 1. *If* $\lambda > 0$ *and*

$$\frac{(\eta - c)^2 \lambda (1+\lambda)}{\eta (1+2\lambda)^2} < F, \tag{20}$$

the optimal price-insurance policy $(\tilde{p}, \tilde{r})$ *is*

$$\tilde{p} = c + \frac{\eta F (1+2\lambda)}{(\eta - c)(1+\lambda)} \tag{21}$$

and

$$\tilde{r} = \frac{\eta F (1+2\lambda)^2 - (\eta - c)^2 \lambda (1+\lambda)}{(1+2\lambda)\left[\eta F (1+2\lambda) + c(\eta - c)(1+\lambda)\right]}. \tag{22}$$

Proof. See Appendix, Proof of Proposition 1.

The optimal price-insurance policy is designed so that it yields zero profit for the firm. The producer price $\tilde{p}$ exceeds the marginal cost of producing the pharmaceutical to cover the fixed *R&D* cost. The condition (20) guarantees that $\tilde{r} > 0$ and the optimal policy is an interior solution. If the condition was not satisfied, the necessary conditions of the regulator's problem (Appendix, Proof of Proposition 1) would support the Ramsey solution.

Proposition 2 below displays the effects of the fixed *R&D* cost and the quality weight on the optimal producer price and health insurance coverage. The results show that an increase in the fixed cost F leads to an increase in the optimal insurance coverage. Intuitively, this finding suggests that the regulator is more likely to introduce greater insurance coverage, the larger the fixed cost. Clearly, the insurance coverage allows the regulator to increase the consumer surplus by reducing the out-of-pocket price of the pharmaceutical. If health insurance was not available, an increase in the fixed cost would, on the contrary, increase the price of the pharmaceutical and decrease the demand for the pharmaceutical and consumer surplus.

Proposition 2. *Suppose that* $\lambda > 0$. *Then*

$$\frac{\partial \tilde{p}}{\partial F} = \frac{\eta(1+2\lambda)}{(\eta-c)(1+\lambda)} > 0;\ \frac{\partial \tilde{r}}{\partial F} = \frac{\eta(\eta-c)(1+\lambda)[\eta\lambda + c(1+\lambda)]}{[\eta F(1+2\lambda)+c(\eta-c)(1+\lambda)]^2} > 0$$

and

$$\frac{\partial \tilde{p}}{\partial \eta} = \frac{-cF(1+2\lambda)}{(\eta-c)^2(1+\lambda)} < 0;\ \frac{\partial \tilde{r}}{\partial \eta} = \frac{-(1+\lambda)\left[F(1+2\lambda)\left(\lambda\eta^2+(1+\lambda)c^2\right)+c(\eta-c)^2\lambda(1+\lambda)\right]}{(1+2\lambda)\left[\eta F\left(1+2\lambda\right)+c(\eta-c)\left(1+\lambda\right)\right]^2} < 0.$$

Proof. See Appendix, Proof of Proposition 2.

The comparative statics results in Proposition 2 also show that a higher quality weight leads to reductions in both the optimal producer price and insurance coverage. An increase in the quality weight moves the inverse demand curve to the right and, in order to price the pharmaceutical according to average costs, the regulator responds by reducing the optimal producer price. At the same time, however, the regulator implements health insurance coverage that increases the patients' co-payment for the pharmaceutical. On the basis of these findings alone, the effect of a higher-quality weight on the out-of-pocket price remains inconclusive. However, our following analysis on the out-of-pocket price shows that a higher-quality weight leads to a higher consumer price for the pharmaceutical (Eq. 23).

The out-of-pocket price that patients pay in the optimal price-insurance policy is

(23) $$\tilde{p}(1-\tilde{r}) = c + \frac{(\eta-c)\lambda}{(1+2\lambda)}.$$

When taxation is distortionary and $\lambda > 0$, the consumer price exceeds the marginal cost of producing the pharmaceutical. This also implies that the demand for the pharmaceutical is below the first-best level, and

(24) $$q(\tilde{p},\tilde{r}) = \frac{(\eta-c)(1+\lambda)}{\eta(1+2\lambda)} < \frac{\eta-c}{\eta} = q(c,0).$$

In addition, one can prove that that the patient's out-of-pocket price (23) in the optimal price-insurance policy is lower than the Ramsey price (17), if the condition for the interior solution (20) holds true (Proof of Lemma 1, Online Appendix). Provided that the demand for the pharmaceutical (5) decreases as the out-of-pocket price increases, such a decrease in the out-of-pocket price also increases the consumption of the pharmaceutical beyond that in the Ramsey solution.

Next, we conduct the welfare analysis by evaluating the consumer surplus, the insurance expenditure and the level of social welfare in the optimal price-insurance policy. Table 1 displays these measures together with the corresponding measures in the first-best and Ramsey solutions. The consumer surplus associated with the optimal price-insurance policy is lower than the consumer surplus in the first-best solution with marginal cost pricing and no insurance coverage due to the positive marginal cost of taxation. On the contrary, it can be shown that the consumer surplus in the optimal price-insurance policy is higher than the consumer surplus in the Ramsey solution, if the condition for the interior solution (20) holds true (Proof of Lemma 2, Online Appendix). The underlying reason for this result is that the out-of-pocket price (23) is lower than the Ramsey price (17).

When the condition for the interior solution (20) is satisfied, the insurance expenditure in the optimal price-insurance policy is positive. Furthermore, we note that in the case of distortionary taxation, the expenditure is less than the fixed cost. On the other hand, when the marginal cost of taxation gets closer to zero, the insurance expenditure approaches the fixed cost. The intuition behind this relationship between the optimal insurance expenditure and the marginal cost of public funds is as follows: the higher (lower) is λ, the less (more) willing the regulator is to use taxation as a means to finance pharmaceutical expenditures via public health insurance.

Table 1: Consumer surplus, profit, insurance expenditure and social welfare

	First-best	Ramsey	Price-insurance policy
CS	$\frac{(\eta-c)^2}{2\eta}$	$\frac{1}{8\eta}\left(\eta-c+\sqrt{(\eta-c)^2-4\eta F}\right)$	$\frac{(\eta-c)^2}{2\eta}(\frac{1+\lambda}{1+2\lambda})^2$
π	0	0	0
IE	n.a.	n.a.	$F-\frac{(\eta-c)^2\lambda(1+\lambda)}{\eta(1+2\lambda)^2}$
W	$\frac{(\eta-c)^2}{2\eta}-F$	$\frac{1}{8\eta}\left(\eta-c+\sqrt{(\eta-c)^2-4\eta F}\right)$	$\frac{(\eta-c)^2}{2\eta}\frac{(1+\lambda)^2}{(1+2\lambda)}-F\left(1+\lambda\right)$

For the purpose of Proposition 3, we denote social welfare in the optimal price-insurance policy (Table 1) as follows:

(25)
$$\tilde{W}=\frac{(\eta-c)^2}{2\eta}\frac{(1+\lambda)^2}{(1+2\lambda)}-F\left(1+\lambda\right).$$

The comparison of the social welfare in the first-best solution and in the optimal price-insurance policy does not to directly reveal that the first-best social welfare exceeds the social welfare in the optimal price-insurance policy (Table 1). However, Proposition 3 below demonstrates that – as expected – this indeed holds true.

Comparing the social welfare under the optimal price-insurance policy (25) with the social welfare in the Ramsey solution (19) leads to a striking observation. The introduction of public health insurance improves welfare because the resulting gain in the consumer surplus exceeds the increase in the publicly funded insurance expenditures (Proposition 3). This result is an illustration of the general theory of second best (Lipsey and Lancaster, 1956), where the introduction of a distortive policy instrument improves the welfare of an inefficient market.

The underlying reason for the finding that health insurance is welfare improving is the fact that the optimal health insurance in our model is combined with regulated producer prices. It is well known in health economics that if health insurance leads to higher prices in the health care market (Pauly, 1968; Feldstein, 1973), the introduction of health insurance is detrimental to welfare. In the context of our model, the introduction of health insurance will decrease the out-of-pocket price and increase the demand for the pharmaceutical but is also associated with a lower producer price due to economies of scale, hence leaving space for a possible welfare improvement (Gaynor et al., 2000).

Proposition 3. *The welfare ranking between the first-best solution, the Ramsey solution and the optimal price-insurance policy is the following:*

(26)
$$\bar{W}_f>\tilde{W}>W_R.$$

Proof. See Appendix, Proof of Proposition 3.

Intuitively, the Ramsey solution produces a smaller welfare than the optimal price-insurance policy, because a great many people are not able to acquire the drug at Ramsey prices. The optimal policy ($\tilde{p}$, $\tilde{r}$), however, does not reach an efficient solution because of the positive marginal cost of taxation.

5. Means-tested price-insurance policy

The previous analysis on the optimal price-insurance policy demonstrated how the introduction of health insurance can improve the efficiency of the pharmaceutical market. From the equity point of view, however, the optimal price-insurance policy has a serious limitation. Patients in the cohort of lowest incomes cannot afford to buy the pharmaceutical even in the presence of the health insurance. The number of such low-income patients is $1 - q(\tilde{p}, \tilde{r}) > 0$. Health is not like any other product, and equity considerations suggest that patients with low ability to pay should also have access to pharmaceutical treatment.

In this section, we examine an approach that adjusts the price-insurance policy to cope with vertical equity. In welfare economics, the idea of equity has been introduced in terms of the Rawlsian welfare criterion. Based on Rawls (1999), it is typically expressed as the maximin rule of the social choice[4]. Accordingly, the policy should aim at considering the utility of the individual who is worst off. In this section, the implications of the Rawlsian equity principle are examined in terms of a means-tested insurance policy that is implemented in the form of a third-degree price discrimination. In particular, we examine an optimal insurance policy that offers a higher insurance coverage for low-income patients who are not able to purchase the pharmaceutical at the out-of-pocket price paid by high-income patients. The advantage of the suggested approach is that it combines a solution for equity with an efficient insurance for those in higher income classes.

We analyse a model where people with high ability to pay and people with low ability to pay are entitled to different coverage rates, say $r_h \leq r_l$, where subscripts h and l refer to high ability to pay (high-income) and low ability to pay (low-income) patients, respectively. Hence, in this section, the focus will be on the price-insurance mechanism (p, r_l, r_h) with the feature $r_h \leq r_l$. Under this mechanism, the regulator offers the price p for the firm and selects the parameters of insurance coverage for high-income and low-income patients so that the out-of-pocket price of low-income patients is lower than that of high-income patients. Since the income variable is a continuous variable, we define low-income patients as the patient group who are not able to purchase a pharmaceutical at price p and insurance coverage r_h. This implies that the groups of low- and high-income patients are determined endogenously on the basis of the policy parameters (p, r_l, r_h) and raises particular questions about where to draw the demarcation lines between those who should have access to medication with price-insurance contract (p, r_l) and those with contract (p, r_h).

In what follows, we assume that the regulator has full information on patient incomes and hence is able to identify the low- and high-income patient groups and offer them different price-insurance contracts. If the regulator did not have full information on patient incomes and offered two price-insurance contracts (p, r_l) and (p, r_h), all patients in the market would prefer the contract offered to low-income patients because of the higher insurance coverage. As a result, the optimal contract to be derived next would not be incentive compatible. To make the contracts implementable, we assume that the regulator is fully informed about the patient incomes.

Given the price-insurance mechanism (p, r_l, r_h), the aggregate consumer surplus is given as follows:

$$CS(p, r_l, r_h) = \int_{\frac{p(1-r_l)}{\eta}}^{\frac{p(1-r_h)}{\eta}} (w\eta - (1-r_l)p)\, dw + \int_{\frac{p(1-r_h)}{\eta}}^{1} (w\eta - (1-r_h)\,p)\, dw. \tag{27}$$

[4] *The Rawlsian view has been widely discussed in welfare economics. For a recent analysis, one can refer to Stark, Jakubek, and Falniowski (2014), for example.*

Under this mechanism, the demand for the pharmaceutical is the sum of the demands of the buying high- and low-income patients:

$$q(p, r_l, r_h) = q_l(p, r_l, r_h) + q_h(p, r_l, r_h)$$

$$= \frac{p(1-r_h)}{\eta} - \frac{p(1-r_l)}{\eta} + 1 - \frac{p(1-r_h)}{\eta} = 1 - \frac{p(1-r_l)}{\eta}, \tag{28}$$

and the profit of the firm is given as follows:

$$\pi(p, r_l, r_h) = (p-c)q(p, r_1, r_h) - F. \tag{29}$$

Aggregate health insurance expenditures consist of the insurance reimbursements paid to subsidize the consumption of high- and low-income patients:

$$IE(p, r_l, r_h) = r_l p\left(\frac{p(1-r_h)}{\eta} - \frac{p(1-r_l)}{\eta}\right) + r_h p\left(1 - \frac{p(1-r_h)}{\eta}\right). \tag{30}$$

The regulator's policy problem is to choose the price and insurance policy (p, r_l, r_h) that maximizes social welfare (10) subject to the profit constraint $\pi(p, r_l, r_h) \geq 0$, the constraint on insurance coverage rates $r_h \leq r_l$, and the feasibility constraints $p \geq 0$ and $0 \leq r_t \leq 1$ for $t = l, h$. The consumer surplus, profit and insurance expenditures in the current problem are defined in expressions (27), (29) and (30), respectively. The following proposition characterizes the optimal means-tested price-insurance mechanism.

Proposition 4. *If* $\lambda > 0$ *and*

$$\frac{(\eta-c)^2 2(1+\lambda)(1+2\lambda)}{\eta(2+3\lambda)^2} < F, \tag{31}$$

the optimal means-tested price-insurance policy is

$$\hat{p} = c + \frac{\eta F\,(2+3\lambda)}{(\eta-c)\,2\,(1+\lambda)} \tag{32}$$

and

$$\hat{r}_l = \frac{\eta F(2+3\lambda)^2 - (\eta-c)^2\,2\lambda\,(1+\lambda)}{(2+3\lambda)\,[\eta F\,(2+3\lambda) + c(\eta-c)2(1+\lambda)]} \tag{33}$$

$$\hat{r}_h = \frac{\eta F\,(2+3\lambda)^2 - (\eta-c)^2\,2\,(1+\lambda)\,(1+2\lambda)}{(2+3\lambda)\,[\eta F\,(2+3\lambda) + c(\eta-c)2(1+\lambda)]}. \tag{34}$$

Proof. See Appendix, Proof of Proposition 4.

The insurance coverage of the low-income group (33) exceeds that of the high-income group (34) in the optimal means-tested price-insurance policy. This follows from the fact that $(1+\lambda)(1+2\lambda) > (1+\lambda)\lambda$. In addition, the comparison of the optimal prices $\hat{p}$ and $\tilde{p}$ demonstrates that $\hat{p} < \tilde{p}$, and the producer price in the means-tested price-insurance policy is lower than the price in the price-insurance policy (Section 4). Therefore, the introduction of the means-tested insurance coverage rates also have implications for the producer price of the pharmaceutical.

The above results will become explicit when we evaluate the welfare properties of the optimal means-tested price-insurance policy. The out-of-pocket price of high-income patients is

(35) $$\hat{p}(1-\hat{r}_h)=\frac{\eta(1+2\lambda)+c(1+\lambda)}{2+3\lambda},$$

and that of low-income patients is

(36) $$\hat{p}(1-\hat{r}_l)=\frac{\eta\lambda+2c(1+\lambda)}{2+3\lambda}.$$

Because $\hat{r}_l > \hat{r}_h$, the buying low-income patients pay less for the pharmaceutical than the buying high-income patients. Straightforward computation shows that the out-of-pocket price of high-income patients (and hence also the producer price) is higher than the monopoly price $(\eta + c)/2$. The out-of-pocket price of low-income patients exceeds marginal cost but is below the monopoly price. More strikingly, the optimal out-of-pocket payments ensure equal access to pharmaceutical treatment, and the low- and high-income patient groups consume the same amount of the pharmaceutical:

(37) $$q_l(\hat{p},\hat{r}_l,\hat{r}_h)=q_h(\hat{p},\hat{r}_l,\hat{r}_h)=\frac{(\eta-c)\,(1+\lambda)}{\eta(2+3\lambda)}\equiv x(\hat{p},\hat{r}_l,\hat{r}_h).$$

The aggregate consumption of the pharmaceutical is then $q(\hat{p}, \hat{r}_l, \hat{r}_h) = 2x(\hat{p}, \hat{r}_l, \hat{r}_h)$. The equal division of the market shows up also in the consumer surplus:

(38) $$CS_l(\hat{p},\hat{r}_l,\hat{r}_h)=CS_h(\hat{p},\hat{r}_l,\hat{r}_h)=\frac{(\eta-c)^2\,(1+\lambda)^2}{2\eta(2+3\lambda)^2}\equiv S^c(\hat{p},\hat{r}_l,\hat{r}_h).$$

The aggregate consumer surplus is $CS(\hat{p}, \hat{r}_l, \hat{r}_h) = 2S^c(\hat{p}, \hat{r}_l, \hat{r}_h)$. We state these findings as follows:

Proposition 5. *Under the Rawlsian principle of equity based on maximizing the aggregate consumer surplus and conditional on the better access to medication by low-income patients by means-tested insurance coverage, the final consumption of the pharmaceutical and the consumer surplus is split equally between low- and high-income patients.*

The result is sharp and it provides a yardstick when alternative equity principles are considered. Hence, and somewhat strikingly, although the patients with low ability to pay obtain the pharmaceutical at the lower out-of-pocket price, their surplus at the optimal solution is no higher than the surplus of the patients with high ability to pay.

By Proposition 5, the high- and low-income patient groups consume the same amount of the pharmaceutical. In addition, since the optimal insurance coverage of low-income patients is higher than that of high-income patients, insurance expenditures that the regulator pays to subsidize the consumption of the low-income group are higher than the corresponding expenditures of the high-income group:

(39) $$\hat{r}_l\hat{p}q_l(\hat{p},\hat{r}_l,\hat{r}_h)>\hat{r}_h\hat{p}q_h(\hat{p},\hat{r}_l,\hat{r}_h).$$

When evaluated in the optimal solution, the aggregate insurance expenditures amount to:

(40) $$IE(\hat{p},\hat{r}_l,\hat{r}_h)=F-\frac{(\eta-c)^2(1+\lambda)(1+3\lambda)}{\eta(2+3\lambda)^2}.$$

Straightforward comparison demonstrates that the insurance expenditure in the optimal means-tested price-insurance policy (40) is less than the insurance expenditure in the optimal price-insurance policy (Table 1). This is because different insurance coverage rates for low- and high-income patients allow the regulator to design financing schemes where the patients with high ability to pay pay a larger share of the pharmaceutical expenditures than the patients with low ability to pay.

Similarly as in the previous sections, the pharmaceutical firm earns zero profit (Proof of Proposition 4) in the means-tested price-insurance policy. The social welfare is then given as follows:

(41) $$\hat{W}=CS(\hat{p},\hat{r}_l,\hat{r}_h)-(1+\lambda)IE(\hat{p},\hat{r}_l,\hat{r}_h)=\frac{(\eta-c)^2\,(1+\lambda)^2}{\eta(2+3\lambda)}-(1+\lambda)F.$$

We next compare the social welfare obtained from the means-tested policy paying explicit attention to equity with the welfare obtained from the optimal price-insurance policy with no concern for low-income patients (Section 4).

Proposition 6. *The optimal means-tested price-insurance policy* ($\hat{p}$, $\hat{r}_l$, $\hat{r}_h$) *yields a strictly higher welfare than the optimal price-insurance policy* ($\tilde{p}$, $\tilde{r}$) *and* $\hat{W} > \tilde{W}$.

Proof. That $\hat{W} > \tilde{W}$ follows directly from the fact $2 + 3\lambda < 2(1 + 2\lambda)$. □

Stated verbally, under the Rawlsian criterion, the social welfare exceeds the social welfare under the optimal price-insurance policy with a uniform coverage rate (Section 4). Third-degree out-of-pocket price discrimination in the form of means-tested insurance benefits increases the consumption possibilities of low-income patients. In addition, different insurance coverage rates for high- and low-income patients allow the regulator to design a price-insurance policy in which the aggregate insurance expenditure is lower than the insurance expenditure in the price-insurance policy with no means-testing. Both of these effects increase the social welfare in comparison to the situation in which no means-testing was available for the regulator.

6. Discussion and concluding remarks

The ability to pay for pharmaceuticals varies among people. A non-trivial fraction of people cannot afford to buy pharmaceutical products at unregulated market prices. Those products are created through expensive *R&D* programs. Expenses associated with *R&D* should subsequently be covered through prices, which, however, may turn out to be too high to be socially acceptable. In the current paper, the question has been raised of how to introduce means-tested subsidies to low-income citizens as part of an optimal regulation and also maintain incentives for pharmaceutical companies to invest in the *R&D*.

The paper has extended the previous work on the optimal price and health insurance regulation of pharmaceuticals in three ways. First, a market has been considered where the ability to pay for pharmaceuticals varies in the patient population. Second, the optimal price regulation together with insurance coverage has been derived and characterized. Third, price and insurance policies improving the access of low-income patients to pharmaceutical treatments has been explored.

The comparison of the social welfare under the optimal price-insurance policy with the social welfare obtained from the Ramsey solution with no health insurance led to a striking observation. The introduction of public health insurance improves welfare because the resulting gain in consumer surplus exceeds the increase in the publicly funded insurance expenditures. This result is an illustration of the general theory of second best where the introduction of a distortionary policy instrument improves welfare in an inefficient market.

The second-best policies explored do not, however, ensure full access to pharmaceutical products for all patients in the lowest income groups. To ensure full access, the regulator should choose full insurance for those low-income patients who are not able to personally finance the consumption of pharmaceuticals with the welfare maximizing price-insurance policy. Therefore, the implications of the Rawlsian equity principle were examined in terms of a means-tested price-insurance policy that is implemented in the form of a third-degree out-of-pocket price discrimination. In particular, we examined an optimal insurance policy that offers a higher insurance coverage for low-income patients. The advantage of the suggested approach is that it combines a solution for equity with efficient insurance for those in higher income classes.

Although it is known that full insurance may create inefficiency in the sense of excessive consumption (Pauly, 1968), the regulator is willing to implement such a policy if the improved access of low-income patients to pharmaceutical treatment is considered socially desirable. The improved access may have social value because of the resulting incremental health gains and the improved productivity of individuals in the labour market. Such social value may, however, not be based on patients' preferences (Brouwer et al., 2008).

In Kanniainen et al. (2020), we explored the welfare implications of price-insurance policies where low-income patients who cannot afford to purchase the pharmaceuticals at the current out-of-pocket price have access to free medication. The regulator assigns a social value to the pharmaceutical consumption of the low-income patients and evaluates the consumption of the pharmaceuticals of the high-income patients based on their consumer surplus. Our analysis suggested that the policy with free medication for low-income patients creates a conflict of interest between the high- and low-income groups. If the health gains in the low-income group are highly valued by the regulator, the out-of-pocket price can be increased leading to a decrease in the consumption and consumer surplus derived from the consumption of the pharmaceutical in the high-income group. The analysis indicates that the regulator wants to implement such a policy with free medication to the low-income group if the social value of health gains exceeds the social marginal cost of producing the pharmaceutical product.

There are other means of improving access to affordable medication besides policies relying on health insurance. One such tool is patent policy, defining the duration of the exclusive rights to sell the originator's drug. Potential entrants producing generic products have strong incentives to challenge such exclusive rights by entering the market before the expiry of the originator firm's patent, particularly when patents are long-lasting. Since the introduction of The Drug Price Competition and Patent Term Restoration Act in 1984, generic products in the US have been given a possibility to enter the pharmaceutical market before the expiry of the originator firm's patent (henceforth early entry). In such a case, a generic firm must file a paragraph IV certification claiming noninfringement or invalidity of the originator firm's patent (Branstetter et al., 2016.).

Izhak et al. (2020) explore the impact of patent length on the early entry of generic products using data from the US pharmaceutical market. They show that adding one year to patent length increases the early entry of generic products by five percentage points. Their findings are consistent with the literature on costly imitation (Gallini, 2002), suggesting that patents in the pharmaceutical sector should have a shorter duration and broader scope than in the current situation. When assessed from the perspective of patients in need of pharmaceutical treatments, a shorter patent duration implies earlier introduction of generic price competition, and also earlier access to affordable medication. The issue of patent length would serve as a fruitful topic for future research. Indeed, the early literature on patent length has suggested that rationally determined imitation makes socially optimal patents longer than what is suggested by models with non-strategic imitation (Kanniainen and Stenbacka, 2000).

Our modelling has some obvious limitations. We have worked with a model with linear demand for the pharmaceutical product and assumed uniform income distribution. In their analysis on public and private interaction, Laine and Ma (2017) illustrate what implications the assumption of a uniform distribution may have. Future work should therefore consider the possibility of generalizing our results. □

References

Abbott, T., and Vernon, J. (2005). *The Cost of U. S. Pharmaceutical Price Reductions: A Financial Simulation Model of R&D Decisions*. NBER Working Papers 11114.

Armstrong, M., and Sappington, D. (2007). "Recent Developments in the Theory of Regulation." In *Handbook of Industrial Organization.* Vol 3, 1557–1700. Eds. M. Armstrong and R. H. Porter.

Baicker, K., Mullainathan, S., and Schwartzstein, J. (2015). "Behavioral Hazard in Health Insurance." *The Quarterly Journal of Economics* 130(4), 1623–1667.

Barros, P. P., and Martinez-Giralt, X. (2008). "On International Cost Sharing of Pharmaceutical R&D." *International Journal of Health Care Finance and Economics* 8, 301–312.

Besley, T. (1988). "Optimal reimbursement health insurance and the theory of Ramsey taxation." *Journal of Health Economics* 7, 321–336.

Borges dos Santos, M.A., dos Santos Dias, L. L., Santos Pinto C. D. B., et al. (2019). "Factors influencing pharmaceutical pricing – a scoping review of academic literature in health science." *Journal of Pharmaceutical Policy and Practice* 2019, 12:24.

Branstetter, L., Chatterjee, C., and Higgins, M. J. (2016). "Regulation and welfare: evidence from paragraph IV generic entry in the pharmaceutical industry." *RAND Journal of Economics* 47(4), 857–890.

Brouwer, W. B. F., Culyer, A. J., van Excel N. Job A., and Rutten F. F. H. (2008). "Welfarism vs. extra-welfarism." *Journal of Health Economics* 28, 325–338.

Diamond, P. (1975). "A many-person Ramsey tax rule." *Journal of Public Economics* 4, 335–342.

Feldstein, M. S. (1973). "The welfare loss of excess health insurance." *Journal of Political Economy* 81, 251–280.

Gallini, N. T. (1992). "Patent policy and costly imitation." RAND Journal of Economics 23(1), 52–63.

Gaynor, M., Haas-Wilson, D., and Vogt, W. B. (2000). "Are invisible hands good hands? Moral hazard, competition, and the second-best in health care markets." *Journal of Political Economy* 108, 992–1005.

Grassi, S., and Ma, C.-t. A. (2011). "Optimal public rationing and price response." *Journal of Health Economics* 30, 1197–1206.

Grassi, S. and Ma, C.-t. A. (2012). "Public sector rationing and private sector selection." *Journal of Public Economic Theory* 14, 1–34.

Grossman, M. (1972). "On the Concept of Health Capital and the Demand for Health." *Journal of Political Economy* 80(2), 223–255.

Izhak, O., Saxell, T., and Takalo T. (2020). Optimal patent policy for Pharmaceutical Industry., VATT Working Papers 131.

Kanniainen, V., and Stenbacka, R. (2000). "Endogenous Imitation and Implications for Technology Policy." *Journal of Institutional and Theoretical Economics* 156(2), 360–381.

Kanniainen, V., Laine, J., and Linnosmaa I. (2020). *Pricing the Pharmaceuticals when the Ability to Pay Differs: Taking Vertical Equity Seriously*. CESifo Working Paper Series 8031, CESifo.

Laffont, J-J., and Martimort, D. (2002). *Theory of Incentives: The Principal-Agent Model*. New Jersey: Princeton University Press.

Laine, L., and Ma, C-t. (2017). "Quality and competition between public and private firms." Journal of Economic Behaviour and Organization 140, 336–353.

Lipsey, R. G., and Lancaster, K. (1956). "The General Theory of Second Best." *Review of Economic Studies* 24(1), 11–32.

Pauly, M. V. (1968). "The Economics of Moral Hazard: Comment." *The American Economic Review* 58(3), 531–537.

Ramsey, F. P. (1927). "A contribution to the theory of taxation." *Economic Journal* 37, 47–61.

Rawls, J. (1999). A Theory of Justice. Cambridge, MA: Harvard University Press.

Stark, O., Jakubek, M., and Falniowski, F. (2014). "Reconciling the Rawlsian and the utilitarian approaches to the maximization of social welfare." Economics Letters 122, 439–444.

Vogler, S., Zimmermann, N., and Haasis M. A. (2019). PPRI Report 2018 – Pharmaceutical pricing and reimbursement policies in 47 PPRI network member countries. WHO Collaborating Centre for Pricing and Reimbursement Policies, Gesundheit Österreich GmbH (GÖG / Austrian National Public Health Institute), Vienna. http://whocc.goeg.at/Publications/CountryReports

Appendix

Result 1: Derivation of the willingness to pay function $\theta(w)$.

Let us consider Cobb-Douglas utility function with constant returns to scale

$$u = x^{1-\alpha} h^{\alpha},$$

where $0 < \alpha < 1$ is the preference weight that patients give to health. The logarithmic transformation of the utility function enables one to present the patients' utility in a separable form:

$$\ln(u) \equiv \tilde{u} = (1-\alpha)\ln(x) + \alpha \ln(h).$$

Given the budget constraint $w = x + (1 - r)pj$, the utility that the patient with income w obtains from the consumption of the pharmaceutical is

$$(1-\alpha)\ln(w-(1-r)pj) + \alpha \ln(h_0 + \Delta j),$$

where $j = 1, 0$. The patient is indifferent between consuming and not consuming the pharmaceutical if

(42) $$(1-\alpha)\ln(w-\theta) + \alpha\ln(h_1) = (1-\alpha)\ln(w) + \alpha\ln(h_0),$$

where θ is the patient's willingness to pay for the pharmaceutical. The equation (42) can be rearranged to obtain

$$\ln\left(\frac{w-\theta}{w}\right) = \ln\left(\left(\frac{h_0}{h_1}\right)^{\frac{\alpha}{1-\alpha}}\right)$$

or

(43) $$1 - \frac{\theta}{w} = \left(\frac{h_0}{h_1}\right)^{\frac{\alpha}{1-\alpha}}.$$

The equation (43) can be solved with respect to θ to obtain

(44) $$\theta = w\left[1 - \left(\frac{h_0}{h_1}\right)^{\frac{\alpha}{1-\alpha}}\right] = w\left[1 - \left(1 + \frac{\Delta}{h_0}\right)^{\frac{-\alpha}{1-\alpha}}\right],$$

where the last equality follows from the fact that $\frac{h_0}{h_1} = \frac{1}{1+\frac{\Delta}{h_0}}$. □

Proof of Proposition 1. The regulator's problem is to find the price-insurance policy (p, r) which maximizes social welfare

$$W = CS(p,r) + \pi(p,r) - (1+\lambda)IE(p,r)$$

subject to the profit constraint

$$-\pi(p,r) \leq 0$$

and the feasibility constraints

$$p \geq 0$$

$$0 \leq r \leq 1.$$

The above problem is called an original problem. In what follows, we analyse the solutions of the original problem without the feasibility constraints. Such a problem is called a relaxed problem. This approach to finding the solution to the regulator's problem through the relaxed problem rests on the intuition that, if solutions of the relaxed problem also satisfy the feasibility constraints, they must also solve the original problem. This approach has become a standard analytical tool in the principal-agent literature (e.g. Laffont and Martimort, 2002).

We assume throughout this proof that the conditions $\lambda > 0$ and

(45) $$\frac{(\eta - c)^2\lambda(1+\lambda)}{\eta(1+2\lambda)^2} < F$$

hold true. Let $(\tilde{p}, \tilde{r})$ denote the price-insurance policy that solves the relaxed problem and μ be the Lagrange multiplier of the profit constraint. The Lagrangian function of the relaxed problem is given as follows:

$$L = CS(p,r) + (1+\mu)\,\pi(p,r) - (1+\lambda)IE(p,r).$$

The solution of the relaxed problem must satisfy the first-order conditions:

$$\frac{\partial L}{\partial p} = -\,(1-r)\left[1 - \frac{p(1-r)}{\eta}\right] + (1+\mu)\left[1 - \frac{2p(1-r)}{\eta} + \frac{(1-r)c}{\eta}\right]$$

(46) $$-\,(1+\lambda)\,r\left[1 - \frac{2p(1-r)}{\eta}\right] = 0$$

$$\frac{\partial L}{\partial r} = p\left[1 - \frac{p(1-r)}{\eta}\right] + (1+\mu)(p-c)\frac{p}{\eta}$$

(47) $$-\,(1+\lambda)\,p\left[1 - \frac{p(1-r)}{\eta} + \frac{pr}{\eta}\right] = 0.$$

Moreover, the solution must satisfy the profit constraint and the complementary slackness conditions $-\pi(p, r) \leq 0$, $\mu \geq 0$ and

(48) $$\mu\left[F - (p-c)\left(1 - \frac{p(1-r)}{\eta}\right)\right] = 0.$$

Lemma 1.1 If $(\tilde{p}, \tilde{r}, \tilde{\mu})$ solves the relaxed problem, then $\tilde{\mu} = \lambda$.

Proof. Contrary to the claim, suppose that $\mu \neq \lambda$ in the solution of the relaxed problem. Then the first-order conditions (46) and (47) have two solutions. The first solution is $\bar{p} = 0$ and $\bar{r} = [\eta\mu + c(1+\mu)]/[\eta\lambda + c(1+\mu)]$ and the second solution is $\check{p} = [\eta(1+\lambda) - c(1+\mu)]/[\lambda - \mu]$ and $\check{r} = [(\eta - c)(1+\mu)]/[\eta(1+\lambda) - c(1+\mu)]$. When evaluated at these two solutions, the profit of the firm is $-\pi(\bar{p}, \bar{r}) = c + F$ and $-\pi(\check{p}, \check{r}) = F$, respectively. Therefore, the solutions of the first-order conditions (46) and (47) never satisfy the profit constraint. This implies that, if $\mu \neq \lambda$, there is no price-insurance pair which would satisfy the necessary conditions of the relaxed problem. For solutions to exist, we must therefore have $\mu = \lambda$. □

Lemma 1.2 If $(\tilde{p}, \tilde{r}, \tilde{\mu})$ solves the relaxed problem, then any pair $(\tilde{p}, \tilde{r})$ satisfying

(49) $$p = \frac{\eta\lambda + c(1+\lambda)}{(1-r)(1+2\lambda)}$$

satisfies both first-order conditions (46) and (47).

Proof. Suppose that $(\tilde{p}, \tilde{r}, \tilde{\mu})$ solves the relaxed problem. Then, the first-order condition (46) holds true for any pair (p, r) for which

(50) $$p = \frac{\eta\tilde{\mu} + c(1+\tilde{\mu}) - r\left[\eta\lambda + c(1+\tilde{\mu})\right]}{(1-r)\left[1+2\tilde{\mu} - r(1+2\lambda)\right]}$$

and the first-order condition (47) is satisfied for any pair (p, r) for which

(51) $$p = \frac{\eta\lambda + c(1+\tilde{\mu})}{1+\tilde{\mu}+\lambda - r(1+2\lambda)} \quad \text{or} \quad p = 0.$$

The solution $p = 0$ can be ruled out because it does not satisfy the profit constraint. By Lemma 1.1, the solution of the relaxed problem must satisfy $\tilde{\mu} = \lambda$. Evaluating the right-hand sides of the equations (50) and (51) at $\tilde{\mu} = \lambda$ yields the equation (49). □

Let us then characterize the solution of the problem. By Lemma 1.1 and the assumption $\lambda > 0$, we must have $\tilde{\mu} = \lambda > 0$. Then the complementary slackness conditions imply that the zero-profit condition $\pi(p, r) = 0$ must hold true at the solution of the regulator's problem. Solving the first-order condition (46) or (47) together with the zero-profit condition yields the optimal price and insurance coverage and the value of the Lagrange multiplier:

$$\tilde{p} = c + \frac{\eta F(1+2\lambda)}{(\eta - c)(1+\lambda)}$$

$$\tilde{r} = \frac{\eta F(1+2\lambda)^2 - (\eta-c)^2\lambda(1+\lambda)}{(1+2\lambda)\left[\eta F\,(1+2\lambda) + c(\eta-c)\,(1+\lambda)\right]}$$

$$\tilde{\mu} = \lambda.$$

When evaluated at the point $(\tilde{p}, \tilde{r}, \tilde{\mu})$, the determinant of the bordered Hessian matrix is

$$\left|\bar{H}\right| = \frac{\left[c(\eta-c)(1+\lambda) + \eta F\,(1+2\lambda)\right]^2}{\eta^3(1+2\lambda)} > 0,$$

which proves that the optimal policy is a local maximum.

Let us finally check that the solution of the relaxed problem satisfies the feasibility conditions. By straightforward calculation, one can demonstrate that the optimal price-insurance policy satisfies the condition $\tilde{r} < 1$. In addition, it holds true that $\tilde{r} > 0$, because the fixed cost satisfies the condition (45). Since the optimal price $\tilde{p}$ is strictly positive, the solution satisfies the feasibility conditions of the original problem. □

Proof of Proposition 2. Let us define

$$A(\eta, F) \equiv \eta F(1+2\lambda)^2 - (\eta-c)^2\lambda(1+\lambda) \text{ and}$$

$$B(\eta, F) \equiv (1+2\lambda)\left[\eta F\,(1+2\lambda) + c(\eta-c)\,(1+\lambda)\right].$$

and let A_η (B_η) and A_F (B_F) denote the partial derivatives of A (B) with respect to η and F.

First, we skip the proof for the expression $\frac{\partial \tilde{p}}{\partial F}$. Secondly, the partial derivative of optimal insurance coverage with respect to the fixed cost is

$$\frac{\partial \tilde{r}}{\partial F} = \frac{A_F B(\eta, F) - B_F A(\eta, F)}{B(\eta, F)^2} = \frac{\eta(1+2\lambda)^2 \left[B(\eta, F) - A(\eta, F)\right]}{(1+2\lambda)^2 \left[\eta F(1+2\lambda) + c(\eta - c)(1+\lambda)\right]^2}$$

$$= \frac{\eta(\eta - c)(1+\lambda)(\eta\lambda + c(1+\lambda))}{\left[\eta F(1+2\lambda) + c(\eta - c)(1+\lambda)\right]^2} > 0.$$

Thirdly, the partial derivative of the optimal producer price with respect to the quality weight is

$$\frac{\partial \tilde{p}}{\partial \eta} = \frac{F(1+2\lambda)(\eta - c)(1+\lambda) - (1+\lambda)(\eta F(1+2\lambda))}{(\eta - c)^2(1+\lambda)^2} = \frac{-cF(1+2\lambda)}{(1+\lambda)(\eta - c)^2} < 0.$$

Finally, the partial derivative of the optimal insurance coverage with respect to the quality weight is

$$\frac{\partial \tilde{r}}{\partial \eta} = \frac{A_\eta B(\eta, F) - B_\eta A(\eta, F)}{B(\eta, F)^2} \tag{52}$$

After some lengthy calculations, the above partial derivative (52) simplifies to

$$\frac{\partial \tilde{r}}{\partial \eta} = \frac{-(1+\lambda)\left[F(1+2\lambda)\left(\lambda\eta^2 + (1+\lambda)c^2\right) + c(\eta - c)^2\lambda(1+\lambda)\right]}{(1+2\lambda)\left[\eta F(1+2\lambda) + c(\eta - c)(1+\lambda)\right]^2} < 0,$$

completing the proof. □

Proof of Proposition 3. Let us assume that $\lambda > 0$ and that the fixed cost satisfies the conditions

$$\frac{(\eta - c)^2\lambda(1+\lambda)}{\eta(1+2\lambda)^2} < F < \frac{(\eta - c)^2}{4\eta}. \tag{53}$$

The above conditions (53) have two implications: first, they ensure that the Ramsey price is well-defined and, secondly, the conditions imply that the optimal price-insurance policy is an interior solution.

We first prove that $\tilde{W} > W_R$. Define the welfare difference

$$DW(F) \equiv \tilde{W} - W_R$$

$$= \frac{(\eta - c)^2}{2\eta}\frac{(1+\lambda)^2}{1+2\lambda} - F(1+\lambda) - \frac{1}{8\eta}\left(\eta - c + \sqrt{(\eta - c)^2 - 4\eta F}\right)^2.$$

The first derivative of the welfare difference with respect to the fixed cost F is given as

$$DW'(F) = -(1+\lambda) + \frac{\eta - c + \sqrt{(\eta - c)^2 - 4\eta F}}{2\sqrt{(\eta - c)^2 - 4\eta F}},$$

and the second derivative is

$$DW''(F) = \frac{\eta(\eta - c)}{\left(\sqrt{(\eta - c)^2 - 4\eta F}\right)^3} > 0.$$

Therefore, the welfare difference is a strictly convex function of the fixed cost F. The strict convexity of the function $DW(F)$ implies that the unconstrained minimum of the welfare difference must be unique. Solving the first-order condition $DW'(F) = 0$ with respect to F yields the minimum point

$$F_1 = \frac{(\eta - c)^2}{\eta}\frac{\lambda(1+\lambda)}{(1+2\lambda)^2} \geq 0, \tag{54}$$

which corresponds to the infimum of the interval of the fixed cost (53). This implies that $DW(F) > DW(F_1)$ for all values of the fixed cost that satisfy the condition (53). When evaluated at the minimum point, the value of the welfare difference is zero:

$$\begin{aligned} DW(F_1) &= \frac{(\eta-c)^2}{2\eta}\frac{(1+\lambda)^2}{1+2\lambda} - F_1(1+\lambda) - \frac{1}{8\eta}\left(\eta - c + \sqrt{(\eta-c)^2 - 4\eta F_1}\right)^2 \\ &= \frac{(\eta-c)^2}{2\eta}\left(\frac{1+\lambda}{1+2\lambda}\right)^2(1+2\lambda) - \frac{(\eta-c)^2}{2\eta}\left(\frac{1+\lambda}{1+2\lambda}\right)^2(2\lambda+1) \\ &= 0. \end{aligned}$$

These observations imply that $DW(F) > DW(F_1) = 0$ and $\tilde{W} > W_R$ for all fixed costs satisfying the conditions (53).

Secondly, we have $\bar{W}_f \geq \tilde{W}$ when

$$\frac{(\eta-c)^2}{2\eta} - F \geq \frac{(\eta-c)^2}{2\eta}\frac{(1+\lambda)^2}{(1+2\lambda)} - F(1+\lambda), \tag{55}$$

which implies that

$$\frac{(\eta-c)^2}{2\eta}\frac{\lambda}{1+2\lambda} \leq F.$$

But now

$$\frac{(\eta-c)^2}{2\eta}\frac{\lambda}{1+2\lambda} = \frac{(\eta-c)^2}{2\eta}\frac{\lambda(1+2\lambda)}{(1+2\lambda)^2} < \frac{(\eta-c)^2}{\eta}\frac{\lambda(1+\lambda)}{(1+2\lambda)^2}, \tag{56}$$

where the last expression corresponds to the infimum of the set of feasible fixed costs (53). Hence, the condition (55) is satisfied as a strict inequality when (53) holds true and $\lambda > 0$, which verifies that $\bar{W}_f > \tilde{W}$. □

Proof of Proposition 4. Let us assume that the conditions $\lambda > 0$ and

$$\frac{(\eta-c)^2 2(1+\lambda)(1+2\lambda)}{\eta(2+3\lambda)^2} < F \tag{57}$$

hold true in this proof.

As above in the proof of Proposition 1, we will start by analysing the relaxed problem in which the feasibility constraints $p \geq 0$ and $0 \leq r_t \leq 1$ for $t = l, h$ are initially ignored. The Lagrangian function of the relaxed problem is given as follows

$$L = CS(p, r_l, r_h) + (1+\mu)\,\pi(p, r_l, r_h) - (1+\lambda)\,IE(p, r_l, r_h) - \kappa\,(r_h - r_l),$$

where the consumer surplus is

$$CS(p, r_l, r_h) = \int_{\frac{p(1-r_l)}{\eta}}^{\frac{p(1-r_h)}{\eta}} (w\eta - (1-r_l)p)\, dw + \int_{\frac{p(1-r_h)}{\eta}}^{1} (w\eta - (1-r_h)\, p)\, dw,$$

the firm's profit is

$$\pi(p, r_l, r_h) = (p-c)\left(1 - \frac{p(1-r_l)}{\eta}\right) - F,$$

and the insurance expenditures are

$$IE(p, r_l, r_h) = r_l p \left(\frac{p(1-r_h)}{\eta} - \frac{p(1-r_l)}{\eta}\right) + r_h p \left(1 - \frac{p(1-r_h)}{\eta}\right),$$

and κ is the multiplier of the constraint $r_h \leq r_l$.

The solution of the relaxed problem $(\hat{p}, \hat{r}_l, \hat{r}_h, \hat{\mu}, \hat{\kappa})$ must satisfy the first-order conditions:

$$\frac{\partial L}{\partial p} = \frac{p\,(r_l - r_h)^2}{\eta} - (1-r_h)\left[1 - \frac{p\,(1-r_h)}{\eta}\right]$$

$$+ (1+\mu)\left[1 - \frac{2p(1-r_l)}{\eta} + \frac{(1-r_l)c}{\eta}\right]$$

(58) $$- (1+\lambda)\left[r_l\left(\frac{2p(r_l - r_h)}{\eta}\right) + r_h\left(1 - \frac{2p(1-r_h)}{\eta}\right)\right] = 0$$

$$\frac{\partial L}{\partial r_h} = \frac{-p^2(r_l - r_h)}{\eta} + p\left(1 - \frac{p(1-r_h)}{\eta}\right)$$

(59) $$-(1+\lambda)\left[\frac{-p^2(r_l - r_h)}{\eta} + p\left(1 - \frac{p(1-r_h)}{\eta}\right)\right] - \kappa = 0$$

$$\frac{\partial L}{\partial r_l} = \frac{p^2(r_l - r_h)}{\eta} + (1+\mu)(p-c)\frac{p}{\eta}$$

(60) $$-(1+\lambda)\left(\frac{p^2(r_l - r_h)}{\eta} + \frac{p^2 r_l}{\eta}\right) + \kappa = 0$$

Moreover, the solution must satisfy the profit constraint and its complementary slackness conditions $-\pi(p, r_l, r_h) \leq 0$, $\mu \geq 0$ and

(61) $$\mu\left[F - (p-c)\left(1 - \frac{p(1-r_l)}{\eta}\right)\right] = 0$$

and the means-testing constraint $r_h \leq r_l$ and its complementary slackness conditions $r_h - r_l \leq 0$, $\kappa \geq 0$ and

(62) $$\kappa(r_h - r_l) = 0.$$

From the perspective of the ensuing analysis it is important to note that effective means-testing, i.e. $r_h < r_l$, occurs in the solution of the regulator's problem only if $\kappa = 0$. If this is not the case and $\kappa > 0$ then by the condition (62) we must have $r_h = r_l$ in the optimal solution. Both low- and high-income patients receive the same insurance reimbursement and means-testing does not take place. Furthermore, it is straightforward to show that the necessary conditions of the problem simplify to those of the optimal price-insurance policy examined in Section 4. Therefore, the following analysis concentrates on the means-testing solution in which $\kappa = 0$.

Lemma 4.1 If $(\hat{p}, \bar{r}_l, \bar{p}_h, \hat{\mu})$ solves the relaxed problem, then $\hat{\mu} = \lambda$.

Proof. Contrary to the claim, suppose that $\mu \neq \lambda$ in the solution of the relaxed problem. Then the first-order conditions (58), (59) and (60) have two solutions $(\bar{p}, \bar{r}_l, \bar{r}_h)$ and $(\check{p}, \check{r}_l, \check{r}_h)$. In the first solution $\bar{p} = 0$ and insurance coverage rates must satisfy the condition (multiple solutions)

$$\bar{r}_h = \frac{1}{\lambda}[\mu + (1+\mu)\frac{c}{\eta}(1-\bar{r}_l)]$$

In the second solution $\check{p} = [\eta(1+\lambda)-c(1+\mu)]/[\lambda-\mu]$ and $\check{r}_h = \check{r}_l = [(\eta-c)(1+\mu)]/[\eta(1+\lambda)-c(1+\mu)]$. When evaluated at these two solutions, the profit of the firm is $-\pi(\bar{p}, \bar{r}_l, \bar{r}_h) = c + F$ and $-\pi(\check{p}, \check{r}_l, \check{r}_h) = F$, respectively. Therefore, the solutions of the first-order conditions (58), (59) and (60) never satisfy the profit constraint. This implies that, if $\mu \neq \lambda$, there are no price-insurance policies that would satisfy the necessary conditions of the relaxed problem. For solutions to exist, we must have $\mu = \lambda$. □

Let us then derive the solution to the regulator's problem. By Lemma 4.1 and the assumption $\lambda > 0$, we must have $\hat{\mu} = \lambda > 0$. Complementary slackness conditions for the profit constraint then imply that $\pi(p, r_l, r_h) = 0$. Solving first-order conditions (58), (59) and (60) together with the zero-profit condition yields the means-tested price-insurance policy and the value of the Lagrange multiplier:

$$\hat{p} = c + \frac{\eta F(2+3\lambda)}{(\eta-c)2(1+\lambda)}$$

$$\hat{r}_l = \frac{\eta F(2+3\lambda)^2 - (\eta-c)^2\, 2\lambda\,(1+\lambda)}{(2+3\lambda)\,[\eta F(2+3\lambda) + c(\eta-c)2(1+\lambda)]}$$

$$\hat{r}_h = \frac{\eta F(2+3\lambda)^2 - (\eta-c)^2\, 2\,(1+\lambda)\,(1+2\lambda)}{(2+3\lambda)\,[\eta F(2+3\lambda) + c(\eta-c)2(1+\lambda)]}$$

$$\hat{\mu} = \lambda.$$

It is worth noting that, because $\lambda < 1 + 2\lambda$, there is effective means-testing and $\hat{r}_h < \hat{r}_l$.

Lemma 4.2 The above solution of the relaxed problem is a local maximum.

Proof. To check that the means-tested price-insurance policy derived above is a local maximum, first note that the relevant bordered Hessian is a 4 × 4 matrix with the profit constraint binding. When evaluated at the solution of the problem, the determinants of the last two (ie. $n - k = 3 - 1 = 2$) leading principal minors of the bordered Hessian are

$$|\bar{H}_4| = \frac{-\lambda\,[2c(\eta-c)(1+\lambda) + F\eta(2+3\lambda)]^4}{4(\eta-c)^2\eta^4(1+\lambda)^2(2+3\lambda)} < 0, \tag{63}$$

and

$$|\bar{H}_3| = \frac{A(F)}{2(\eta-c)^2\eta^3(1+\lambda)^2(2+3\lambda)^2}, \tag{64}$$

where $A(F) \equiv B + CF + DF^2$ is a quadratic function in the fixed cost. The expressions for B, C and D are defined as follows:

$$B \equiv (1+\lambda)^4(1+2\lambda)\left[8c^2(c^4+6c^2\eta^2+\eta^4)-32c^3\eta(c^2+\eta^2)\right]$$

$$C \equiv 4\eta c(1+\lambda)^2(2+3\lambda)[-c^3(1+\lambda)(2+5\lambda)+\eta^3(2+\lambda)(1+2\lambda)$$

$$-\eta^2 c(6+\lambda(17+9\lambda))+\eta c^2(6+\lambda(19+12\lambda))]$$

$$D \equiv \eta^2(2+3\lambda)^2[c^2(1+\lambda)^2(2+7\lambda)-2c\eta(1+\lambda)(2+\lambda(5+\lambda))$$

$$+\eta^2(2+\lambda(7+4\lambda(2+\lambda)))]$$

To show that the proposed solution is a local maximum point, we need to show that $\bar{H}_3 > 0$. To do this it suffices to show that $A(F) > 0$ for all relevant values. We do this in two steps.

Step 1. First, we first show that $A(F)$ is a strictly convex function of the fixed cost by showing that $D > 0$ for all relevant values. Define $D_p(\eta) \equiv \frac{D}{\eta^2(2+3\lambda)^2}$. Then

$$D_p(\eta) = c^2(1+\lambda)^2(2+7\lambda)-2c\eta(1+\lambda)(2+\lambda(5+\lambda))+\eta^2(2+\lambda(7+4\lambda(2+\lambda)))$$

Since $\eta^2(2+3\lambda)^2 > 0$, to prove that $D > 0$ it suffices to demonstrate that $D_p(\eta) > 0$ for all relevant values of η. The expression $D_p(\eta)$ is a strictly convex function in η with a unique minimum point η_m, which can be found by solving the condition $D'_p(\eta) = 0$ with respect to η. When evaluated at η_m, the value of $D_p(\eta)$ is

$$D_p(\eta_m) = \frac{c^2\lambda(1+\lambda)^2(2+3\lambda)^3}{2+\lambda(7+4\lambda(2+\lambda))} > 0$$

where the strict inequality holds true by the assumptions $c > 0$ and $\lambda > 0$. This implies that $D_p(\eta) \geq D_p(\eta_m) > 0$ for all η and hence also for parameter values $\eta > c$.

Step 2. By the first step, the expression $A(F)$ has a unique minimum point with respect to F, denoted as F_m, which can be found by solving the condition $A'(F) = 0$ with respect to F. When evaluated at the minimum point, the value of the function $A(F)$ is

$$A(F_m) = \frac{4c^2(\eta-c)^2\lambda(1+\lambda)^4(2+3\lambda)(c(1+\lambda)+\eta(1+2\lambda))^2}{E}$$

where

$$E \equiv c^2(1+\lambda)^2(2+7\lambda)-2c\eta(1+\lambda)(2+\lambda(5+\lambda))+\eta^2(2+\lambda(7+4\lambda(2+\lambda))).$$

Note that $E = D_p(\eta)$. Step 1 above thus showed that the denominator of $A(F_m)$ is strictly positive for all relevant parameter values. Similarly, the numerator of the $A(F_m)$ is strictly positive because $c > 0$ and $\lambda > 0$. Therefore $A(F) > 0$ and $|\bar{H}_3| > 0$ when $\lambda > 0$ and $c > 0$, which verifies that the solution of the regulator's relaxed problem is a local maximum. □

To check that the solution of the relaxed problem $(\hat{p}, \hat{r}_l, \hat{r}_h)$ satisfies the feasibility conditions, note that the solution satisfies the constraint $p \geq 0$. The condition (57) ensures that $\hat{r}_h > 0$. That $\hat{r}_l > 0$ follows then from the fact that $\hat{r}_h < \hat{r}_l$. Straightforward calculation shows that $\hat{r}_t < 1$ for both $t = l, h$. Hence, the solution of the relaxed problem also solves the original problem. □