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Statistical characteristics of L1 carrier phase observations from four low-cost GPS receivers

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Abstract. *Statistical properties of L1 carrier phase observations from four low-cost GPS receivers are investigated through a case study. The observations are collected on a zero baseline with a frequency of 1 Hz and processed with a double difference model.*

The carrier phase residuals from an ambiguity fixed solution and zero baseline coordinate components computed on an epoch-by-epoch basis are used for accessing the statistical properties of the observations. Graphical statistical methods are used for characterizing the statistical properties.

For each type of receiver, the residuals have a sample mean value close to zero and the sample variance is time invariant. The residuals from one type of receiver deviate from being normally distributed, whereas the residuals from the remaining receivers are close to being normally distributed. Two of the receivers deliver uncorrelated carrier phase observations. Some of the carrier phase observations from the other two receivers are serially correlated. The correlation is receiver specific and is related to the individual channels of the receivers.

Key words. *Low-cost GPS, L1 carrier phase observations, white gaussian noise, normal probability plot, sample autocorrelation function, Garmin 12XL, Magellan AC12, ublox ANTARIS TIM-LP, ublox ANTARIS 4 LEA-4T.*

1 Introduction

It is possible to access the raw L1 carrier phase observations from some low-cost GPS receivers. Therefore, these receivers could for instance be used for precise relative phase positioning. Before doing so it is desirable to assess some statistical properties of the observations.

The article presents a case study of single frequency carrier phase observations collected with four pairs of low-cost GPS receivers (Garmin 12XL, Magellan AC12, ublox ANTARIS TIM-LP, and ublox ANTARIS 4 LEA-4T). The Magellan

receivers and the ublox receivers cost approximately €1000 apiece and €300 apiece. The Garmin receivers are no longer sold. The observations are collected using zero baselines where each pair of receivers is connected to the same antenna.

When processing carrier phase observations it is often assumed that the observations are normally distributed and only affected by white noise. The aim of the case study is to determine whether the collected observations possess these statistical characteristics.

The zero baseline carrier phase observations are processed using least squares estimation and a double difference model which eliminates most systematic errors (orbital errors, propagation errors, antenna phase centre errors, and multi-path). The remaining errors reflect the noise of the observations. For each zero baseline, the double difference phase residuals are computed. Likewise, for each zero baseline, baseline vector components are estimated epoch-by-epoch. Graphical statistical methods are applied to the residuals and vector components to determine whether the observations from the four types of low-cost receivers possess the above-mentioned statistical characteristics.

The case study shows that most of the collected data sets are not completely normally distributed and not only affected by white noise. About 10% of the carrier phase observations from the Garmin 12XL receivers deviate from the normal distribution. Some of the carrier phase observations collected with the Magellan AC12 receivers and the ublox ANTARIS 4 LEA-4T receivers show signs of time dependent correlation. Other researchers have conducted similar studies of survey grade receivers, see (Bona, 2000), (Borre and Tiberius, 2000), (Tiberius and Borre, 1999), and (Amiri-Simkooei and Tiberius, 2007). Nor the survey grade receivers deliver normally distributed observations only affected by white noise.

The article is divided into seven parts. The first part is this introduction. The second part presents the double difference model for processing the carrier phase observations. The third part presents the statistical methods used for assessing the statistical properties of the observations. Part four and five describe the conducted experiment and the data processing. Part six and seven present the results and the conclusions of the research.

2 Double difference model for processing GPS carrier phase observations

All the GPS receivers collect both L1 carrier phase observations and L1 code observations. In this case study, it is chosen only to focus on the carrier phase observations. For each zero baseline (in all four zero baselines, one for each type of receiver) the carrier phase observations are processed using a double difference model as most systematic errors are eliminated. It is chosen to parameterize the baselines in baseline components and double difference ambiguities. Thus, the unknowns are the baseline components of the zero baseline and the double difference ambiguities. For each baseline the coordinates of the two antennas are identical, as it is a zero baseline. Therefore, these coordinates could be elimi-

nated from the model. However, it is chosen to keep the coordinate components of the zero baseline as unknowns in the adjustment. This model is often used for survey applications. The functional model is described in many text books, e.g. (Leick, 2004) and (Hofmann-Wellenhof et al., 2008). The ambiguities are estimated using the LAMBDA method (Teunissen, 1994). After the double difference ambiguities are estimated, an epoch-by-epoch ambiguity constrained solution is computed in which only the baseline components are unknowns. These baseline components together with the double difference carrier phase residuals from the ambiguity fixed adjustment are used for accessing the statistical properties of the carrier phase observations.

A very basic stochastic model is used for processing the zero baselines, as it is assumed that all the observed carrier phase observations have equal precision and are only affected by white gaussian noise. Thus, all physical conditions that affect the precision of the observations and the correlation among them are ignored. Although these assumptions do not hold, they are often applied for baseline processing. The applied stochastic model only accounts for the mathematical correlation among the observations coming from forming the double difference observations. This mathematical correlation is for example described in (Hofmann-Wellenhof et al., 2008).

3 Statistical assessment of GPS carrier phase observations

It is often assumed that the carrier phase observations are affected by noise coming from a white gaussian noise process. Thus, it is assumed that the noise is normally distributed with zero mean, has a constant variance, and is serially uncorrelated. A white gaussian noise process is completely characterized by its mean, variance, and autocorrelation function (Box et al., 2008). The double difference phase residuals from the ambiguity fixed adjustment and the baseline components from the epoch-by-epoch baseline calculations are regarded as time series samples realized from such processes.

Sample mean, sample variance, and sample autocorrelation functions of the time series are used to decide whether the above-mentioned assumptions hold good. The normality of the time series is assessed using normal probability plots.

3.1 Sample mean and sample standard deviation

The sample mean \bar{z} and the sample variance s^2 of a time series of N successive observations $\mathbf{z} = z_1, z_2, \dots, z_N$ are

$$\bar{z} = \frac{1}{N} \sum_{t=1}^N z_t, \quad \text{and} \quad (1)$$

$$s^2 = \frac{1}{N-1} \sum_{t=1}^N (z_t - \bar{z})^2. \quad (2)$$

3.2 Sample autocorrelation function

The sample autocorrelation function is used to evaluate whether the time series is serially uncorrelated. An estimate of the sample autocorrelation coefficient r_k between any two observations from the time series separated in time by lag k is (Box et al., 2008)

$$r_k = \frac{\sum_{t=1}^{N-k} (z_t - \bar{z})(z_{t+k} - \bar{z})}{\sum_{t=1}^N (z_t - \bar{z})^2}. \quad (3)$$

The sample autocorrelation function is a scatter plot of the sample autocorrelation coefficients ($k = 0, 1, 2, \dots$) of the time series against lag, see figure 1. The 0th lag sample autocorrelation coefficient is unity by definition, see equation 3. If the time series is serially uncorrelated, all other sample autocorrelation coefficients are expected to be close to zero, as the autocorrelation coefficient of a white noise process is zero for $k \neq 0$ (Box et al., 2008). If the time series is realized from a white noise process, 95% of the sample autocorrelation coefficients r_k should lie in the interval $[-2/\sqrt{N}, 2/\sqrt{N}]$ for $k > 0$ (Box et al., 2008).

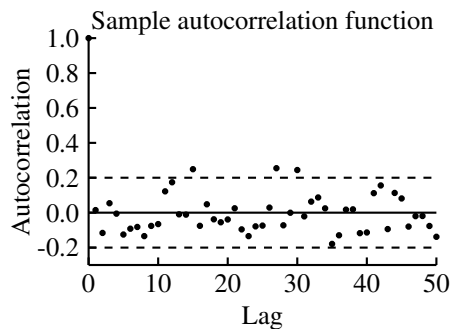


Figure 1. Example of sample autocorrelation function. Broken lines indicate 95% bounds.

3.3 Normal probability plot

The normal probability plot is a graphical tool for assessing whether a time series is normally distributed. The normal probability plot is constructed in a few steps (Montgomery and Runger, 2003), (Thode, 2002):

1. Rank the observations in the time series from smallest to largest. Number the ordered observations $i = (1), (2), \dots, (N)$. The smallest observation is $z_{(1)}$ and the largest observation is $z_{(N)}$.
2. Compute the cumulative probability $F_{(i)}$ of the ordered observation for $i =$

$1, 2, \dots, N$

$$F_{(i)} = \frac{i - 0.5}{N}.$$

3. Compute the inverse of the normal cumulative distribution function corresponding to the cumulative probability computed above, $\Phi^{-1}(F_{(i)})$, for $i = 1, 2, \dots, N$.
4. Plot $\Phi^{-1}(F_{(i)})$ against the ordered observations $z_{(i)}$ for $i = 1, 2, \dots, N$; see figure 2.

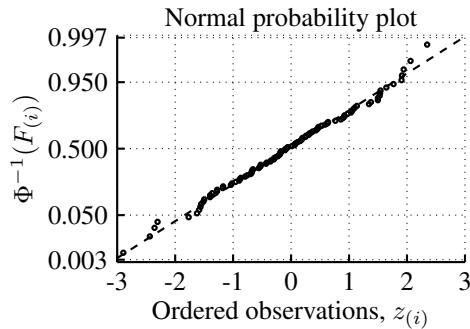


Figure 2. Example of normal probability plot.

For a normally distributed time series, the plot is linear except for random fluctuations in the time series. Systematic deviation from linearity indicates that the time series is not normally distributed.

If the normal probability plot is linear, it is possible to get a rough visual estimate of the sample mean value and sample standard deviation as a by-product of the plot. The sample mean is the intersection of a best-fit straight line through the observations and a horizontal line representing 50% probability. The inverse of the slope of the best-fit straight line is proportional to the sample standard deviation of the time series (Thode, 2002).

4 Experiment

The carrier phase observations were collected with four different kinds of receivers: Garmin 12XL, Magellan AC12, ublox ANTARIS TIM-LP, and ublox ANTARIS 4 LEA-4T. Two receivers of each kind were used. All the receivers are single frequency receivers. The Garmin and the Magellan receivers have 12 channels each, whereas the ublox receivers have 16 channels each. The Magellan receivers and the ublox receivers are designed to output raw carrier phase observations. This is not the case with the Garmin receivers. It is only possible to log carrier phase observations from the Garmin receivers thanks to software written

by Antonio Taberero Galán (Galán, 2002) using undocumented features of the receiver.

The observations were collected at Aalborg University, Denmark, April 14th, 2009. The logging started at 11.45 GPST and lasted for 30 minutes. The observations were logged at a 1 Hz frequency resulting in 1800 epochs of observations.

Observations were collected simultaneously from all eight receivers in order to be able to compare the performance of the receivers. Ideally, all the receivers should have been connected to the same antenna. However, this was not possible. Therefore, two identical low-cost Gilsson antennas were applied. Each antenna was connected to a GPS Networking ALDCBS1X4 antenna splitter which splits the signal to four receivers. Each pair of identical receivers was connected to the same antenna splitter/antenna. The two antennas were placed on the roof of the university building with a clear view of the sky. All the observations were stored on a PC.

The observations from the Garmin receivers were logged using software written by Antonio Taberero Galán (Galán, 2002). The observations from the Magellan and the ublox receivers were logged using *acom32* version 3.00.02 and *u-center* version 5.06 that is serial data collection software provided by Magellan and ublox, respectively.

5 Data processing

The logged observations were converted to RINEX observation files that were used for the subsequent post processing. The Garmin observations were converted using software written by Antonio Taberero Galán (Galán, 2002). The Magellan observations were converted using *log2be* version 1.0.13 and *Rinex Converter* version 2.70 provided by Magellan and Thales Navigation, respectively. Finally, the ublox observations were converted using *TEQC* version 2008Oct2 (UNAVCO, 2008). A RINEX broadcast ephemeris file was downloaded from NGS (National Geodetic Survey - CORS Group, 2009).

5.1 Timing issues

Forming double differences between carrier phase observations eliminates many systematic errors if the observations are taken simultaneously. However, the observations in two RINEX observation files collected during the same time span are not in general taken simultaneously as the observations are affected by the receiver clock errors, see (Gurtner, 2001). Therefore, prior to processing the carrier phase observations, the receiver clock errors were estimated on an epoch-by-epoch basis using pseudo range point position solutions. Next, the time-tags of the observations and the observations themselves were corrected for the estimated receiver clock errors, see (Gurtner, 2001). After having dealt with the receiver clock errors, the observations were synchronous with GPST. However, all the observations were not necessarily taken at exactly the same time. Finally, the observations were

interpolated to coincide with the nearest full second. Figure 3 shows the estimated receiver clock errors of the eight receivers. The manufacturers of the GPS receivers handle the receiver clock errors differently. Garmin lets the clock errors drift freely, Magellan continuously resets the clocks to GPST, and ublox resets the clocks an integer number of milliseconds when necessary to keep the clocks close to GPST.

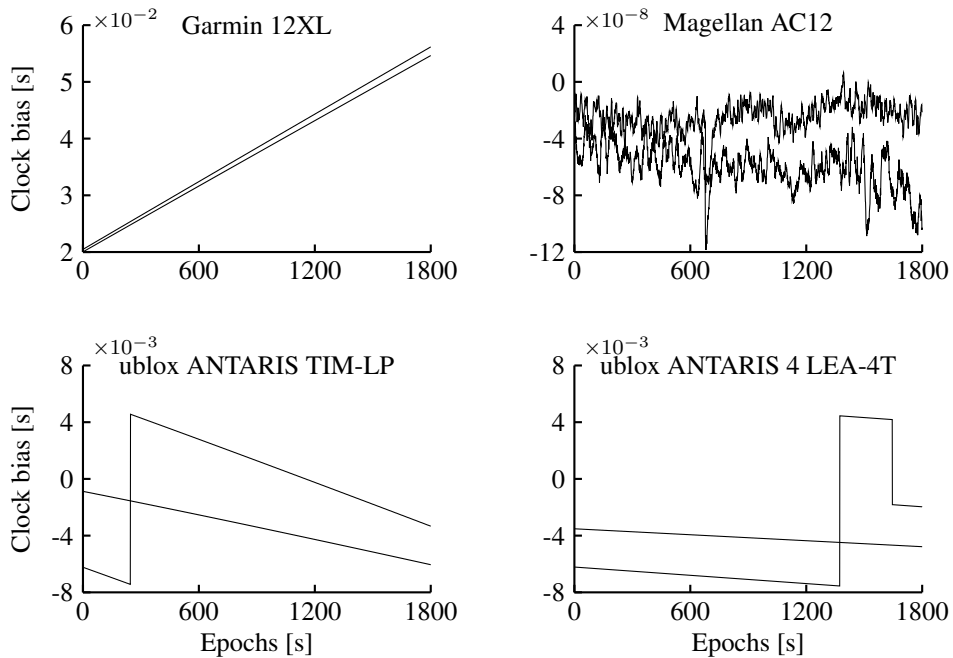


Figure 3. Receiver clock errors. Note the different scales on the vertical axes.

5.2 Processing

All the zero baselines were processed with a cut-off angle of 15 degrees and with PRN 23 as reference satellite through the whole observation period. The elevation angle of PRN 23 was minimum 65 degrees through the entire period. None of the collected observations were affected by cycle slips. For each zero baseline, a float solution was computed based on all the collected carrier-phase observations. All the computed double differenced carrier phase observations were close to being either integers or .5 valued numbers. This indicates that all four types of receivers collect half-cycle carrier phase observations. Next, the ambiguities were estimated using the LAMBDA method (Teunissen, 1994), and a fixed solution was computed. The double difference residuals from this fixed solution were used for assessing the statistical properties of the observations.

After the double difference ambiguities were estimated, an epoch-by-epoch ambiguity constrained solution was computed in which only the baseline com-

ponents were unknowns. This resulted in 1800 estimates of each zero baseline. All of these baselines were transformed to a local topocentric cartesian coordinate system with the axes pointing towards east, north, and up, respectively. These baseline components were also used for accessing the statistical properties of the observations.

6 Results

6.1 Sample mean and sample standard deviation

The double difference phase residuals from the fixed solution are shown in figure 4, and the sample mean and sample standard deviation of the residuals are shown in table 1. Note, that the denominator in equation 2 is $(N - 3)$ when the sample variance and subsequently the sample standard deviation is computed as three coordinate unknowns is being estimated. All the time series have mean values of a few 10th of a millimetre. The standard deviation of the Garmin 12XL double difference residuals is close to 3 millimetres whereas the standard deviations of the residuals from the other receivers are close to 1 millimetre or less. Figure 4 suggests that the variances of the time series are time invariant. Furthermore, the precision of the residuals are illustrated in figure 5.

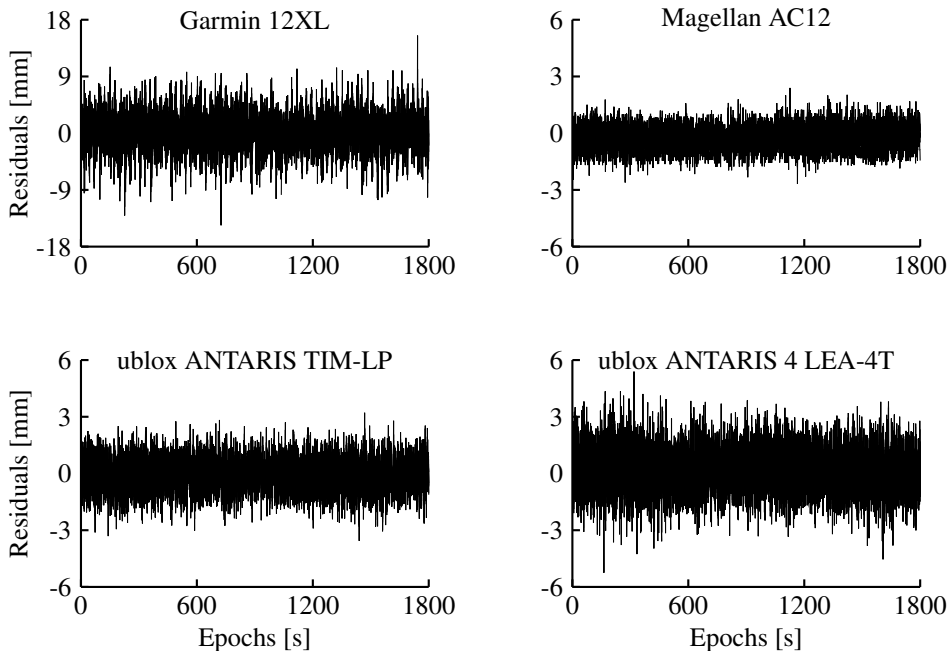


Figure 4. Double difference carrier phase residuals from all satellites. Note that the scale on the vertical axis in the upper left figure is different from the corresponding scales in the other figures.

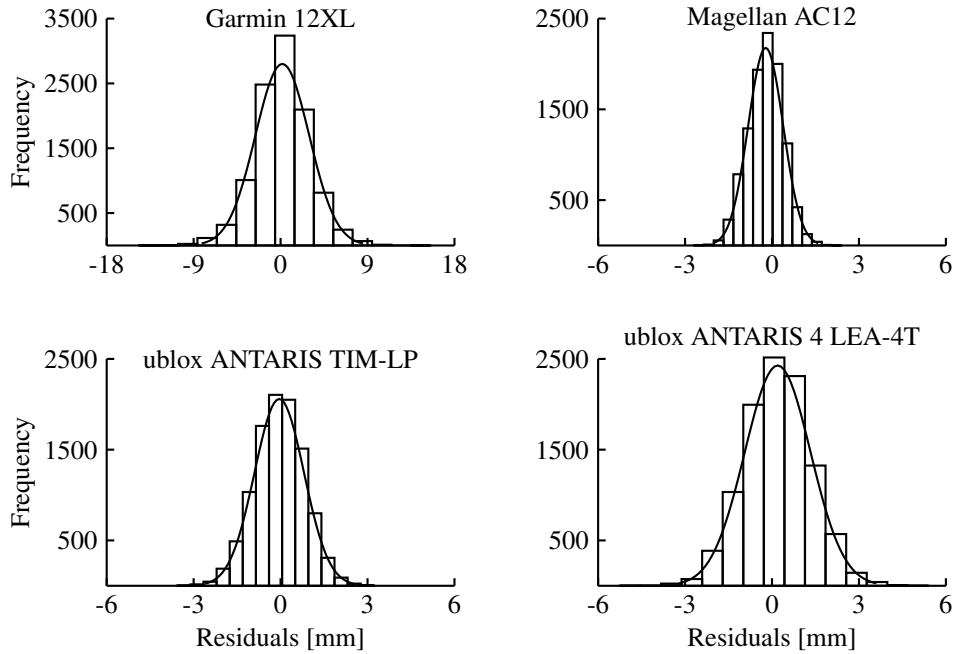


Figure 5. Histograms of double difference carrier phase residuals from all satellites. Note that the scales in the upper left figure are different from the corresponding scales in the other figures.

A double difference observation is composed from adding/subtracting four carrier phase observations. As the number of observations is much larger than the number of unknowns, the sample variance of the double difference phase residuals estimated by equation 2 closely approximates four times the sample variance of a single carrier phase observation (s_{Φ}^2). Hence, the sample standard deviation of a single carrier phase observation is approximated as

$$s_{\Phi} \approx \sqrt{\frac{s_{DD}^2}{4}} = \frac{s_{DD}}{2}. \quad (4)$$

The sample standard deviation of a single carrier phase observation is shown in the last column of table 1. Equation 4 is only valid when it is assumed that

1. the number of unknowns (coordinates) is much smaller than the number of observations,
2. the satellite elevation angle does not affect the precision of the phase observation, and
3. both receivers used for observing a zero baseline have the same effect on the precision of the phase observations.

The first and the second assumption hold good. For each data set about 10000 double difference carrier phase observations are used for estimating 3 unknowns. Figure 6 illustrates that the magnitudes of the residuals do not depend significantly on the elevation angles of the satellites. The validity of the third assumption is not tested. However, the assumption is kept as identical receivers are used for observing each zero baseline.

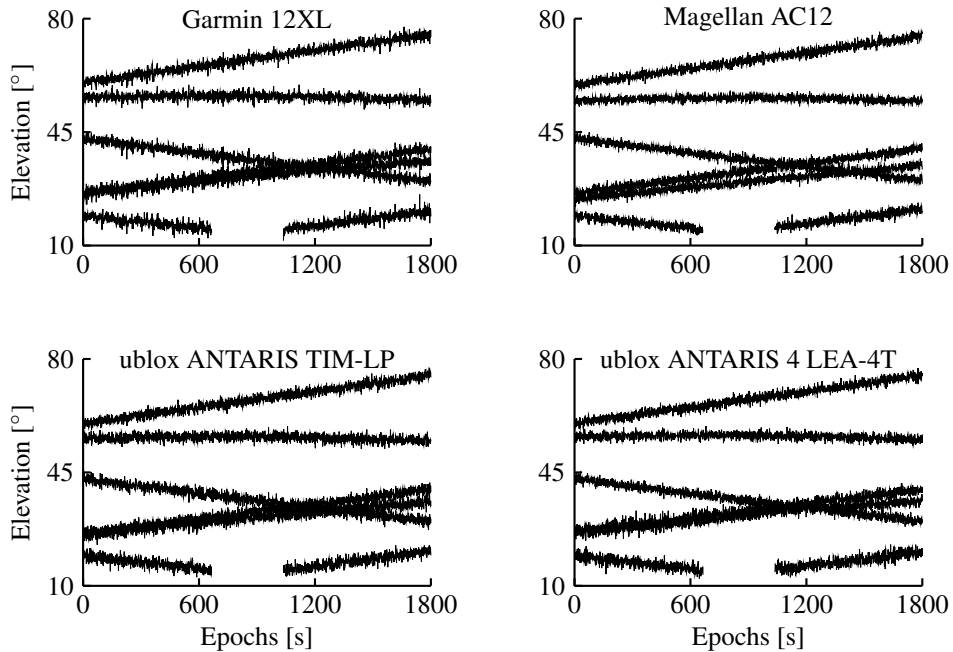


Figure 6. Double difference carrier phase residuals from all satellites superimposed on the elevation graphs of the satellites. The scales of the residuals are not identical in the four subfigures. Therefore, the magnitude of the residuals cannot be compared across the subfigures; the residuals can only be compared within each subfigure.

Table 1. Sample mean and sample standard deviation of double difference phase residuals. Sample standard deviation of carrier phase observations

Receiver	\bar{v}_{DD} [mm]	$s_{v_{DD}}$ [mm]	s_{Φ} [mm]
Garmin 12XL	0.18	2.78	1.40
Magellan AC12	-0.22	0.60	0.30
ublox ANTARIS TIM-LP	-0.05	0.85	0.43
ublox ANTARIS 4 LEA-4T	0.20	1.10	0.55

The zero baseline coordinate components from the epoch-by-epoch ambiguity constrained solution are shown in figure 7, and the sample mean and sample standard deviation of the coordinate components are shown in table 2. The mean values of the coordinate components are all a 10th of a millimetre or less. The less precise phase observations from the Garmin 12XL receiver are carried on to the baseline components, as the Garmin 12XL baseline components are less precise than the baseline components from the other receivers. The precision of the coordinate components is illustrated in figure 8. It applies to all four types of receivers that $s_E < s_N < s_H$. This pattern is caused by the receiver satellite geometry and is typical for GPS positioning at this latitude ($\varphi \approx 57^\circ$).

Table 2. Sample mean and sample standard deviation of zero baseline coordinate components.

Receiver	\bar{E} [mm]	s_E [mm]	\bar{N} [mm]	s_N [mm]	\bar{H} [mm]	s_H [mm]
Garmin 12XL	0.01	1.78	0.01	2.27	0.09	3.77
Magellan AC12	-0.02	0.30	-0.05	0.43	-0.03	0.74
ublox ANTARIS TIM-LP	0.00	0.56	0.00	0.72	0.03	1.17
ublox ANTARIS 4 LEA-4T	-0.01	0.71	0.02	1.10	0.10	1.60

6.2 Normal probability plot

The normal probability plot of the double difference phase residuals are shown in figure 9. The Garmin 12XL normal probability plot is clearly not linear indicating that the distribution of the time series is not normal. A visual inspection of the figure shows that about 10% of the Garmin residuals deviate from the normal distribution. The S-shape of the plot with observations on the left being above the straight line and observations on the right being below the straight line indicates that the time series is realized from a symmetric distribution with longer tails than the normal distribution (Montgomery and Runger, 2003), (Thode, 2002). The plot of the ublox ANTARIS 4 LEA-4T residuals shows the same tendency but less distinct. Only a few per mille of the observations deviate from the straight line. The residuals from the other two receivers seem to be normally distributed.

Comparison of figure 9 and figure 5 shows that histograms are not well suited to identify whether or not a time series is normally distributed; it is difficult to tell from figure 5 that the Garmin 12XL residuals are not normal.

Figure 10 shows the normal probability plot of the zero baseline coordinate components. The same tendencies that apply to the normal probability plot of the double difference phase residuals apply to figure 10. The Garmin 12XL time series is not normally distributed. Except from the north components coming from the ublox ANTARIS 4 LEA-4T receiver, the other normal probability plots are close to being linear.

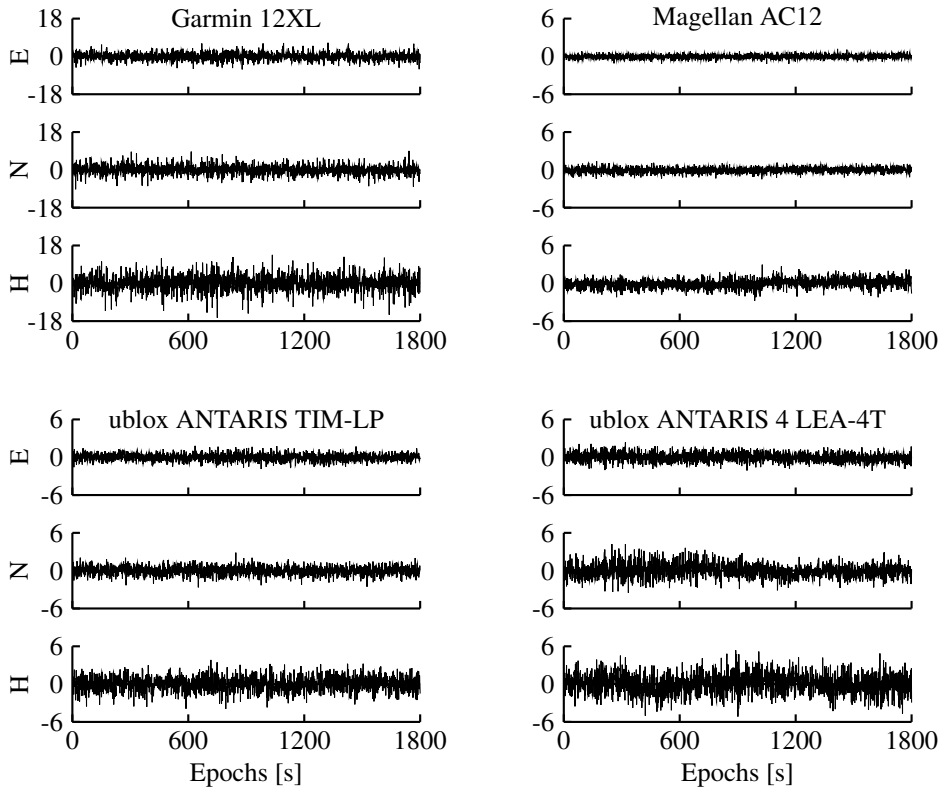


Figure 7. Zero baseline coordinate components. Note that the scales on the vertical axes in the upper left figure are different from the corresponding scales in the other figures.

6.3 Sample autocorrelation function

The sample autocorrelation functions of the double difference phase residuals from all the satellites have been investigated. The sample autocorrelation functions are computed up to lag 1600. Only the sample autocorrelation functions of PRN 2, 4, and 13 are shown in figure 11. These sample autocorrelation functions are chosen as examples of correlated and uncorrelated residuals, respectively. The sample autocorrelation functions of the residuals from the remaining satellites indicate that these residuals are uncorrelated.

All the residuals from the Garmin 12XL receivers and the ublox ANTARIS TIM-LP receivers are uncorrelated. The residuals from PRN 2 and 4 from the Magellan AC12 are slightly correlated whereas the residuals from PRN 2 from the ublox ANTARIS 4 LEA-4T receivers show a more significant correlation.

Correlation among the residuals from for example PRN 2 is only evident on two of the four types of receivers. Therefore, the correlation is caused by the receivers. Further, the correlation among the Magellan AC12 and ublox ANTARIS 4 LEA-4T observations is caused by the individual channels in the receivers as only

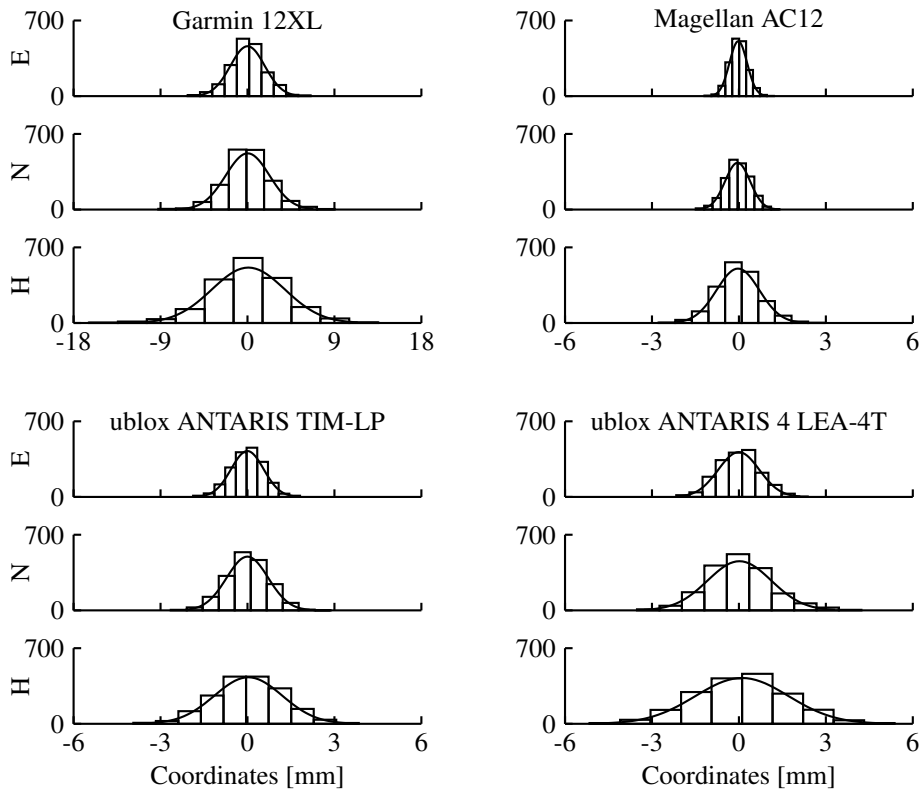


Figure 8. Histograms of zero baseline coordinate components. Note that the scales on the horizontal axes in the upper left figure are different from the corresponding scales in the other figures.

observations coming from some channels show signs of correlation; observations coming from other channels do not show signs of serial correlation.

The sample autocorrelation functions of the zero baseline coordinate components are shown in figure 12. The coordinate components from the Garmin 12XL receivers and the ublox ANTARIS TIM-LP receivers are uncorrelated. The correlations among the carrier phase observations from the Magellan AC12 receivers and the ublox ANTARIS 4 LEA-4T receivers are carried on to the estimated baseline components.

7 Conclusions

The research presented in this article is a case study of the statistical properties of carrier phase observations from four pairs of single frequency low-cost GPS receivers (Garmin 12XL, Magellan AC12, ublox ANTARIS TIM-LP, and ublox ANTARIS 4 LEA-4T).

For each pair of receivers, L1 carrier phase observations were collected on a zero baseline for 30 minutes with a frequency of 1 Hz. Each pair of collected car-

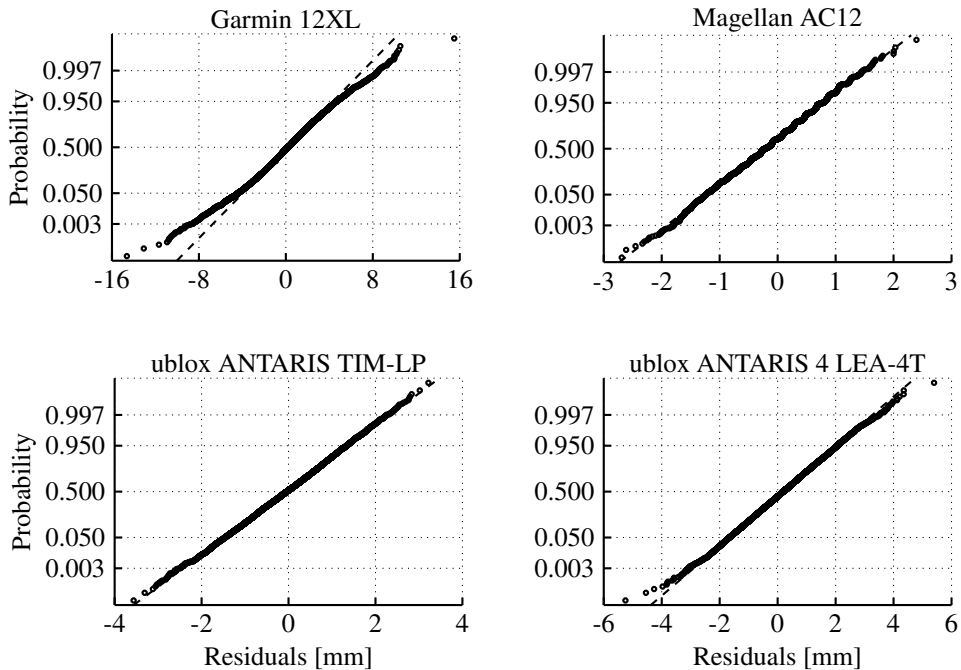


Figure 9. Normal probability plot of all double difference carrier phase residuals. Note the different scales on the horizontal axes.

rier phase observations were processed with a double difference model in order to eliminate the effect of most of the systematic errors. The double difference carrier phase residuals from a ambiguity fixed solution are measures of the capabilities of the receivers and the precision of the carrier phase observations.

The double difference residuals and coordinate components from an ambiguity constrained epoch-by-epoch estimation of the zero baseline are regarded as time series samples coming from a white gaussian noise process. Thus, it is assumed that the noise is normally distributed with zero mean, has a constant variance, and is serially uncorrelated. Sample mean, sample standard deviation, sample autocorrelation functions, and normal probability plots of the time series are used to decide if these assumptions hold good.

The sample mean of all the time series is close to zero. Likewise, the variation of all the time series is almost time invariant. Thus, it is a fair assumption that each of the time series has a zero mean and a constant variance. The variation of the time series is at the millimetre level.

Normal probability plots of the Garmin 12XL time series show that these are not normally distributed. About 10 % of the Garmin 12XL time series deviates from the normal distribution. The time series from the other receivers seem normally distributed.

Sample autocorrelation functions show that the time series from the Garmin

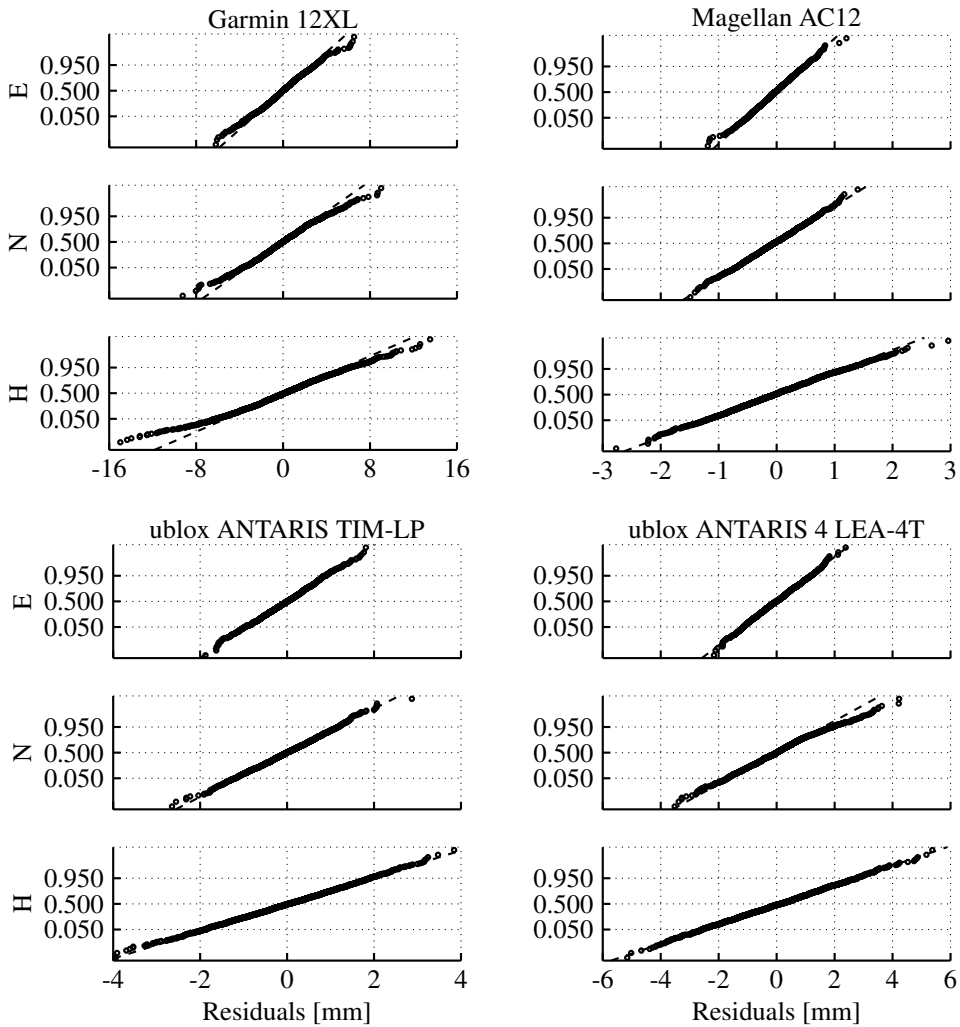


Figure 10. Normal probability plot of zero baseline coordinate components. Note the different scales on the horizontal axes.

12XL receivers and the ublox ANTARIS TIM-LP receivers are uncorrelated at the 1 second lag. Sample autocorrelation functions show that carrier phases observed on some channels of the Magellan AC12 receivers and the ublox ANTARIS 4 LEA-4T receivers are serially correlated, while carrier phases collected on other channels are uncorrelated. These correlations are caused by the receivers.

The case study shows that the Garmin 12XL carrier phase observations are less precise than the carrier phase observations from the other receivers. Likewise, the Garmin 12XL carrier phase observations deviate more from a normal distribution than the carrier phase observations from the other receivers. However, it is fair to mention that the Garmin 12XL receiver is not meant to deliver raw carrier phase

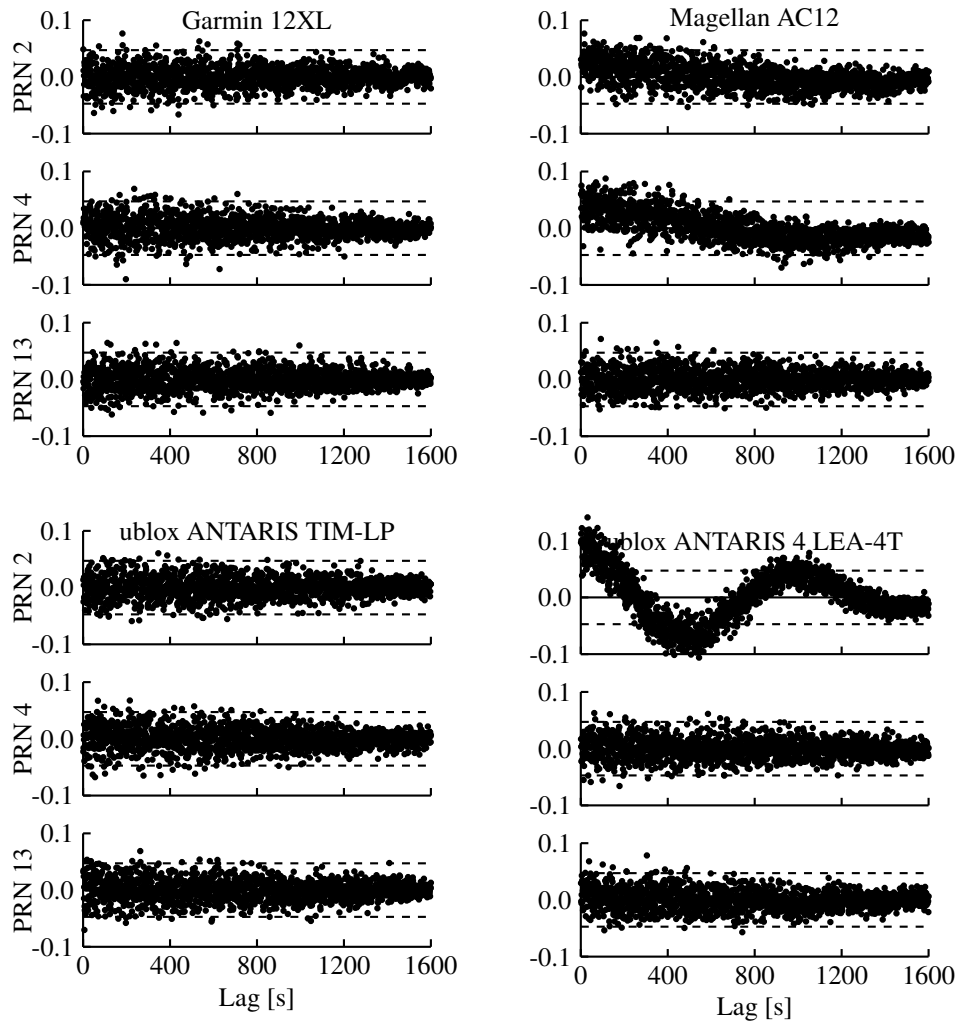


Figure 11. Sample autocorrelation function of double difference carrier phase residuals from PRN 2,4,13. Lag zero is not shown as by definition it equals one. Broken lines indicate 95 % bounds.

observations, which is the case for all the other receivers. It is only possible to access the raw carrier phase observations from the Garmin 12XL receiver thanks to software written by Antonio Taberero Galán (Galán, 2002).

The findings presented in this article are based on residuals from zero baselines. Zero baselines yield more optimistic results than non-zero baselines, as the correlation among the systematic errors weakens with growing distance between the two antennas. Therefore, findings based on residuals from non-zero baselines would probably be less optimistic than the finding presented here. Further studies are needed to shed light on the capabilities of the receivers in real-world position-

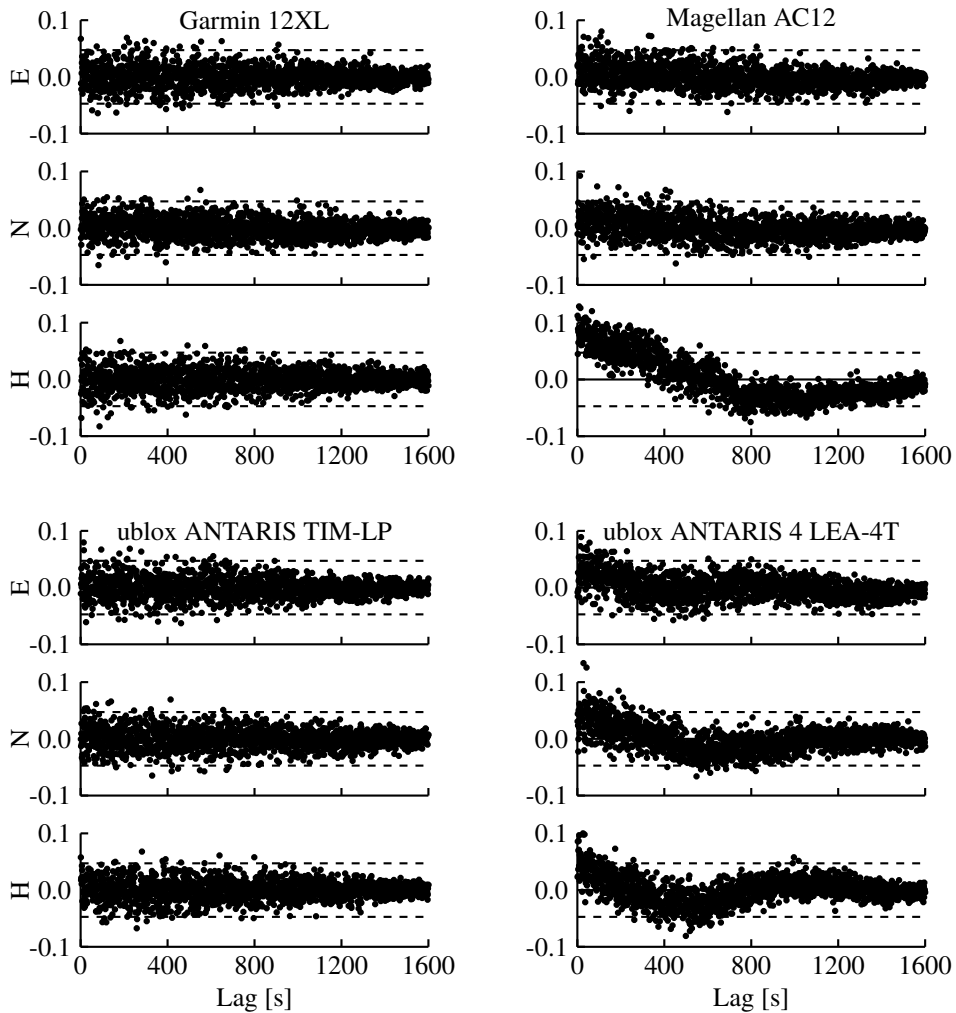


Figure 12. Sample autocorrelation function of zero baseline coordinate components. Lag zero is not shown as by definition it equals one. Broken lines indicate 95 % bounds.

ing applications with non-zero baselines.

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References

- Amiri-Simkooei, A. and Tiberius, C. (2007). Assessing receiver noise using GPS short baseline time series. *GPS Solutions*, 11(1):21–35.
- Bona, P. (2000). Precision, Cross Correlation, and Time Correlation of GPS Phase and Code Observations. *GPS Solutions*, 4(2):3–13.

- Borre, K. and Tiberius, C. (2000). Time Series Analysis of GPS Observables. In Proceedings of The 13th International Technical Meeting of the Satellite Division of the Institute of Navigation GPS 2000, pages 1885 – 1894.
- Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (2008). Time Series Analysis - Forecasting and Control. John Wiley & Sons, 4 edition.
- Galán, A. T. (2002). Obtaining raw data from some Garmin units. <http://artico.lma.fi.upm.es/numerico/miembros/antonio/async/index.html>.
- Gurtner, W. (2001). RINEX: The Receiver Independent Exchange Format Version 2.10. Technical report, Astronomical Institute, University of Berne.
- Hofmann-Wellenhof, B., Lichtenegger, H., and Wasle, E. (2008). GNSS - Global Navigation Satellite Systems GPS, GLONASS, Galileo & more. Springer-Verlag, 1 edition.
- Leick, A. (2004). GPS Satellite Surveying. John Wiley & Sons, 3 edition.
- Montgomery, D. C. and Runger, G. C. (2003). Applied Statistics and Probability for Engineers. John Wiley & Sons, Inc.
- National Geodetic Survey - CORS Group (2009). CORS DATA and Related Information. <http://www.ngs.noaa.gov/CORS/Data.html>.
- Teunissen, P. (1994). A New Method for Fast Carrier Phase Ambiguity Estimation. In Proceedings IEEE Position, Location and Navigation Symposium PLANS'94, pages 562–573.
- Thode, Jr., H. C. (2002). Testing For Normality. Marcel Dekker, Inc.
- Tiberius, C. and Borre, K. (1999). Are GPS Data Normally Distributed? In Schwartz, K.-P., editor, Geodesy Beyond 2000 - The Challenges of the First Decade, volume 121, pages 245–248. International Association of Geodesy, Springer.
- UNAVCO (2008). TEQC - The Toolkit for GPS/GLONASS/Galileo/SBAS Data. <http://facility.unavco.org/software/teqc/teqc.html>.