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Development of the Finnish Height Conversion Surface FIN2005N00

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Abstract. With the new Finnish height system, N2000, came the need for a new national height conversion surface, with which ellipsoidal EUREF-FIN heights, as measured with GPS in Finland, can be transformed into N2000 heights, as measured by levelling. The conversion surface was calculated by fitting a correction surface to the Nordic NKG2004-geoid model using 50 EUVN-DA-points, for which both levelled and GPS-determined heights were available.

Polynomial surfaces with varying degrees were fitted to the data, as well as least-squares collocation surfaces using varying parameter values. The surfaces were analysed using cross-validation. Collocation surfaces performed better than the polynomial surfaces, resulting in smoother surfaces and better fit statistics.

Best results were obtained using a correlation length of 200 km and 2 cm noise level in the least-squares collocation. The resulting surface was added to the NKG2004 geoid model to form the new height conversion surface for Finland: FIN2005N00. The standard deviation of the cross-validation residuals indicates that with the new surface heights can be converted with an accuracy better than 2 cm.

Keywords: geoid model, height conversion surface, Finland, FIN2005N00

1 Introduction

After completion of the third precise levelling of Finland a new national height system, N2000, was introduced to the public in autumn 2007 (JHS163, 2007; Lehmuskoski et al., 2008). The new height system brought the need for a new height conversion surface to be used in GPS-levelling in Finland. This new surface is the link between N2000-heights and ellipsoidal heights in the national ETRS89 realization EUREF-FIN, as measured by GPS. The new height conversion surface was developed by fitting the Nordic NKG2004 geoid model to GPS/levelling data.

This article describes how the fitting to GPS/levelling data was done and how the final solution was chosen. The NKG2004 geoid model and the GPS/

levelling data are described in section 2. Possible correction surfaces were fitted to the NKG model using two different techniques, polynomial fitting and least-squares collocation. Both techniques and the correction surfaces they produced are described in section 3. This section also analyses the possible correction surfaces using cross-validation. The results are then discussed in section 4. The conclusions in Section 5 present the final height conversion surface solution and its accuracy, which is given by the standard deviation of the cross-validation residuals.

2 Data

The new height conversion surface of Finland was calculated by fitting the Nordic geoid model, NKG2004, to GPS/levelling data. The NKG 2004 model is described in section 2.1. and the GPS/levelling data in section 2.2.

2.1 NKG2004 geoid model

The NKG2004 geoid model (figure 1) is the latest geoid model covering the Nordic countries. It was developed by the working group for geoid determination of the Nordic Geodetic Commission. It is a follow up on the NKG96 model (Forsberg et al., 1996). Relative to the NKG96 model, new gravity data were included for Sweden, Norway, Russia, and the Baltic States, as well as aerial gravimetry data over the Baltic Sea. Additionally, an improved digital terrain model was used. The background model for the NKG2004 model was a combination of the global models GGM02S (Tapley et al., 2005) and EGM96 (Lemoine et al., 1998). The accuracy of the NKG2004 model is better than 10 cm for most of the Nordic area.

2.2 EUVN-DA GPS/levelling data

To calculate the height conversion surface, data was needed of points with GPS coordinates in the national coordinate system EUREF-FIN, as well as levelled heights in the new N2000 height system. EUREF-FIN is the Finnish national ETRS89 realization. It is in the ETRF96 system at epoch 1997.0 (Ollikainen et al., 2000). The only GPS/levelling data available with levelled N2000-heights were the results of the EUVN-DA (European Vertical Network – Densification Act) campaign (Ollikainen, 2006).

The Finnish EUVN-DA dataset consists of 50 GPS/levelling points, of which 20 were part of the EUREF-FIN network that consists of 100 points, measured in Finland in 1996–1997. The remaining 30 points were new and measured in the EUVN-DA GPS-campaign in 2005. All points are either first order levelling points or new points that were directly connected to the first order levelling network. As a result, they all have levelled heights in the new N2000 height system (figure 2) in addition to the GPS coordinates. The accuracy of the new GPS coordinates was estimated to be \pm 3.2 mm, \pm 3.4 mm, and \pm 3.8 mm in north, east, and height direction (Ollikainen, 2006).

For the final EUVN-DA solution in Ollikainen (2006) all GPS-coordinates were reduced to the ETRF2000 system at epoch 2000.0. However, the Finnish national ETRS89 realization EUREF-FIN is in the ETRF96 system at epoch

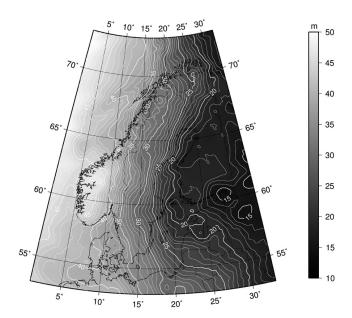


Figure 1. NKG2004 geoid model.

1997.0. Therefore, to produce a height conversion surface to be used in national GPS-levelling, the EUVN-DA coordinates had to be transformed to the ETRF96 system at epoch 1997.0.

The coordinates of the first 20 points that were part of the 1996–1997 GPS-campaign were already available in the EUREF-FIN system. For the 30 new points, intermediate results of the 2005 campaign were first reduced from the ITRF2000 system to the ETRF2000 system using the parameters in Boucher and Altamimi (2001). Then the coordinates were reduced from epoch 2005.58 to the EUREF-FIN epoch 1997.0 to remove the effect of the intra-plate deformation. For this reduction the NKG_RF03vel velocity model developed by the working group for Reference Frame and Positioning of the Nordic Geodetic Commission (Nørbech et al., 2006) was used. In this model the horizontal motions are derived from the BIFROST-model (Milne et al., 2001) and from new velocities calculated from GPS-data (Lidberg et al., 2007). Vertical velocities were taken from the NKG2005LU land uplift model (Vestøl, 2007; Ågren and Svensson, 2007).

In the end, the data of the 2005 campaign was fitted to the earlier EUREF-FIN data by calculating a Helmert transformation using results of the permanent GPS stations that were part of the 1996–1997 campaigns as well as the 2005 campaign. The result was a dataset containing 50 points with EUREF-FIN GPS coordinates as well as N2000 heights from the precise levelling (figure 2).

3 Calculations

To fit the NKG2004 model to the EUVN-DA GPS/levelling data, geoid heights for each point were interpolated from the NKG2004 model and subtracted from the difference between the GPS- and levelled heights:

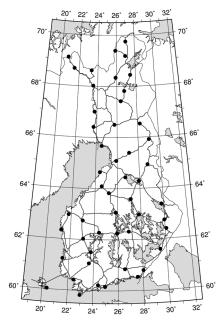


Figure 2. EUVN-DA GPS/levelling points and the Finnish first order levelling network.

$$\Delta N = h_{GPS EUREF-FIN} - H_{levelling N2000}^* - \zeta_{NKG2004}. \tag{1}$$

The resulting geoid height differences, ΔN , had an average of -3.2 cm and a standard deviation of 4.1 cm (case 0 in tables 1 and 2).

To correct the NKG2004 model, a correction surface was fitted through the geoid height differences, ΔN . Two different methods for surface fitting were tested: polynomial fitting and least-squares collocation. The methods are explained in sections 3.1 and 3.2. The sections give also the results using different parameters. The results were tested using cross-validation, which is discussed in section 3.3.

3.1 Polynomial surface fitting

The first method tested was fitting a polynomial through the geoid height differences:

$$\Delta N = \sum_{i=0}^{n} \sum_{j=0}^{n-i} a_{ij} \Delta \phi^i \Delta \lambda^j, \tag{2}$$

where *n* is the order or degree of the polynomial, and $\Delta \varphi$ and $\Delta \lambda$ are scaled coordinates, given by:

$$\Delta \phi = \frac{2\phi - (\phi_{\text{max}} + \phi_{\text{min}})}{\phi_{\text{max}} - \phi_{\text{min}}} \text{ and } \Delta \lambda = \frac{2\lambda - (\lambda_{\text{max}} + \lambda_{\text{min}})}{\lambda_{\text{max}} - \lambda_{\text{min}}}.$$
 (3)

Table 1 shows the statistics of the fit residuals for polynomials with orders ranging from zero (offset only) to five. As can be seen, the number of polynomial coefficients, a_{ij} , grows fast with increasing order. When only a tilted plane (first order

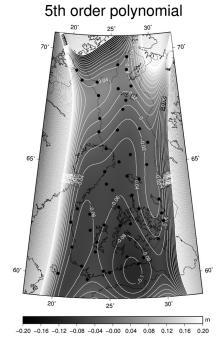


Figure 3. 5th order polynomial surface fit to the geoid height differences (case P5 in tables 1 and 3).

polynomial) is fitted through the data, the standard deviation is reduced considerably from 4.1 cm to 2.3 cm. When the order increases, the standard deviation decreases only a little for each extra order. In table 1 the best fit is obtained using the highest order, five (case P5). However the fitted surface is not very smooth and changes rapidly outside the area covered by the dataset, as can be seen in figure 3.

Table 1. Statistics of fit residuals in polynomial surface fitting, with minimum (min),									
	maximum (n	nax), average (av) and star	ıdard deviai	tion (σ) .				
Case		Number of	` ′	Max (m)	Av (m)	σ (m)			

Case	Polynomial	Number of	Min (m)	Max (m)	Av (m)	σ (m)
	degree	coefficients				
0	_	0	-0.119	0.069	-0.032	0.041
P0	0	1	-0.087	0.101	0.000	0.041
P1	1	3	-0.042	0.055	0.000	0.023
P2	2	6	-0.040	0.062	0.000	0.019
P3	3	10	-0.036	0.047	0.000	0.017
P4	4	15	-0.035	0.038	0.000	0.015
P5	5	21	-0.030	0.032	0.000	0.013

3.2 Least-squares collocation

The second method tested was least-squares collocation (Moritz, 1989). Here, the surface values at grid points are estimated from the known geoid height differences using a covariance function. The relationship is given by:

$$\hat{s} = C_{xx} C_{xx}^{-1} x, \tag{4}$$

where x contains the geoid height differences, ΔN , and \hat{s} the geoid height differences at grid points forming the correction surface. C(r) is a covariance function that describes the relationship between two points at distance r. For this study the covariance function was approximated by a second order Gauss-Markov covariance function. This function is commonly used in geoid surface fitting and readily implemented in the GRAVSOFT-software package used for the calculations (Forsberg, 2003):

$$C(r) = C_0 \left(1 + \frac{r}{\alpha}\right) e^{(-r/\alpha)}.$$
 (5)

The input parameter α is the correlation length and the variance C_0 is given by the standard deviation of the data.

To find the optimal solution for the surface fit, different correlation lengths and noise levels were tested. The noise level defines how tight the surface is fitted to the known values. In combination with the least-squares collocation either an offset or a tilt were estimated as well. The results of the tests are given in table 2.

As can be seen in table 2, estimated surfaces fit best to the known data when short correlation lengths and low noise levels are used. Applying a tilt instead of an offset makes the fit worse. The best fit is obtained with case C1, with 50 km

Table 2. Statistics of fit residuals in least-squares collocation, with correlation length (α) , minimum (min), maximum (max), average (av) and standard deviation (σ) .

		Collocation parameters					
Case	Trend type	α (km)	noise (m)	min (m)	max (m)	av (m)	σ (m)
0	_	_	_	-0.119	0.069	-0.032	0.041
C1	Offset	50	0.01	-0.005	0.006	0.000	0.002
C2	Offset	100	0.01	-0.010	0.008	0.000	0.003
C3	Offset	100	0.02	-0.021	0.021	0.000	0.007
C4	Tilt	50	0.01	-0.010	0.011	0.000	0.004
C5	Tilt	100	0.01	-0.016	0.017	0.000	0.006
C6	Tilt	100	0.02	-0.025	0.033	0.000	0.011
C7	Offset	100	0.008	-0.007	0.006	0.000	0.002
C8	Tilt	100	0.008	-0.013	0.013	0.000	0.005
C9	Offset	80	0.01	-0.008	0.007	0.000	0.002
C10	Offset	150	0.01	-0.014	0.014	0.000	0.005
C11	Offset	150	0.02	-0.025	0.028	0.000	0.009
C12	Offset	200	0.01	-0.018	0.019	0.000	0.007
C13	Offset	200	0.02	-0.028	0.034	0.000	0.011
C13a	As C13, point A left out			-0.018	0.034	0.000	0.011
C13b	As C13, point B left out			-0.029	0.033	0.000	0.010
C13c	As C13, point C left out			-0.029	0.041	0.000	0.011
C13d	As C13, points A & B left out			-0.017	0.034	0.000	0.010
C14	Offset	250	0.02	-0.031	0.038	0.000	0.013
C15	Offset	200	0.027	-0.033	0.039	0.000	0.013
C16	Tilt	100	0.027	-0.029	0.039	0.001	0.014

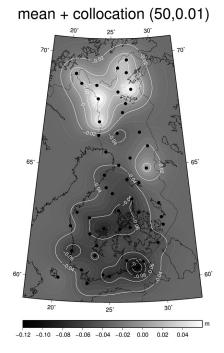


Figure 4. The surface that was fitted to the geoid height differences by least-squares collocation using parameters of case C1: 50 km correlation length, 1 cm noise level, and an offset (see tables 2 and 4).

correlation length and 1 cm noise level in combination with an offset. In that case the standard deviation of the fit residuals is only 2 mm. The surface is shown in figure 4.

Comparing table 1 and 2, it is clear that the least-squares collocation method gives better fits to the known geoid heights differences than the polynomial fits.

3.3 Cross-validation

The results in sections 3.1 and 3.2 were analysed by looking at the characteristics of the fit residuals. The standard deviations of the fit residuals tell how well the surfaces fit to the given data, but do not indicate how well the surfaces perform on other locations. At the time of the calculations, the new height system had just been established and no other independent dataset of GPS/levelling points with N2000 heights was available for testing besides the EUVN-DA dataset. Therefore cross-validation was applied to address the performance of the fitted surfaces at unknown locations.

The 50 EUVN-DA points were already quite sparsely distributed over the country and dividing the dataset into a test set and a training set of e.g. 20 and 30 points would result in a too sparse distribution of the data to give a reliable result. The surface fitted to the training set would not anymore be comparable to the surface fitted through the complete dataset. Therefore so-called leave-one-out cross-validation was applied, leaving one point at a time out of the surface fitting

procedure. The value for the left out point was then calculated from the fitted surface and compared to the known value. This way all points were left out in turn, resulting in cross-validation residuals for all points. Table 3 gives the results for the polynomial fitting and table 4 for the least-squares collocation method.

Table 3. Statistics of cross-validation residuals in polynomial surface fitting, with minimum (min), maximum (max), average (av) and standard deviation (σ).

Case	Polynomial	number of	min (m)	max (m)	av (m)	σ (m)
	degree	parameters				
P0	0	1	-0.089	0.103	0.000	0.041
P1	1	3	-0.045	0.059	0.000	0.024
P2	2	6	-0.049	0.067	0.000	0.023
P3	3	10	-0.044	-0.059	0.000	0.021
P4	4	15	-0.048	0.066	0.002	0.025
P5	5	21	-0.059	0.135	0.002	0.033

Table 4. Statistics of cross-validation residuals in least-squares collocation, with correlation length (α) , minimum (min), maximum (max), average (av) and standard deviation (σ) .

		Collocation					
		parameters					
Case	Trend type	a (km)	noise (m)	min (m)	max (m)	av (m)	σ (m)
C1	Offset	50	0.01	-0.054	0.077	0.001	0.025
C2	Offset	100	0.01	-0.045	0.060	0.001	0.020
C3	Offset	100	0.02	-0.049	0.064	0.001	0.020
C4	Tilt	50	0.01	-0.041	0.059	0.001	0.021
C5	Tilt	100	0.01	-0.040	0.057	0.001	0.020
C6	Tilt	100	0.02	-0.042	0.058	0.001	0.020
C7	Offset	100	0.008	-0.045	0.059	0.001	0.020
C8	Tilt	100	0.008	-0.039	0.056	0.001	0.020
C9	Offset	80	0.01	-0.047	0.064	0.001	0.021
C10	Offset	150	0.01	-0.043	0.056	0.001	0.020
C11	Offset	150	0.02	-0.048	0.060	0.001	0.020
C12	Offset	200	0.01	-0.043	0.056	0.001	0.019
C13	Offset	200	0.02	-0.047	0.060	0.001	0.019
C13a	As C13, point A left out			-0.032	0.060	0.001	0.018
C13b	As C13, point B left out			-0.048	0.059	0.001	0.018
C13c	As C13, point C left out			-0.048	0.074	0.001	0.019
C13d	As C13, points A & B left out			-0.032	0.059	0.001	0.016
C14	Offset	250	0.02	-0.048	0.059	0.001	0.020
C15	Offset	200	0.027	-0.050	0.061	0.001	0.020
C16	Tilt	100	0.027	-0.042	0.058	0.001	0.021

When looking at the cross-validation results in table 3, the 5th order polynomial fit is not anymore performing best. Instead, in cross-validation the 3rd order polynomial (case P3, see figure 5) proves to give the best predictions at unknown locations, with a standard deviation of 2.1 cm.

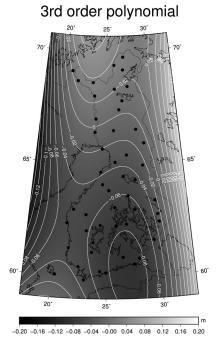


Figure 5. 3rd order polynomial surface fit to the geoid height differences (case P3 in tables 1 and 3).

mean + collocation (200,0.02)

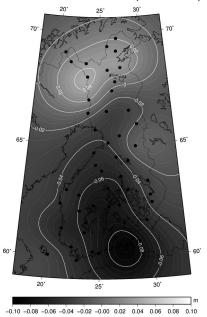


Figure 6. The surface fitted to the geoid height differences by least-squares collocation using parameters of case C13: 200 km correlation length, 2 cm noise level, and an offset (see tables 2 and 4)

The cross-validation results of the least-squares collocation in table 4 are about two times worse than the fit residuals given in table 2. However, also in the cross-validation, the least-squares collocation results are slightly better than the polynomial fitting results. In the case of least-squares collocation cross-validation shows that longer correlation distances give better results in contrast to the fit results of table 2, where shorter correlation distances gave best results. The best performance is obtained with a correlation length of 200 km (e.g. case C13 in table 4, see figure 6).

4 Discussion of the results

The standard deviations of the cross-validation residuals in tables 3 and 4 are overall 2 times worse than those of the fit residuals in tables 1 and 2. However, the fit residuals only tell how well the fitted correction surfaces perform at the locations of the EUVN-DA points. The cross-validation residuals give a better impression of the expected performance of the fitted surfaces at unknown locations and are therefore used to choose the best fit for the new height conversion surface.

Using polynomial fitting, the 3rd order fit (case P3) gives the best cross-validation results with a standard deviation of 2.1 cm (see table 3). However, figure 5 shows that, although the 3rd order polynomial surface is quite stable within the area covered by the GPS/levelling points, it changes rapidly outside the area

close to and outside the Finnish borders. This is not a desirable characteristic, as it means that the errors of the height conversion surface would grow fast near the borders when this correction surface would be used. In addition, the 3rd order polynomial fit performs worse than most of the surfaces calculated using least-squares collocation. Therefore, the choice for the final height conversion surface goes to the correction surfaces fitted using least-squares collocation. These surfaces also behave more natural outside the area covered by the GPS/levelling points where they smoothly reduce to the offset or tilt values.

Best collocation results are obtained with a collocation length of 200 km. The cross-validation results of both cases C12 and C13, with noise levels of 1 and 2 cm respectively, give a standard deviation of 1.9 cm (see table 4). The minimum and maximum residuals of case C12 are slightly smaller than those of C13. However, the correction surface of case C12 shows irregular behaviour around two of the points. When the noise level is increased to 2 cm, case C13, this irregular behaviour of the surface becomes much smaller and the surface itself much smoother. Therefore, case C13 is preferred above case C12.

Figures 7 and 8 show the distribution of the fit and cross-validation residuals of case C13, note the scale difference. There is a clear division between the south, where the residuals are overall smaller, and the north, where the residuals are larger and more diverse. This indicates that the final height conversion surface will perform better in the south and than in the north.

A few points with extreme residuals are indicated by A, B, and C in figure 8. It is tempting to remove these points from the calculations and this is done in cases C13a, C13b, C13c, and C13d (see tables 2 and 4). When point A or point B is left out of the calculations, in cases C13a and C13b, the standard deviation of the cross-validation residuals drops from 1.9 to 1.8 cm. Only the residuals of the points directly surrounding the left-out point change. Leaving point C out, in case C13c, does not improve the test statistics. In case C13d, leaving out both points A and B, the standard deviation of the cross-validation residuals drops from 1.9 to 1.6 cm. This can, however be mainly explained by the fact that these left out points were also left out of the cross-validation. The standard deviation of the fit residuals only drops from 1.1 cm to 1.0 cm between cases C13 and C13d (see table 2), which indicates that the correction surface of case C13d does not fit the geoid height differences much better than the correction surface of case C13. In addition, neither the results of the levelling nor those of the GPS campaigns indicate any problems with points A and B that may support the decision to leave those points out of the solution. Leaving point B out, would also lead to a large area in the north without a point. It was therefore decided to keep the points in the solution.

The fitted surface of case C13 was in the end chosen to be the best correction surface. It was then added to the NKG2004 model resulting in the new height conversion surface for Finland. The new height conversion surface was named FIN2005N00. The standard deviation of the cross-validation residuals indicates that with the new conversion surface heights can be converted with an accuracy better than 2 cm. The FIN2005N00 surface is shown in figure 9.

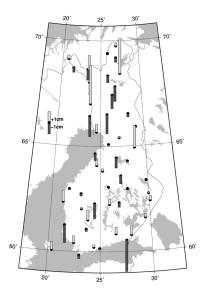


Figure 7. Fit residuals for the least-squares collocation surface fit case C13, using 200 km correlation length, 2 cm noise level, and an offset (see table 2).

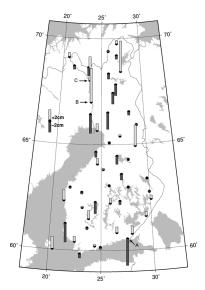


Figure 8. Cross-validation residuals for the least-squares collocation surface fit case C13, using 200 km correlation length, 2 cm noise level, and an offset (see table 4).

5 Conclusions

A new height conversion surface for Finland was calculated to link levelled heights in the new national height system, N2000, and heights measured by GPS in the national EUREF-FIN system.

A correction surface was fitted to the NKG2004 geoid model using the 50 GPS/levelling points of the Finnish EUVN-DA GPS campaign. Correction surfaces were calculated using polynomial fitting and least-squares collocation using a second order Gauss-Markov covariance function and testing different parameter values.

Cross-validation proved to be a good tool for testing the different correction surfaces in a case like this, where no other independent GPS/levelling data was available.

A 3rd degree polynomial gave good results, but the resulting correction surface changed rapidly outside the area close to and outside the Finnish borders. Better results were obtained using least-squares collocation, resulting in smooth surfaces, also outside the area covered by the points. A correlation length of 200 km and noise level of 2 cm gave the best results. The correction surface obtained with these parameters was added to the NKG2004 geoid model.

The new height conversion surface for Finland was named FIN2005N00. The standard deviation of the cross-validation residuals indicates that with the surface EUREF-FIN heights can be converted to N2000-heights and vice versa with an accuracy better than 2 cm. The FIN2005N00 model should be used for transformations between heights in the N2000-system and heights in the EUREF-FIN system in Finland.

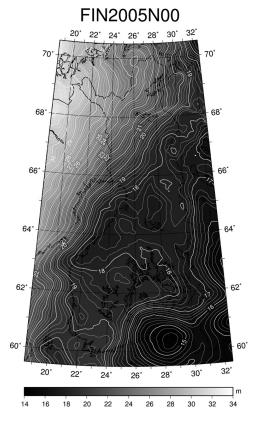


Figure 9. The new height conversion surface for Finland FIN2005N00.

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