

THE ELEPHANT'S FOOTPRINT: AN ANCIENT INDIAN LOGIC DIAGRAM

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A seminal article by Margaret Baron, published in 1969, explored the history of set diagrams (Venn diagrams). However, Baron did not look beyond the evidence of European sources. This article presents evidence of a literary simile from ancient India that exemplifies the idea of a larger circle including within it many smaller circles, each circle standing for an ethical concept. The simile – an elephant's footprint enclosing the footprints of smaller animals – first appears in the Buddhist Canon, and it was used occasionally in South Asian literature through the following millennia until the eighteenth century. I argue that the Elephant's Foot simile can be added to Baron's catalogue of historical cases where ancient authors were using language that implied a simple concept of logical sets.

BACKGROUND

In 1969, Margaret Baron published a study on the development of logic diagrams of the type that has become associated with the name of John Venn (1834–1923) (Baron 1969; Venn 1880). Baron's excellent study has been widely cited in subsequent literature and has become a standard reference. However, Baron did not look beyond the European tradition of Venn's immediate predecessors. This short article expands the evidence for thinking diagrammatically in sets to include textual descriptions from South Asia that are older than Baron's earliest sources.

Baron speculated that spatial logic diagrams of some type might be as old as Aristotle, although we have no concrete evidence for this. She also sketched the contributions of some historical precursors to Venn's formalized system, including those of Llull (1232/33–1315), Weise (1642–1708), Leibniz (1646–1716), Lange (1669–1756), Euler (1707–1783), Ploucquet (1716–1790), Lambert (1728–1777), and Gergonne (1771–1859) (Baron 1969: 114–121).

In what follows, I shall present evidence from Sanskrit and Middle Indo-Aryan sources, but it should be noted that these are all textual descriptions of set diagrams. The earliest manuscripts in which these descriptions are transmitted do not include such set diagrams in geometrical form. It is a known feature of the manuscript tradition of South Asia that drawings or diagrams of technical or scientific topics are rare or non-existent (Wujastyk 2014: 165). This situation is not different than the case of Aristotle, cited in Baron's (1969: 114–115) introduction, where she notes that the *Organon* and the *Doctrine of the Syllogism* contain no diagrams:

Nonetheless, so suggestive is the language and manner of presentation of the syllogistic scheme, that many logicians have speculated as to the possibility that Aristotle made use of spatial concepts in his actual lectures.

It is implausible that in forty years of preaching, the Buddha – whom, as we shall see below, used set imagery – never once used a stick to sketch an image on the ground, just as Aristotle might have done.

THE BUDDHIST CANON

The Buddha lived to the age of eighty and died, according to the revised consensus, in about 400 BCE (Cousins 1996). Soon after his death, his followers gathered to recite and memorize his sermons. These group recitations became formalized, and after a series of Buddhist councils a canon emerged that collected the Buddha's teachings and associated materials (Norman 1983; von Hinüber 1982). At first these recitations were memorized and transmitted orally, but eventually the Buddhist Canon was committed to writing, perhaps in Sri Lanka during the first century BCE. There has been much debate about the degree to which the Buddha's sermons as recorded in the Canon represent his own words (Wynne 2005), but there is no indisputable argument against accepting the Canon as being close to his voice. Detailed philological arguments aside, many readers agree that the canonical writings breathe with a spirit of freshness and individuality that makes it hard not to feel the force of an original personality behind the scripture.

One of the books forming part of the core of the Buddhist Canon is the *Great Elephant Footprint Simile*.¹ Written in the Pāli language, it preserves a sermon given by Sāriputta, one of the Buddha's chief monks. The text begins as follows:

This is what I heard. On one occasion, the Blessed One was staying at Jeta Park in Sāvatti, in the garden of Anāthapiṇḍika. At that place, the Venerable Sāriputta spoke to the monks as follows.

“Greetings, Brother Monks!”

“Greetings to you, Brother Monk,” the monks replied to Venerable Sāriputta.

Then Venerable Sāriputta said the following: “Brothers, all the footprints of moving animals are contained inside the footprint of an elephant, and the elephant's footprint is considered to be the first amongst them because of its size. In just the same way, Brothers, all good things are included within the Four Noble Truths.”²

The text continues with descriptions of various Buddhist doctrinal categories. But the striking image here is that of the small footprints of animals contained within the large, all-encompassing footprint of the elephant, which is reckoned to be the most important of beasts because of its size. This simile has a charming and vivid visual impact. It effortlessly conveys the concept of a large set containing subsidiary entities.

If this text recounts a real sermon by Sāriputta delivered in the presence of the Buddha, then it is datable to the decades before the Buddha's death in 400 BCE. This is, in my view, the likeliest case. But if it was retrospectively attributed to Sāriputta by some later author, it

1 Pāli *Mahāhatthipadopamasutta*, *Majjhimanikāya* 28. Pāli text: (Trenckner & Chalmers 1888–1925, I: 184):

1. *Evaṃ me sutam: ekaṃ samayaṃ bhagavā sāvattiyam viharati jetavane anāthapiṇḍikassa ārāme. Tatra kho āyasmā sāriputto bhikkhū āmantesi: 'āvuso bhikkhavo'ṭi. Āvusoti kho te bhikkhū āyasmato sāriputtassa paccassosum. Āyasmā sāriputto etadavoca. 2. Seyyathāpi āvuso yāni kānici jaṅgalānaṃ pānānaṃ padajātāni sabbāni tāni hatthipade samodhānaṃ gacchanti, hatthipadaṃ tesam aggamakkhāyati yadidaṃ mahattanena, evameva kho āvuso ye keci kusalā dhammā sabbe te catusu ariyasaccesu saṅgahaṃ gacchanti.*

2 Translation my own; for another translation, see Nānamoḷi & Bodhi 1995: 276.

could be dated to as late as the first century BCE, when the Canon had been formalized and was committed to writing.

THE GREAT EPIC

The Great Epic of India, the *Mahābhārata*, is a giant work telling the heroic and dastardly adventures of gods, demons, royal warriors and villains, goddesses, sages and supernatural beings of all kinds. Composed by bards over a long period during the centuries between 200 BCE and 200 CE, it has continued to inspire popular culture in India up to the present time (Brockington 1998). In Book Twelve, the *Book of Peace*, the Elephant Footprint simile is used, but this time in a non-Buddhist context.

All the other footprints made by the feet of creatures that walk are placed inside the elephantine Elephant's Footprint. In just the same way, the whole of virtue and profit are placed inside non-violence. The person who practises non-violence is immortal. He lives forever.³

Notice that in the Buddhist tradition, the outer set, the Elephant's Footprint, is Dharma, and all other virtues are subsumed within that. For the *Mahābhārata*, however, the outer set is non-violence (*ahiṃsā*), and other virtues, including Dharma, are subsets of that. The *Mahābhārata* is in the Sanskrit language, not Pāli, and the text is at least two centuries removed from the time of the Buddha, but the simile is identical to the original Buddhist one, even if the terms have been inverted.

The language is slightly different in subtle respects. The verb that I have translated as “are placed inside” (archaic Sanskrit *apīdhīyante*) later came to mean (as *pidhā-*) ‘to obscure, cover, blot out’. As we shall see, in later texts this usage is brought to the fore, and as a result the meaning of the simile changes.

One other term may be worth noting. The expression “he practises” uses a Sanskrit word (*pratīpadyate*) that is one of the key terms of Buddhist doctrine. When the Buddha taught the Middle Way, this was the word he used – as recorded in the Pāli Canon – for “Way”. It is perhaps a stretch, but it is possible that the language of the *Mahābhārata* has retained a faint echo of the Buddhist context in which this simile was originally embedded.

LATER USES OF THE SIMILE

The *Mahābhārata* was such an influential and widely distributed work that its use of the Elephant's Footspread to several other branches of Sanskrit literature. Thus, the simile occurs in the *Agnipurāṇa*, a large compendium of traditional lore that is roughly datable to the end of the first millennium.⁴

3 *Mahābhārata*, vol. 12, ch. 237, verse 18 f. repeated at 13.115.6 f. (Sukthankar, Belvalkar et al. 1933–1959): *yathā nāgapade 'nyāni padāni padagāminām | sarvāṅy evāpidhīyante padajātāni kauñjare evaṃ sarvam ahiṃsāyāṃ dharmārtham apīdhīyate | amṛtaḥ sa nityaṃ vasati yo 'hiṃsāṃ pratīpadyate |* Translation my own. For another translation, see Ganguli 1884–1894, VII: 300, based on a variant text.

4 Ch. 372, verses 4cd–5ab (Ānandāśramasthapaṇḍitāḥ 1900: 466): *yathā gajapade 'nyāni padāni pathagāminām ||4cd|| evaṃ sarvam ahiṃsāyāṃ dharmārtham abhidhīyate ||5ab||*. On the text and its dating, see Rocher 1986: 134–137 *et passim*.

The simile also occurs in the *Bhṛgusaṃhitā*, “The Compendium of Bhṛgu”, a work on Vaiṣṇava theology datable before 1100 CE.⁵

The simile also found its way into the literature of Yoga. The earliest Indian treatise on classical Yoga, datable to about 400 CE, is called *The Doctrine of Yoga According to Patañjali* (Sanskrit: *Pātañjalayogaśāstra*).⁶ Patañjali’s system of Yoga expounds a path of ascetic discipline that has eight components, starting with the practice of personal and social virtues, and ascending through several types of meditation and concentration to a final state of serene introversion called “wholeness”.⁷ These virtues are a prerequisite for meditation, according to Patañjali. The first of these is non-violence (Skt.: *ahiṃsā*). The commentator Vijñānabhikṣu, who lived in northern India during the last half of the sixteenth century (Nicholson 2007; 2010) and discussed non-violence in Patañjali’s treatise on Yoga, explicitly cites the *Mahābhārata* passage from the *Book of Peace* (Paṭavardhana & Keśavaśāstrī 1884: 161). In this way, Vijñānabhikṣu brought the ancient simile of the Elephant’s Foot into the discourse of Yoga philosophy, and thus into the early modern world of Indian religious and philosophical thought.

One might reasonably imagine that the Elephant’s Foot simile would have been taken up by early South Asian logicians, but a search of the Sanskrit literature on logic (*nyāya*) has so far drawn a blank. Even the earliest authors on Indian logic used expressions that strongly suggest a diagrammatic approach; for example, one can cite Dinnāga’s “circle of reasons” (*hetucakra*) or “four-pointed [set of alternatives; tetralemma]” (*catuṣkoṭi*). Furthermore, early Sanskrit treatises on logic deal with topics that modern interpreters often feel are best expounded using Venn diagrams (Matilal 1998: 17; Chi 1969: 17 *et passim*). This instinct that diagrammatic interpretation somehow informs or illuminates Indian logical thought is not new. The seventh-century Chinese interpreter of Indian logic, Lü Ts’ai (600–665 CE), wrote a treatise on Indian logic entitled *Explanations and Diagrams on Logical Demonstration and Refutation* that is said to have indeed contained diagrams (Chi 1969: lxxvii, citing Waley 1952: 107 ff.). As Chi (1969: lxxvii) says,

It is most unfortunate that his book is lost, we cannot tell what his diagrams were like; possibly they were something like Euler’s.

In spite of these tantalizing hints of diagrams being used in the seventh-century Chinese reception of Indian logic, I have so far found no use of the Elephant’s Foot simile in the Sanskrit literature on logic.

5 Gonda 1977: 145. *Bhṛgusaṃhitā, prakīrṇādhikāra* 36.439 (Ramakrishna Kavi 1929: 496): *yathā nāgapade ’nyāni padāni padagāminām ||36.438|| sarvānyevāpidhīyante padajātāni kaiñjare | evaṃ lokeṣvahiṃsā tu nirdiṣṭādharmataḥ purā || 36.439||*.

6 In the last century, it was assumed by many scholars that the sutras were written by an otherwise unknown philosopher called Patañjali, while the commentary (Sanskrit: *bhāṣya*) was composed at a later time by another person called Vyāsa. The work was thus often described as “The Yoga Sutras of Patañjali with the commentary by Vyāsa”. However, recent breakthroughs in historical and philological work have compellingly shown that the whole work is by one person, Patañjali, and that this single work – including both sutras and commentary – can be dated to the period 325–425 CE (Maas 2006; 2013, para. 2).

7 Gelblum has convincingly argued for “wholeness” as the best translation of the Sanskrit term *kaivalya*, which has sometimes been rendered as “aleness” (Gelblum 1970).

LATER REINTERPRETATION

Later Indian philosophers used the image of the Elephant's Foot in a slightly different way as a simile for a larger, more powerful system of thought that absorbs a lesser one, "just as the tracks of all other animals disappear within the tracks of the elephant" (Halbfass 1988: 366). Thus, the *Mahānirvāṇatantra*, a tantric text datable to the late eighteenth century (Goudriaan & Gupta 1981: 98–99), which uses the simile in this way, says:

There is no higher way than the Kaula Way (*kaula-dharma*), O Lotus Lady. [...] Just as the footsteps of all creatures melt away in the elephant's footprint, so, my Dear, all other ways are submerged in the Kaula Way.⁸

If the Buddha's original idea was that small footprints were visible inside the larger one, in the tantric presentation the small footprints have been flattened and obliterated by a new, larger footprint. This new interpretation of the simile subverts its original meaning: instead of symbolizing inclusion, the Elephant's Footprint now signifies occlusion.

CONCLUSION

For two and a half millennia, the Elephant's Foot simile has occasionally been used by Indian philosophical and religious writers as a way of expressing the image of a circular set encompassing smaller sets. Examples are not common, and extensive literature searches have turned up only the examples cited above. Nevertheless, there is a thematic continuity across centuries of literature. Apparently invented by the Buddha, the simile continued to be used in Sanskrit texts until at least the late eighteenth century. Furthermore, it was always expressed in language; no manuscripts known to me contain drawings of this image. Nevertheless, I think it is reasonable to add the Elephant's Foot simile found in India to Margaret Baron's catalogue of historical cases where ancient authors were using language that implied a simple concept of logical sets.

REFERENCES

- ĀNANDĀŚRAMASTHAPANDITĀH 1900. *ŚrīmadDvaipāyanamunipraṇītam Agnipurāṇam*. (Ānandāśrama Sanskrit Series 41) Pune: Hari Nārāyaṇa Āpaṭe at the Ānandāśrama Press. ark:/13960/t4rk0z28x.
- BARON, Margaret E. 1969. A Note on the Historical Development of Logic Diagrams: Leibniz, Euler and Venn. *The Mathematical Gazette* 53(383): 113–125. doi: 10.2307/3614533.
- BHAṬṬĀCĀRYA, Jīvānānanda Vidyāsāgara 1884. *Mahānirvāṇatantram. Kulāvadhūta śrīmdhariharānandanāthabhārātī-Viracitayā ṭikayā Sahitam*. Kalikātā: Jīvānānanda Vidyāsāgara. <goo.gl/BLkgbm>, accessed 9 Aug. 2018.
- BROCKINGTON, John L. 1998. *The Sanskrit Epics*, II.12. (Handbuch Der Orientalistik) Leiden: Brill.
- CHI, Richard See Yee 1969. *Buddhist Formal Logic, 1: A Study of Dignāga's Hetucakra and K'uei-Chi's Great Commentary on the Nyāyapraveśa* (Dr. B.C. Law Trust Fund 1) London: Royal Asiatic Society of Great Britain.
- COUSINS, Lance S. 1996. The Dating of the Historical Buddha: A Review Article. *Journal of the Royal Asiatic Society* 6(1): 57–63. doi: 10.1017/s1356186300014760.
- DUTT, Manmatha Nath 1900. *A Prose English Translation of Mahanirvana Tantram*. Calcutta: H.C. Dass. ark:/13960/t6935z78r.
- GANGULI, Kisari Mohan 1884–1894. *The Mahabharata of Krishna-Dwaipayana Vyasa Translated into English Prose*. Calcutta: Protap Chandra Roy at the Bharata Press. ark:/13960/t0tq5zz0h.

⁸ *Mahānirvāṇatantra* 14.179–180 (Bhaṭṭācārya 1884: 446): *kauladharmāt paro dharmo nāstyeva kamalānane | antyaḥpi yamāśritya pūtaḥ kaulapadaṃ vrajet || 179 || karipāde vilīyante sarvaprānīpādā yathā | kuladharme nīmajjanti sarve dharmāstathā priye || 180 ||* Translation my own. For another translation, see Dutt 1900: 312.

- GELBLUM, Tuvia 1970. Sāṃkhya and Sartre. *Journal of Indian Philosophy* 1: 75–82. doi: 10.1007/bf02346509.
- GONDA, Jan 1977. *Medieval Religious Literature in Sanskrit*, II.1. (A History of Indian Literature) Wiesbaden: Harrassowitz.
- GOUDRIAAN, Teun & Sanjukta GUPTA 1981. *Hindu Tantric and Śākta Literature*, II.2 (A History of Indian Literature) Wiesbaden: Harrassowitz.
- HALBEASS, Wilhelm 1988. *India and Europe*. Albany: SUNY Press.
- VON HINÜBER, Oscar 1982. Pali as an Artificial Language. *Indologica Taurinensia* 10: 133–140. <asiainstitutetorino.it/Indologica/volumes/vol10/vol10_art10_hinuber.pdf>, accessed 27 Aug. 2018.
- MAAS, Philipp A. 2006. *Samādhipāda: das erste Kapitel des Pātañjalayogaśāstra zum ersten Mal kritisch ediert = the First Chapter of the Pātañjalayogaśāstra for the First Time Critically Edited*. Aachen: Shaker.
- MAAS, Philipp 2013. A Concise Historiography of Classical Yoga Philosophy. In: E. FRANCO (ed.), *Historiography and Periodization of Indian Philosophy*. Vienna: Verein Sammlung de Nobili. <academia.edu/3520571/>, accessed 27 May 2018.
- MATILAL, Bimal Krishna 1998. *The Character of Logic in India*. Ed. Jonardon Ganeri & Heeraman Tiwari. Albany: SUNY Press.
- ÑĀNAMOLI, Bhikkhu & Bhikkhu BODHI 1995. *The Middle Length Discourses of the Buddha: A New Translation of the Majjhima Nikaya*. Boston: Wisdom Publications.
- NICHOLSON, Andrew J. 2007. Reconciling Dualism and Non-Dualism: Three Arguments in Vijñānabhikṣu's Bhedābheda Vedānta. *Journal of Indian Philosophy* 35(4): 371–403. doi: 10.1007/s10781-007-9016-6.
- NICHOLSON, Andrew J. 2010. *Unifying Hinduism: Philosophy and Identity in Indian Intellectual History*. NY: Columbia University Press.
- NORMAN, Kenneth Roy 1983. *Pāli Literature, Including the Canonical Literature in Prakrit and Sanskrit of All the Hīnayāna Schools of Buddhism*, VII.2. (A History of Indian Literature) Wiesbaden: Harrassowitz. ark:/13960/t6c29c94g.
- PAṬAVARDHANA, Rāmakṛṣṇaśāstrī & Keśavaśāstrī (eds) 1884. *Yogavārtikam. Mahāmahopādhyāyaśrīvijñānabhikṣuviracitaṃ Yogabhāṣyavyākhyārūpaṃ ... Paṭavardhanopanāmakena Rāmakṛṣṇaśāstrīnā ... Keśavaśāstrīnā ca Pariṣkṛtya Saṃsodhitam Tayoḥ Saṃmatyā ca*. Kāśī: Dr E.J. Lazarus (Lājarasa) at The Medical Hall. ark:/13960/t0wq61w9s.
- RAMAKRISHNA KAVI, M. 1929. *Śrī Vaikhānasa Bhagavacchāstrē Bhagavadbhṛgumaharṣi Prōktāsu Saṃhitāsu Prakīrṇādhikārah*. Madras: Śailanāthulu and Sons. ark:/13960/t8sb3xx2z.
- ROCHER, Ludo 1986. *The Purāṇas*. (A History of Indian Literature) Wiesbaden: Harrassowitz.
- SUKTHANKAR, Sitaram Vishnu, Shripad Krishna BELVALKAR et al. (eds) 1933–1959. *The Mahābhārata*. Poona: Bhandarkar Oriental Research Institute. <goo.gl/CBEJjk>, accessed 24 Sept. 2018.
- TRENCKNER, Vilhelm & Robert CHALMERS 1888–1925. *The Majjhima-Nikāya*. (Pali Text Society Text Series 60) London: Pali Text Society (H. Frowde). ark:/13960/t50g88k99.
- VENN, John 1880. On the Diagrammatic and Mechanical Representation of Propositions and Reasonings. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 9(59): 1–18. doi: 10.1080/14786448008626877.
- WALEY, Arthur 1952. *The Real Tripitaka and Other Pieces*. NY: Macmillan.
- WUJASTYK, Dominik 2014. Indian Manuscripts. In: J. QUENZER, D. BONDAREV & J.-U. SOBISCH (ed.), *Manuscript Cultures: Mapping the Field*: 135–157. (Studies in Manuscript Cultures 1) Berlin: De Gruyter. doi: 10.1515/9783110225631.159.
- WYNNE, Alexander 2005. The Historical Authenticity of Early Buddhist Literature: A Critical Evaluation. *Wiener Zeitschrift Für Die Kunde Südasiens* 49: 35–70. doi: 10.1553/wzksxli35.