A Simplified Method for Computing the Lethality of Fragmenting Munitions Based on Physical Properties

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Abstract

This paper describes a computational model for assessing the lethality of fragmenting ammunition. The model is based on the physical properties of fragmenting ammunition and target elements and physical phenomena, including retardation of fragments due to air resistance and fragment perforation. The purpose of this paper is not to provide a detailed description of the model, but rather to provide a summary of the algorithm and the basic equations. An extended version of the model is in use in the Finnish Defence Forces.

1 Introduction

This paper provides a foundation for calculating the probability of kill for a single target element due to fragmenting munitions. The computational model is based on the physical properties of munitions and target elements.

A numerical model for simulating fragmenting ammunition has been developed and used in the Finnish Defence Forces for over a decade. An initial version was presented by Heininen (2006). This model was extended by Lappi, Pottonen, Mäki, Jokinen, Saira, Åkesson, and Vulli (2008) to include handling of blast damage and direct hits and a model for delivery accuracy of multiple rounds. Further extensions of the model have been to take into account the shape of the terrain (Lappi, Sysikaski, Åkesson, and Yildirim, 2012) and the effect of forest environment (Roponen, 2015).

The fragmenting ammunition model has been validated using field experiments, in which 66 mortar bombs (120 mm high explosive) were fired in flat terrain, in three separate experiments (Åkesson, Lappi, Pettersson, Malmi, Syrjänen, Vulli, and Stenius, 2013). The physical model gives reliable predictions in all three test cases (mean absolute error (MAE) = 1.1 %, no systematic error detected). The errors in the Carleton (MAE = 4.1 %) and the cookie cutter (MAE = 7.2 %) were threefold and sixfold, respectively.

The lethality model can be used directly as part of an effectiveness model, as shown in Figure 1, instead of using it to generate lethal areas or damage matrices. The Sandis combat model (Lappi, 2012), developed at the Finnish Defence Forces Technical Research Centre (now the Finnish Defence Research Agency), has incorporated an implementation of the effectiveness model since 2006. A full software package, called EETU, which was designed specifically for indirect fire effectiveness assessment, was released in 2016. The effectiveness model implemented into EETU was designed to be modular and extensible, having interchangeable submodels. This paper presents a simplified version of the lethality model used in EETU.

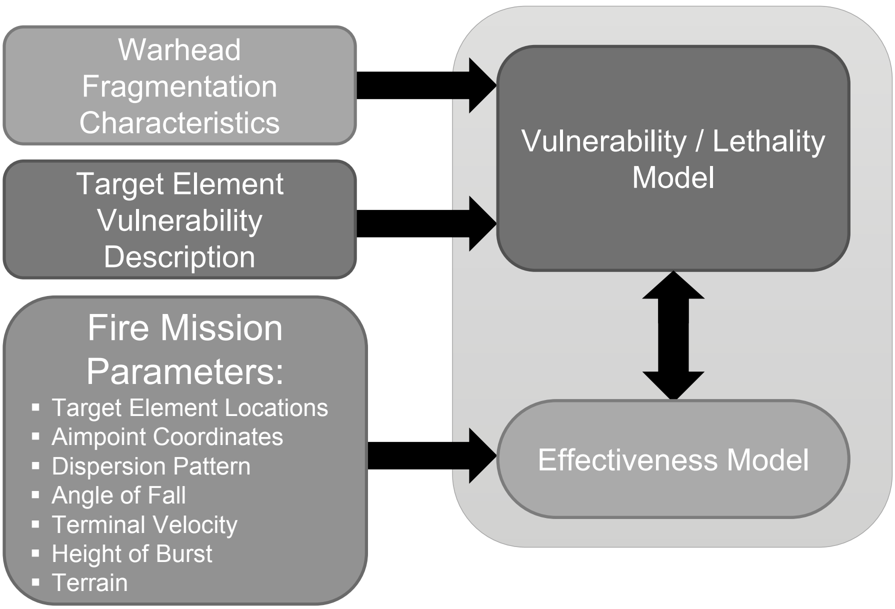

Figure 1. The lethality model described in this paper can be connected to an effectiveness model, or be used as part of one. The Finnish Defence Forces has two implementations of the effectiveness model: one is incorporated into the Sandis combat model (Lappi, 2012) and a newer one into the EETU software package for indirect fire effectiveness assessment.

The fragment effect model consists of four components: fragment patterns, a fragment drag model, a fragment perforation model and a target element model. A fragmentation warhead is characterized by fragment zones (also called fragment sprays and fragment fans), which are modelled as spherical zones, as illustrated in Figure 2. Fragmentation arena tests can provide experimental data on the warhead fragmentation patterns (US Army Test and Evaluation Command, 1993).

A target element is described by a collection of armour segments facing different directions. The simple kill criterion used in this paper states that the target is considered killed if any of its armour segments are sufficently perforated by fragments or damaged by blast. The armour segments are considered independent of each other. More elaborate target element models with advanced kill rules can be constructed.

Since the lethality model is based on physical properties, it can also be used to investigate how changes in physical properties of munitions and target elements influence the overall weapon effectiveness. This can, e.g., be used to study weapons under development, or completely hypothetical weapons, in tactical scenarios or cost-effectiveness studies. An example of this type of study was presented by Haataja, Lappi and Åkesson (2017).

This paper is organized as follows. The next section describes input data for model and presents example input data for a high explosive (HE) shell and a target element representing a prone soldier. This is followed by an outline of the general algorithm for computing the kill probability of a single fragmenting warhead to a single target element. Basic equations are presented in the following section. The paper ends with some concluding remarks.

2 Input Data

2.1 Parameters for Fragmenting Munitions

A fragmenting munition can be described by the following set of parameters.

- Explosive fill in TNT equivalent mass / alternatively the mass and type of explosive. This is used for determining the blast effect.
- An arbitrary number of fragment zones (also called fragment sprays), modelled as spherical zones, each having the following information
  - start and end angles with respect to the warhead nose
  - fragment mass distribution (in tabular or functional form)

- initial fragment speed
- fragment shape factor or, more generally, a drag model
- fragment perforation equation (Rilbe, THOR, etc.), chosen based on the shape and material of the fragments

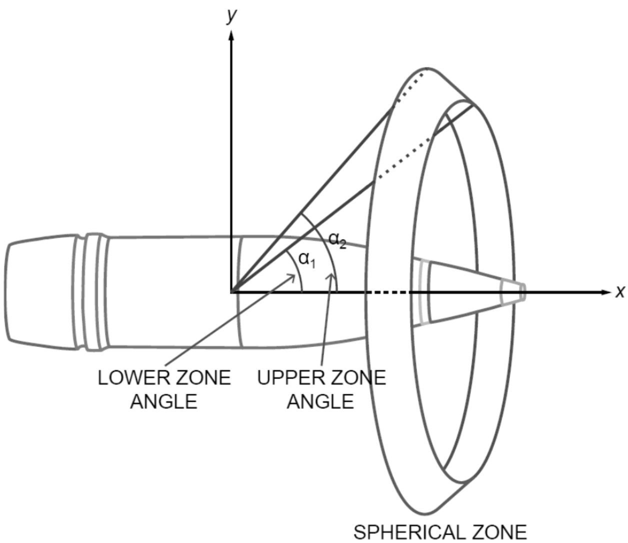

Figure 2. Fragment zone described as a spherical zone. Figure adapted from Yager (2013).

2.1.1 Example: 155 mm HE Shell M107

The shell has an explosive fill of 6.6 kg TNT (Dullum, 2008). Illustrative parameters relating to fragmentation are presented in Table 1. The total mass of the shell casing is divided over the zones as follows: 15 % in the nose zone, 80 % in the side zone and 5 % in the base zone. The angles of the fragment zones and the fraction of fragments in each zone are based on data for a generic HE shell given in *Kenttätykistöopas I: Ampumaoppi* (1990).

Table 1. Fragmentation characteristics of a 155 mm HE shell M107, based on open source data. The initial fragment speed and average fragment mass are based on data by Krauthammer (2008).

Parameter	Nose zone	Side zone	Base zone
Lower zone angle	0°	65°	170°
Upper zone angle	10°	115°	180°
Initial fragment speed at start angle	1030 m/s	1030 m/s	1030 m/s
Initial fragment speed at end angle	1030 m/s	1030 m/s	1030 m/s
Fragment distribution	Mott distribution, 381 fragments with average mass 14.34 g	Mott distribution, 2030 fragments with average mass 14.34 g	Mott distribution, 127 fragments with average mass 14.34 g
Fragment drag model	Irregular fragments	Irregular fragments	Irregular fragments
Fragment perforation model	Rilbe, steel fragments	Rilbe, steel fragments	Rilbe, steel fragments

The warhead data can be stored in an arbitrary format. One such format is the ZDATA file format (Yager, 2013), in which the fragment mass distribution for each zone is given in tabular form and the fragments have a shape factor used for computing drag.

2.2 Target Element Parameters

The target elements can be represented in three dimensions by a set of armour segments, each having a relative position, a normal vector and an area. Each segment is given a thickness value and material type, e.g. mild steel. Additionally, criteria for blast damage may be added to each segment.

This model has the advantage that personnel and vehicles can be handled in a similar manner. It also makes it straightforward to model the effect of protective gear for personnel, as well as different postures.

2.2.1 Example: Prone Soldier

Dimensions of a prone soldier are presented in Table 2.

Table 2. Dimensions of a prone soldier. A fragment capable of perforating 1.5 mm of mild steel is considered sufficient of causing incapacitation. Source of areas: Saarelainen (2007).

Aspect	Area [m^2]	Equivalent steel thickness [mm]
Front/Rear	0.08	1.5
Left/Right	0.38	1.5
Top	0.61	1.5

3 Calculating Fragment and Blast Effects to a Single Target Element

This section presents a simple algorithm for calculating the kill probability of a single fragmenting warhead to a single target element. It is assumed that the warhead detonates at ground level or above the ground. The geometry of the warhead/target element interaction is illustrated in Figure 3.

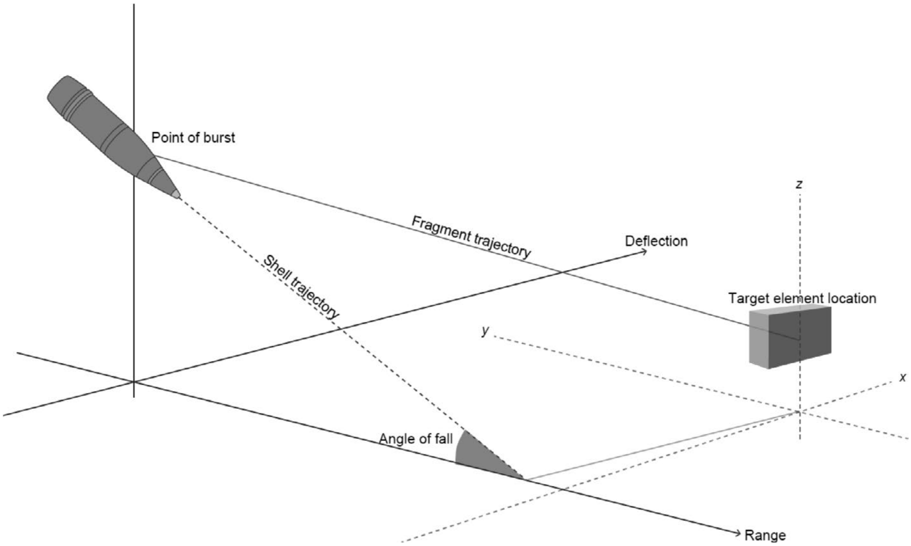

Figure 3. Terminal ballistics geometry. Figure adapted from Driels (2004).

A simple kill rule is to consider the target element killed if any of its armour segments are damaged by either blast or fragments. This kill rule is generally sufficient for target elements representing personnel. Let $p_{\mathrm{k},j}$ be the probability that the jth armour segment is killed. The overall kill probability for the target element is then

$$P(\text{kill}) = 1 - \prod_j (1 - p_{\mathrm{k},j}). \tag{1}$$

Let $p_{\mathrm{blast},j}$ be the probability that the jth target segment is killed by blast and let $p_{\mathrm{frag},i,j}$ be the probability that the jth target segment is killed by fragments from the ith fragment zone. Then $p_{\mathrm{k},j}$ can be computed from

$$p_{\mathrm{k},j} = 1 - (1 - p_{\mathrm{blast},j}) \prod_i (1 - p_{\mathrm{frag},i,j}). \tag{2}$$

The algorithm is outlined in Figure 4.

The following inputs are needed:

- Target element location and orientation
- Warhead velocity vector and desired point of burst
- Warhead and target element parameters, see section "Input Data"
- Optional: digital elevation model of the target area

The algorithm for computing the probability of kill is outlined as follows.

1. Determine the point of burst based on fuze settings and terrain
2. For all armour segments j in target element:
    - 2.1 Calculate distance from point of burst to segment
    - 2.2 Calculate blast kill probability $p_{\mathrm{blast},j}$ for segment j, see section "Blast Damage"
    - 2.3 For all fragment zones i in the warhead:
        - 2.3.1 Calculate dynamic zone angles, Eq. (10)
        - 2.3.2 Check that armour segment is within the fragment zone
        - 2.3.3 Calculate projected area of armour segment
        - 2.3.4 Check that armour segment is facing the point of burst
        - 2.3.5 Check for line of sight from point of burst to armour segment
        - 2.3.6 Calculate surface area of fragment zone, Eq. (9)
        - 2.3.7 Calculate minimum mass capable of perforation, Eq. (15)
        - 2.3.8 Calculate the number of effective fragments, Eq. (14)
        - 2.3.9 Calculate fragment kill probability $p_{\mathrm{frag},i,j}$, Eq. (11)
    - 2.4 Calculate kill probability $p_{\mathrm{k},j}$ for segment j, Eq. (2)
3. Calculate kill probability for target element, Eq. (1)

Figure 4. Algorithm for computing the kill probability of a single warhead to a single target element.

4 Basic Equations

4.1 Fragment Mass Distributions

We define the complementary cumulative distribution function (CCDF) of the fragment mass distribution as

$$N(M > m) = F_{\mathrm{M,c}}(m). \tag{3}$$

This is the cumulative number of fragments having a mass greater than a given mass m.

Here, three fragment mass distributions are presented: a discrete (categorical) distribution, the Mott distribution (Mott, 1943) and the Held distribution (Held, 1990). Several other distributions are available as well, see e.g. Elek and Jaramaz (2009).

4.1.1 Categorical Distribution

A straightforward way of describing a fragment mass distribution in a fragment zone is to divide the fragment masses into n_g mass groups. Each group i contains n_i fragments with average mass m_i.

In this case, the CCDF is

$$F_{\mathrm{M,c}}(m) = \sum_{m_i > m} n_i, \qquad i = 1,2,\ldots,n_{\mathrm{g}} \tag{4}$$

4.1.2 The Mott Distribution

The Mott distribution has the following parameters

- N_0 – Total number of fragments
- m_{avg} – Average mass of fragments [kg]

The total mass of fragments in the distribution is given by

$$M_0 = N_0 m_{\mathrm{avg}}. \tag{5}$$

The CCDF of the Mott distribution is given by

$$F_{\mathrm{M,c}}(m;\ N_0, m_{\mathrm{avg}}) = N_0 \exp\left(-\sqrt{\frac{2m}{m_{\mathrm{avg}}}}\right) \tag{6}$$

4.1.3 The Held Distribution

The Held distribution has the following parameters

- M_0 – Total mass of fragments in distribution [kg]
- B – Scaling factor
- λ – Form factor

The CCDF of the Held distribution is an implicit function and has to be solved numerically with respect to N for a given mass m.

$$m = M_0 B \lambda N^{\lambda-1} \exp\left(-B N^{\lambda}\right) \tag{7}$$

4.2 Fragment Kill Probability

4.2.1 Fragment Hit Probability

Assume an area A perpendicular to the fragment path. If the area of the fragment zone A_{zone} is large compared to the area A, the probability of fragment hitting the area is

$$p_{\mathrm{hit}} = \frac{A}{A_{\mathrm{zone}}}. \tag{8}$$

The fragment pattern of an HE shell can be modelled as a spherical zone, defined by an upper and a lower angle. Due to the velocity of the projectile, the angles of the zones will change and the total initial velocity of the fragments will be the resultant of the projectile velocity and the initial velocity in the static case. An illustration of fragment zones for a shell at rest and a shell in motion is shown in Figure 5. The static angles, when the shell is at rest, are denoted by α and the corresponding dynamic zones, when the shell is in motion, by β. The area of a spherical zone is

$$A_{\mathrm{zone}} = 2\pi x^2(\cos(\beta_{\mathrm{start}}) - \cos(\beta_{\mathrm{end}})) \tag{9}$$

where x is the distance and β_{start} and β_{end} are the start angle and end angle of the fragment zone, respectively.

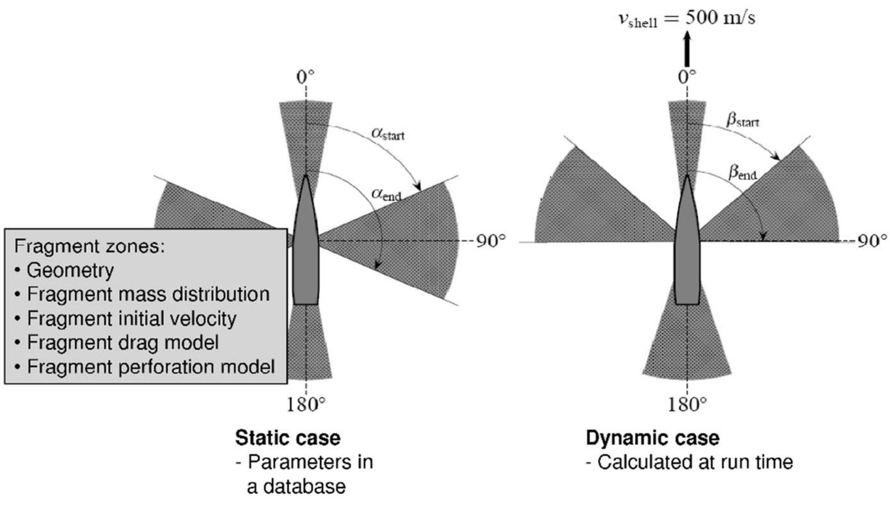

Figure 5. Fragment zone angles for a shell at rest and in motion. Figure adapted from Åkesson et al. (2013)

The angles are defined such that $0° \leq \alpha_{start} \leq \alpha_{end} \leq 180°$, where 0° is in the direction of the shell's nose.

Given the static angle α, the fragment initial speed in the static case and the shell velocity, the dynamic angle can be calculated from

$$\beta = \arccos\left(\frac{v_{\text{shell}} + v_{\text{frag}}\cos(\alpha)}{v_{\text{tot}}}\right) = \arccos\left(\frac{v_{\text{shell}} + v_{\text{frag}}\cos(\alpha)}{\sqrt{v_{\text{frag}}^2 + 2v_{\text{frag}}v_{\text{shell}}\cos(\alpha) + v_{\text{shell}}^2}}\right) \quad (10)$$

with:

β	=	dynamic fragment zone angle
α	=	static fragment zone angle
v_{shell}	=	shell speed
v_{frag}	=	fragment speed in the static case
v_{tot}	=	total fragment speed

4.2.2 A Simple Kill Rule Based on Fragment Perforation

The probability of at least k perforating fragment hits is calculated using the binomial distribution

$$P(\text{at least } k \text{ fragment hits}) = 1 - F_{\text{X,Bin}}(k-1; n_{\text{eff}}, p_{\text{hit}}), \quad (11)$$

where n_{eff} is the number of effective, i.e., perforating, fragments. $F_{X,Bin}(k; n, p)$ is the cumulative distribution function of the binomial distribution.

In the special case where we calculate the probability of at least one perforating fragment, Eq. (11) simplifies to

with:

$$P(\text{at least one perforating fragment hit}) = 1 - \left(1 - \frac{A}{A_{\text{zone}}}\right)^{n_{\text{eff}}} \quad (12)$$

$$\approx 1 - \exp\left(-n_{\text{eff}}\frac{A}{A_{\text{zone}}}\right) = 1 - e^{-\rho_{\text{frag}}A} \quad (13)$$

A	=	area of target segment perpendicular to the fragment path [m^2]
A_{zone}	=	area of fragment zone [m^2]
n_{eff}	=	number of effective fragments
ρ_{frag}	=	areal density of fragments [$1/m^2$]

Small fragments will lose speed faster than larger ones, which means that large fragments will remain effective over greater distances. Therefore, we first need to find the smallest effective fragment. The number of fragments with a mass greater than or equal to a minimum mass m_{min} is then calculated from the fragment mass distribution

$$n_{\text{eff}} = F_{\text{M,c}}(m_{\text{min}}). \tag{14}$$

The effective fragment is here taken as a fragment capable of perforating an armor plate of a certain thickness e_{min} from a given distance x with an initial speed v_0 and can be computed using the following procedure.

The aim is to solve the optimization problem

$$\underset{m}{\text{minimize}}\, m \tag{15}$$

subject to the following constraints

$$m > 0 \tag{16}$$

$$f_{\text{e}}(m, v_{\text{s}}) \geq e_{\text{min}} \tag{17}$$

where $f_e(\cdot)$ is a perforation equation. The striking speed v_s is given by a drag model

$$v_{\text{s}} = v(m;\ x, v_0). \tag{18}$$

This can be solved as a constrained nonlinear program. It can also be set up as a nonlinear root search problem.

Instead of using a perforation model, the fragment lethality can, e.g., be based on its kinetic energy. In that case the effectiveness threshold e_{min} is defined as a minimum kinetic energy and function $f_e(\cdot)$ as the formula for kinetic energy. However, when using kinetic energy as the lethality criterion, the influence of different fragment shapes and materials cannot be observed from the results.

4.3 Fragment Drag Models

The drag models provide the fragment speed at a distance x, given an initial speed v_0. They are used for calculating the striking speed v_s of the fragment, when the point of detonation and the position of the target element are known.

Two drag models are presented here. The first one has a general form and is applicable to fragments with regular shape. The second one is intended for irregular fragments, formed by natural fragmentation.

4.3.1 General Drag Model

A general drag model can be derived from the drag equation and Newton's second law,

$$v(x) = v_0 \exp\left(-\frac{\rho_a C_d A x}{2m}\right) \tag{19}$$

with:

$v(x)$	=	speed at distance x [m/s]
x	=	the distance traversed [m]
v_0	=	the initial speed [m/s]
ρ_a	=	density of air [kg/m^3] ($\rho_a \approx 1.2$ kg/m^3)
C_d	=	(unitless) drag coefficient
A	=	average cross-sectional area of the fragment [m^2]
m	=	fragment mass [kg]

The drag coefficient C_d depends on the shape and orientation of the fragment and on the Mach number M and the Reynolds number Re. The value of the Reynolds number gives an indication about the type of fluid flow around an object. The variation with Reynolds number is usually small within practical regions of interest, and the dependency is therefore ignored. Examples of drag coefficients for cubes and spheres are listed in Table 3.

Table 3. Drag coefficients for various shapes. Mach region indicates the Mach values for which the drag coefficient has been defined. Source: Janzon (1971) and US Army Test and Evaluation Command (1993).

Shape	C_d	**Mach Region**
Cube	0.83	$M \leq 0.9$
Cube	1.14	$M > 0.9$
Sphere	0.49	$M \leq 0.9$
Sphere	0.93	$M > 0.9$

4.3.2 Drag Model for Irregular Fragments

A fragment shape factor f_k can be introduced,

$$f_\mathrm{k} = \frac{A}{m^{2/3}} \tag{20}$$

with:

f_k	=	fragment shape factor [$\mathrm{m}^2/(\mathrm{kg})^{2/3}$]
A	=	average cross-sectional area of the fragment [m^2]
m	=	fragment mass [kg]

Substituting Eq. (20) into Eq. (19), the following equation is derived

$$v(x) = v_0 \exp\left(-\frac{kx}{\sqrt[3]{m}}\right) \tag{21}$$

where

$$k = \frac{\rho_\mathrm{a} C_\mathrm{d} A}{2m^{2/3}} = \frac{1}{2}\rho_\mathrm{a} C_\mathrm{d} f_\mathrm{k}. \tag{22}$$

The value of coefficient k is determined experimentally and represents an average for all fragments in the fragment zone. In *Technical Manual TM 5-855-1: Fundamentals of protective design from conventional weapons* (1986) and *United Facilities Criteria (UFC): Structures to resist the effects of accidental explosions, Change 2* (2014) the value k = 0.004 $(\mathrm{kg})^{1/3}/\mathrm{m}$ is given for a steel fragment of some standard shape. The following parameter values for a steel fragment are given in by Noopila (1984) and Nilsson (2010), the original sources being reports published by the Swedish National Defence Research Institute (FOA) in the 1970s

$$k = \begin{cases} 0.00264, & M \leq 0.9 \\ 0.00456, & M > 0.9 \end{cases} \tag{23}$$

4.4 Fragment Perforation Equations

The fragment perforation equations provide the perforation capacity e of a fragment, given a fragment mass m and a striking speed v_s.

4.4.1 The Rilbe Formula

The Rilbe formula (Rilbe, 1970) is used to compute the perforation capacity of a fragment with a certain mass and striking speed. The formula is expressed as

$$e = q v_s m^{1/3} \tag{24}$$

with:

e	=	the target thickness [m]
q	=	a parameter that depends on the fragment material, target material and fragment shape [s(kg)$^{-1/3}$], see Table 4
v_s	=	the striking speed [m/s]
m	=	the fragment mass [kg]

Table 4 lists values for constant q for a few combinations of fragment and target material.

Table 4. Rilbe constants for Eq. (24). Source: Rilbe (1970).

		q [s(kg)$^{-1/3}$]	
Fragment		**Target material**	
Material	**Shape**	**Mild steel (SIS 1311)**	**Dural**
Steel	Soft sphere (HRC 12)	$56 \cdot 10^{-6}$	$115 \cdot 10^{-6}$
	Cube	$42 \cdot 10^{-6}$	$90 \cdot 10^{-6}$
	Natural fragment	$39 \cdot 10^{-6}$	$82 \cdot 10^{-6}$ (calculated) / $70 \cdot 10^{-6}$ (experimental)
Tungsten	Small sphere (diam. ≤12 mm)	$72 \cdot 10^{-6}$	$190 \cdot 10^{-6}$
	Cube	$61 \cdot 10^{-6}$	$150 \cdot 10^{-6}$

4.4.2 The THOR Equations

There is a number of variations of the THOR formula (Ballistic Analysis Laboratory, 1959; Ballistic Analysis Laboratory, 1961; Crull and Swisdak, Jr., 2005; Dusenberry, 2010). One can use the formula to estimate the residual speed of the fragment after exiting the target plate or the striking speed necessary to perforate a target plate of a specific thickness. The equations can be simplified by making assumptions about the fragment shape.

By setting the residual fragment speed to zero, we obtain the ballistic limit for a general fragment shape

$$v_s = 10^{c_{1,SI}} (eA)^{\alpha_1} m^{\beta_1} (\sec(\theta))^{\gamma_1} \tag{25}$$

and for a specific fragment

$$v_s = 10^{c_{1,SI}}(eA)^{\alpha_1} m^{\beta_1} (\sec(\theta))^{\gamma_1} \tag{26}$$

with:

v_s = the fragment striking speed [m/s]
e = the target thickness [m]
A = the average impact area of the fragment [m^2]
m = the mass of the fragment [kg]
θ = the angle between the trajectory of the fragment and the normal to the target material
$c_{1,SI}$, $c_{1,SI}^*$, α_1, β_1, β_1^*, γ_1 = constants determined separately for each material, see Table 5

Constants for the THOR equations are given in Table 5.

Table 5. THOR constants for equations (25) and (26). The fragment material is steel. Source: Ballistic Analysis Laboratory (1959), Ballistic Analysis Laboratory (1961) and Crull and Swisdak, Jr. (2005).

Target Material	β_1	$c_{1,SI}$	c_1^*	β_1^*	$c_{1,SI}^*$	γ_1	α_1
Aluminium alloy (2024-T3)	-0.941	6.049	4.276	-0.399	3.781	1.098	0.903
Cast iron	-2.204	10.867	5.533	-0.747	5.375	2.156	2.186
Copper	-3.687	14.950	6.823	-1.403	5.975	4.270	3.476
Lead	-2.753	12.037	5.175	-0.930	3.127	3.590	2.735
Magnesium	-1.076	6.141	4.226	-0.406	3.611	0.966	1.004
Steel, face-hardened	-1.397	7.026	5.178	-0.603	4.036	1.747	1.191
Steel, mild homogenous	-0.963	6.309	4.608	-0.359	4.034	1.286	0.906
Steel, hard homogenous	-0.963	6.387	4.685	-0.359	4.111	1.286	0.906
Titanium alloy	-1.314	7.873	4.753	-0.431	4.545	1.643	1.325

4.5 Blast Damage

A blast wave generated in air and transmitted through the air is characterized primarily by a peak overpressure and a specific impulse, the latter being the integral of the overpressure over the positive phase time duration.

There are diagrams and numerical models available for TNT for determining the peak overpressure and impulse as a function of the distance from the point of detonation. Such diagrams are given e.g. in *United Facilities Criteria (UFC): Structures to resist the effects of accidental explosions, Change 2* (2014). For explosives other than TNT, one can convert the mass to a TNT equivalent by multiplying with a scaling factor (Cooper, 1996). As some of the energy released by the detonation goes into fracturing the shell casing, this also needs to be considered, by converting the mass into bare equivalent charge.

Threshold values for various levels of damage to personnel and structures from overpressure and impulse can be found in literature, enabling a simple three-dimesional cookie cutter damage function to be used for detonations in free air.

5 Conclusion

In this paper a computational model for assessing the lethality of fragmenting ammunition based on physical properties was presented. The model can be used as part of an effectiveness model, to estimate losses caused by fire missions on targets consisting of multiple target elements. The model is in use in the Finnish Defence Forces in two software implementations: the Sandis combat model and the newer indirect fire effectiveness assessment software EETU.

Remark

The author wrote the original manuscript, *A Physics-based Lethality Model for Fragmenting Ammunition*, in 2014–2015 as a white paper with the purpose of providing a summary of the algorithm and key equations and parameters of the lethality model implemented into the EETU software package. During that time the author was a member of the team of experts responsible for updating NATO standard 4654 (*Indirect Fire Appreciation Modelling*) and the white paper was used when writing the new version of the standard. The original white paper has not been published, but it has been included in the material distributed with EETU. The model description in this paper is included in the new version of the standard, titled AOP-4654 Edition A Version 1, which was released in June 2021.

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