DETERMINATION OF TAPERED ROLLER BEARING SYSTEM LIFE INDEX CONSIDERING BEARINGS PRELOAD AND SHAFT STIFFNESS

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ABSTRACT

Rolling bearings are considered to be rigid supports of a pinned nature. In fact, rolling bearings develop displacements that depend on the external load and the stiffness of the shaft on which they are supported. In addition, tapered roller bearings use preloading, which has an additional effect on the displacements in the bearings. these displacements affect the internal deformations in the bearings and thus the actual load on the bearing.

This paper presents a methodology for determining displacements in bearings and actual loads in bearings, taking into account shaft stiffness and preload clamping. Then the bearing life was determined. Based on the obtained bearing life, the bearing life index was determined for the adopted load variants.

On the basis of the results obtained it can be concluded that the consideration of the durability of individual bearings is insufficient, because it usually happens that when the durability of one bearing increases, the durability of the other bearing decreases. According to the author, the best solution is to combine the durability of both bearings and to consider the durability of the whole bearing system, not the individual bearings.

Keywords: tapered roller bearings, preload, durability of bearing, bearing life, shaft stiffness

1. Introduction

According to the literature [1], [2], [3] the main reasons for using initial tension are its following profitable effects:

- increasing the rigidity of the bearing system,
- noise reduction during operation,
- increase in the shaft guiding accuracy,
- compensation of wear and settling processes during operation,
- ensuring longer service life.
- reducing vibration

The modern scientific literature provides a lot of information on the influence of preload on bearings and bearing systems. The initial clamp occurs not only in the case of angular contact bearings, but also in the case of other bearings, such as in [4], where the influence of the preload on the operation of the slewing ring bearing is presented.

Much attention was paid to the influence of the preload on the stiffness of machine tool spindles in relation to thermal effects, e.g. [5], which presents a method of predicting the thermal characteristics of a high-speed spindle bearing subjected to preload. As a result of the analysis, it was noticed that the preload and bearing stiffness change non-linearly with the temperature increase.

On the other hand, [6] presents experimental studies of the influence of preload on the rotational performance of the bearing-spindle system at various rotational speeds.

The preload of the angular contact bearing is extremely important for the operation of a high-speed spindle and has a large impact on the dynamic and thermal characteristics of the spindle [7], [8], vibrations [9], the work of the spindle and bearing systems in the machine tool [10], [11], dynamic characteristics of the rotor [12], on the dynamic properties of the bearing system [13] and spindles [14].

The preload has a significant impact on the axial stiffness of the machine tool spindle, which was shown in the works [15], [16], where its influence on the spindle stiffness was investigated.

Wear between balls and races has significant effects on the dynamic characteristics of bearing which is the main reason to cause bearing failure. Some existing contact stiffness models were established to study the dynamic characteristics of bearing [17]. However the wear of bearing has been rarely investigated due to the complexities of contact load and wear mechanism.

The life of the angular contact bearing system can be determined to a good approximation using the catalogue method. However, this method is not suitable for

calculations which take into account the influence of preload. Therefore in the presented work a method based on the value of the average rolling element load [18] is used.

It is not possible to exactly determine forces acting in bearings if elasticity of bearings and shaft are not taken into account at the same time. The bearings together with the shaft are a coupled system in which a shaft deflection forces an angular deflection of the bearing rings but the angular deflection of the bearing rings causes arising of a reaction torque in the bearings (except for self-aligning bearings). This reaction torque impacts reducing the shaft deflection. On all of this a preload.

Preload is understood in this paper as the axial displacement of one bearing relative to the other such that the internal force in the bearings increases. This displacement is measured from the neutral state. The neutral state is defined in that if no external force (not even gravity) acts on the bearing arrangement, then in this state all rolling parts are adjacent to both rings of each bearing without contact pressure. It follows that a positive preload must cause contact pressures of the rolling parts to occur even when no external force is acting. A negative preload then causes the rolling elements to move away from the bearing raceway.

If the bearings are loaded by radial forces, a positive preload increases the number of rolling elements subjected to contact pressure and a negative preload decreases this number. The larger the preload (in absolute value), the stronger the described phenomenon occurs.

It is obvious that the preload affects both bearings simultaneously, but in the general case where the bearing arrangement is subjected to axial force, these effects are differentiated. Greater axial load can occur in both left and right bearings. This depends on which direction the shaft is "pushed" together with the inner bearing rings. The resultant of the axial forces is decisive in this regard [19].

2. Assumptions and method

2.1. Assumptions for analysis

In the presented work a modelling method devised by the author is described, it is similar to the method for angular contact ball bearings and used in works [20], [21], [22], [23], [24]. Due to the complexity of the problem the following most important simplifying assumptions were made:

1) bearing material is isotropic and subject to Hooke's law;
2) work surfaces of bearings are perfectly smooth;
3) there are no shape errors of rolling elements or bearing rings or shaft;
4) axes of bearing outer rings are always in one straight line (geometrically flawless seating of bearings, no clearances connected with bearing fit);
5) mass forces, cage effects and lubricant resistance are not taken into account in rolling load analysis;
6) tangential forces have no significant effect on elastic deformations and are ignored;

7) the pressure distribution in the point of contact of rolling elements and raceways is the same in motion as in static load;
8) elastic deformations of bearing elements occur only in points of contact of rolling element and rings.

2.2. Calculation method

It is not possible to exactly determine forces acting in bearings if elasticity of bearings and shaft are not taken into account at the same time. The bearings together with the shaft are a coupled system in which a shaft deflection forces an angular deflection of the bearing rings but the angular deflection of the bearing rings causes arising of a reaction torque in the bearings (except for self-aligning bearings). This reaction torque impacts reducing the shaft deflection. On all of this a preload, i.e. a close-up of the outer bearing rings towards each other, must be applied. This clamp will manifest itself as the sum of axial deflections in both bearings but it is not known in advance how this sum is distributed between the two bearings of the arrangement.

The influence of the following factors on the rolling element load in the bearing was therefore considered:

- radial and axial load acting on the bearing shaft,
- elastic deflection of the shaft causing deflection of the inner bearing rings,
- preload.

Figure 1 shows the set of loads taken into account. The external loads from hypothetical gear wheels (F_x, F_y) are the basis for calculating the R_x and R_y forces on the bearings. When calculating these forces it was assumed that the bearing reactions are concentrated at the points of intersection of the lines of action of the rolling elements.
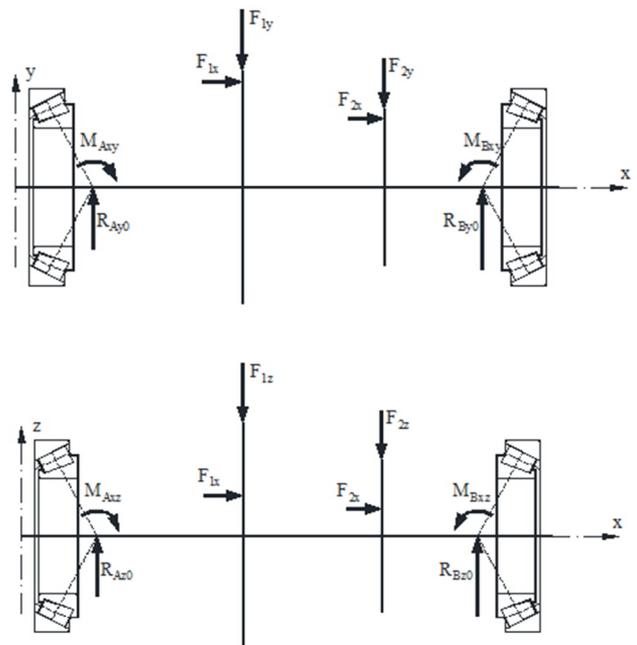
Figure 1. Calculation scheme of transverse support reactions

Figure 2. Calculation scheme of axial support reactions

The determination of forces in bearings is statically impossible. To solve it a procedure used earlier by the author [19], [20], [21] was adopted. This procedure relies on iterative search for such displacements in bearings (displacements and deflections) that all equilibrium conditions are fulfilled:

- compliance of the radial bearing reactions with the radial forces influencing the bearings according to the scheme in figures 1 and 2,
- compliance of the sum of bearing axial reactions with the sum of external axial loads,
- compatibility of bearings' deflection angles and shaft's deflection angles under bearings,
- compatibility of the reaction torques arising in the bearings and the bending moments taken into account while calculating the shaft deflection line,
- compatibility of the sum of axial displacements in the bearings with the value of the preload.

From the knowledge of the Q forces acting between the bearing shafts and rings it is possible to determine all bearing reactions understood in general, i.e. reaction forces in three directions of the coordinate system and reaction torques regarding the axes which are perpendicular to the bearing axis of rotation. Creation of reactions and marking them down is illustrated in Figure 3.
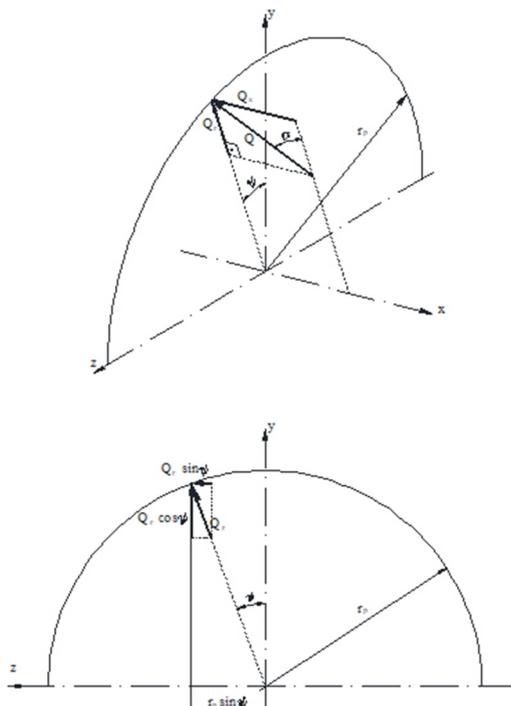
Figure 3. Illustration of determination of bearing reactions

In the upper part of the drawing there is shown in perspective an arc of the centres of shafts' points of contact with the bearing outer ring raceway. Reaction Q forces acting between the shaft and the raceway of the ring are applied to the points lying on this arc. The forces are deflected from the y-z plane by α bearing action angle. From the projection of the Q force on x-y-z directions formulae result:

$$Q_x = Q \cdot \sin \alpha \qquad (1)$$
$$Q_r = Q \cdot \cos \alpha \qquad (2)$$
$$Q_y = Q_r \cdot \cos \psi = Q \cdot \cos \alpha \cdot \cos \psi \qquad (3)$$
$$Q_z = Q_r \cdot \sin \psi = Q \cdot \cos \alpha \cdot \sin \psi \qquad (4)$$

The bending moment resulting from Q force is calculated in regard to the nodal point of the nominal bearing reaction because according to assumptions this point is assumed to be the place where the shaft is supported by the bearing. This calculation is illustrated in Figure 4.
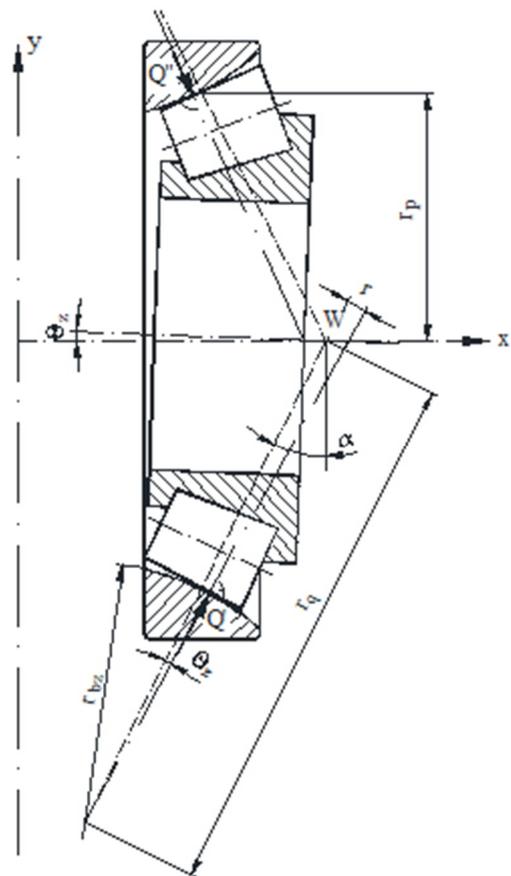
Figure 4. Illustration of determination of bearing reaction torque

The nodal point is denoted by W. Q force is the reaction of the outer ring on the shaft so it is applied at the centre of the shaft's point of contact with the outer ring raceway. The moment of Q force coming from an arbitrarily chosen shaft (here the shaft lying in the plane of the figure is chosen) is equal:

$$M = Q \cdot r$$

r arm of Q force in regard to O point is r segment. The length of this segment is:

$$r = r_q \, \Theta_z$$

where:

$$r_q = r_{bz} + \frac{r_p}{\cos \alpha}$$

The r_p dimension is determined by formula:

$$r_p = 0,5 \cdot d_{wm} + D_{km} \cos(\alpha - \beta)$$

And r_{bz} dimension (outer ring correction radius) is the roller bearings execution parameter.

The reaction torque acting in regard to the z-axis, resulting from the tilt by an angle Θ_z and coming from one shaft lying in the x-y plane, is equal:

$$M_{\Theta z} = Q \cdot r_q \, \Theta_z$$

Shafts lying outside x-y plane due to the same Θ_z tilt generate smaller reaction torques and if the shaft lies in x-z plane, $M_{\Theta z}$ reaction torque is zero. This dependence looks as follows:

$$M_{\Theta z i} = Q_i \, r_q \, \Theta_z \, \cos \psi_i$$

The vectors of moments coming from all the shafts are directed circumferentially. Therefore the resultant M_z moment must be determined by projecting all the individual moments onto z-axis. At the same time from the same individual $M_{\Theta z}$ moments the resultant My moment is created (with regard to y-axis), deriving from the same Θ_z tilt. This is illustrated in the first quadrant of the y-z system in Figure 5.
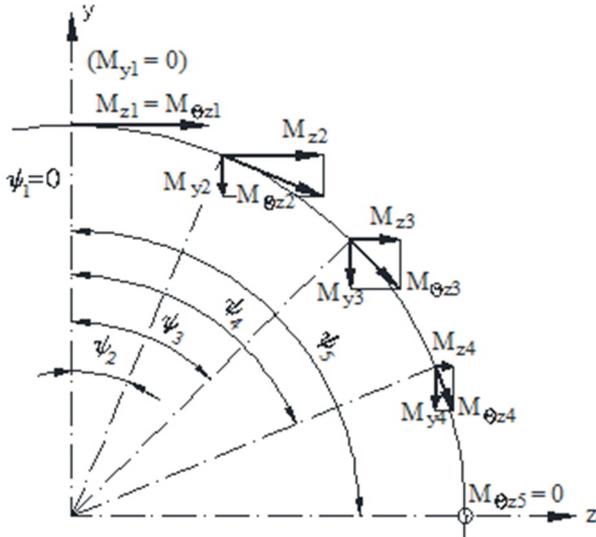
Figure 5. Vectors of reaction torques resultant from tilt Θ_z

Therefore as a result of Θ_z tilt at the location of each shaft, identified by number "i", moments arise:

$$M_{zi} = Q_i \, r_q \, \Theta_z \, (\cos \psi_i)^2$$
$$M_{yi} = Q_i \, r_q \, \Theta_z \, \cos \psi_i \, \sin \psi_i$$

Arising of reaction torque proceeds analogously due to Θy tilt. According to the illustration presented in Figure 6 each shaft can be assigned to generate a moment particle of value:

$$M_{\Theta y i} = Q_i \, r_q \, \Theta_y \, \sin \psi_i$$
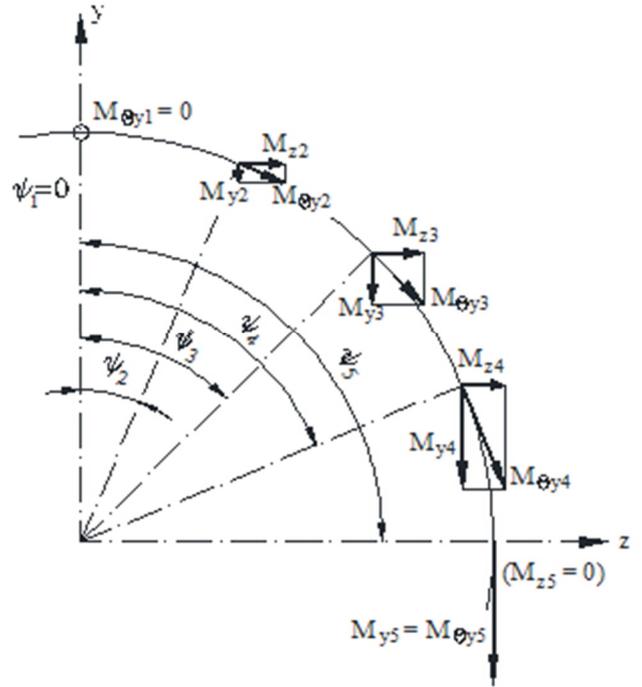
Figure 6. Vectors of reaction torques resultant from Θ_y tilt

The vectors of moments coming from all the shafts are directed circumferentially. As a result of the projection on y and z axes two resultant moments arise: with regard to y axis M_y moment and with regard to z axis M_z moment. This is illustrated in the first quadrant of y-z system in Figure 6.

Thus at the location of each shaft, identified by number "i", due to Θ_y tilt moments arise:

$$M_{yi} = Q_i \, r_q \, \Theta_y \, (\sin \psi_i)^2$$
$$M_{zi} = Q_i \, r_q \, \Theta_y \, \sin \psi_i \, \cos \psi_i$$

Considering the general case when bearing loads cause deflections of the shaft in both planes (i.e. Θ_y and Θ_z tilts occur simultaneously), the following expressions are obtained, describing the particles of the reaction torque, generated at the point determined by ψ_i angle:

$$M_{yi} = Q_i \, r_q \, [\Theta_y \, (\sin \psi_i)^2 + \Theta_z \, \sin \psi_i \, \cos \psi_i] \quad (6)$$

$$M_{zi} = Q_i \, r_q \, [\Theta_z \, (\cos \psi_i)^2 + \Theta_y \, \sin \psi_i \, \cos \psi_i] \quad (7)$$

Expressions (1)÷(7) define the forces and moments coming from one (any) shaft. The total bearing reactions and the total bearing reaction torques are the result of the interaction of all the shafts. These quantities are calculated by summing the forces and moments caused by all those shafts which are subjected to normal deformations, i.e. are loaded with non-zero forces.

$$R_x = \Sigma(Q \cdot \sin \alpha) \quad (8)$$
$$R_r = \Sigma(Q \cdot \cos \alpha) \quad (9)$$
$$R_y = \Sigma(Q \cdot \cos \alpha \cdot \cos \psi) \quad (10)$$
$$R_z = \Sigma(Q \cdot \cos \alpha \cdot \sin \psi) \quad (11)$$
$$M_y = \Sigma\{ Q_i \, r_q \, [\Theta_y \, (\sin \psi_i)^2 + \Theta_z \, \sin \psi_i \, \cos \psi_i]\} \quad (12)$$
$$M_z = \Sigma\{ Q_i \, r_q \, [\Theta_z \, (\cos \psi_i)^2 + \Theta_y \, \sin \psi_i \, \cos \psi_i]\} \quad (13)$$

The formulae presented were used to determine the reactions of both shaft bearings separately, on the basis of their separate internal deformations.

Similarly to the case of angular contact ball bearings and tapered roller bearings the substitutive bearing load is determined based on the average load of the rolling element in its motion around the bearing axis. The method of proceeding is based on the handbook [1] where the values of coefficients are as follows: rolling elements load distribution coefficient Jr (Sjövall integrals)

$J_{r}(0.5) = 0.2453$ and load function $J1(0.5) = 0.6495$.

Thus the substitutive load of a journal bearing with linear contact is calculated from the formula:

$$P = \frac{J_{r(0,5)}}{J_{1(0,5)}} Z\ Q_{śr} = \frac{0,2288}{0,5625} Z\ Q_{śr} = 0,4068\ Z\ Q_{śr} \quad (14)$$

The fatigue strength of the raceways and shafts of a tapered roller bearing depends on their average load in the same way as in a cylindrical roller bearing provided that the total load, not only its radial component, is taken into account. In the procedure devised it is the total loads that are calculated). Therefore it was assumed to use the formula (14) for the tapered roller bearing.

The number of millions of revolutions the bearing is likely to complete before significant damage to the raceway or rolling parts occurs is determined by the formula:

$$L = \left(\frac{C}{P}\right)^{3,333} \quad (15)$$

3. Calculation example

Taper roller bearings of the basic type, i.e. series 302, were used as the subject of calculation. Bearing No. 30209 was selected for the calculation example. The dimensions of the bearings' work surfaces are required for the calculation. They were assumed according to the archival documentation of CBKŁT. There were no contemporary data available as regards this topic (bearing manufacturers keep these dimensions as production secrets) but contemporary deviations from these archival data cannot be very big (overall dimensions of bearings are unchangeable). Therefore these possible deviations cannot qualitatively influence calculation results. Dimensions of work surfaces of selected bearings are shown in Table 1.

For the selected bearings a model shaft was determined the dimensions of which are defined according to Figure 7 and given in Table 2. It was assumed that in each calculation case the shaft is supported by two identical tapered roller bearings being in X arrangement. The shape of the shaft was selected according to the typical shape of gear shafts in which the bearings are placed at the ends of the shaft and the diameters at the individual segments correspond to the theoretical outline being built on the principle of equal bending strength. In technical reality there are infinitely many types of shaft shape.

Table 1. Dimensions of work surfaces of bearings taken into account for calculation

Bearing	30209
D_k [mm]	10,495
L_n [mm]	13,8
r_e [mm]	150
s_f [mm]	0,5
Z	19
D_1 [mm]	57,945
r_{bz} [mm]	7500
α [rad]	0,2637096
δ [rad]	1,562070
β [rad]	0,0349066
e	0,40
Y	1,5
C [N] [3]	66000
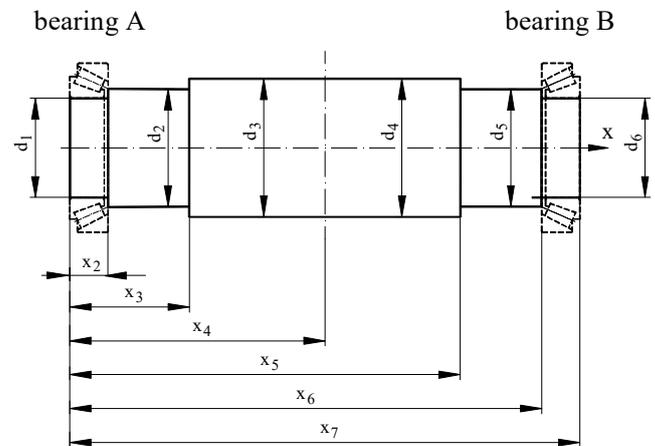
Figure 7. Model shaft design

For all model shafts the shaft beginning coordinate x_1 was assumed as equal to zero.

The bearing was calculated with loads varying in value. A variants of the load location is shown in Figure 8.
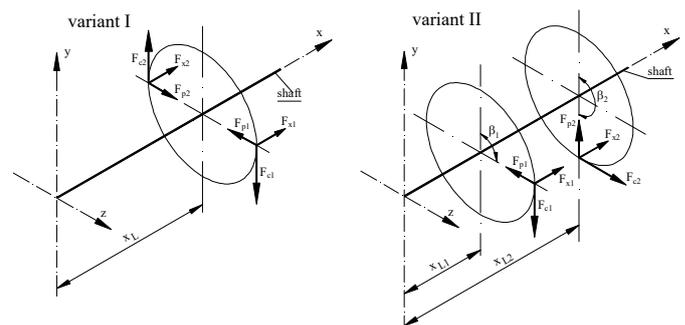
Figure 8. Assumed variants of bearing load

Table 2. Dimension parameters of models shafts [mm]

Bearing	30209
x_2	21
x_3	75
x_4	150
x_5	225
x_6	279
x_7	300
d_1	45
d_2	52
d_3	60
d_4	60
d_5	52
d_6	45

The loading forces (circumferential, radial and axial) are applied at points marked by a straight line running through the wheel parallel to the "z" axis, so the circumferential forces F_c are parallel to the "y" axis, the radial forces F_p are directed towards the center of the wheel, axial forces F_x according to the axis "x". The positions of the load application points are defined by the angles β_1 and β_2. The directions of the radial and axial forces are determined in the same way as in the drawing. The position of the load planes in variant I is assumed in relation to the shaft length L_w, equal to the dimension x_7, which is $x_L = 0.6\,L_w$, and in variant II: $x_{L1} = 0.4\,L_w$, $x_{L2} = 0.6\,L_w$.

Roller diameter = 150 mm.

The loads shown in Figure 10 were assumed to be identical ($F_{c1} = F_{c2}$, $F_{p1} = F_{p2}$, $F_{x1} = F_{x2}$).

Firstly it was established that the circumferential force on the alleged F_{c1} gear wheel would be taken at three levels: as 0.075 dynamic load capacity C, 0.1 dynamic load capacity C or as 0.125 dynamic load capacity C. Assuming that the interlocking buttress angle of the gears is 20° the radial Fp force was set as approximately 0.36 of the circumferential force. The axial Fx force was assumed in five values in the fixed relations to the circumferential force. However due to the greater weight of the axial force in tapered roller bearings than in angular contact ball bearings the axial force contributions were reduced accordingly. This was done on the principle of inverse proportionality to the thrust load Y factor. In the tapered roller bearings of the 302 series – slightly more than 1.4. The contribution of the axial force was therefore assumed smaller for the tapered roller bearings in the ratio 0.57:1.4 = 0.41. Hence the following relative values of the axial forces for the tapered roller bearings of 302 series resulted: 0, 0.03 F_c, 0.05 F_c, 0.0874 F_c, 0.16 F_c. Since in the

assumed load variant two equal axial forces and two equal circumferential forces act on the bearing, the ratio of the sum of axial forces to the sum of circumferential forces is described by the same series of numbers. The circumferential Fc force is not the only transverse load (the other one is the radial force) but due to the assumed constant ratio of the radial force to the circumferential one it is assumed to treat the ratio of the circumferential force to the bearing Fc/C carrying capacity as a parameter characterizing the level of the transverse load in the bearing arrangement.

The load values summarized in Table 3 result from the above findings.

Table 3. Load values assumed for calculation

Bearing		30209		
F_c/C		0,075	0,100	0,125
F_c [N]		4950	6600	8250
F_p [N]		1802	2402	3003
F_x [N]	1	0	0	0
	2	149	198	248
	3	248	330	413
	4	433	577	721
	5	792	1056	1320

4. Calculation results and discussion

Figs 9÷14 show the fatigue life characteristics of the left (A) and right (B) bearing as a function of Zc preload for assumed load variants.

First, the influence of the initial clamp on the durability of the bearing "A" and "B" was determined, where it was found that the life of one bearing decreases while the other increases. From the performed observation, it was not possible to unequivocally state what value of the initial clamp is optimal for the bearing system. Therefore, the characteristics of the W_T index, combining the durability of both bearings, were determined:

$$W_T = \frac{L_{hA}}{L_{hA0}} \cdot \frac{L_{hB}}{L_{hB0}} \tag{16}$$

where:

L_{hA} – fatigue life of the bearing A determined in specific conditions with applied preload

L_{hB} – fatigue life of the bearing B determined in the same conditions with applied preload

L_{hA0} – fatigue life of the bearing A determined in specific conditions without preload

L_{hB0} – fatigue life of the bearing B determined in the same conditions with without preload
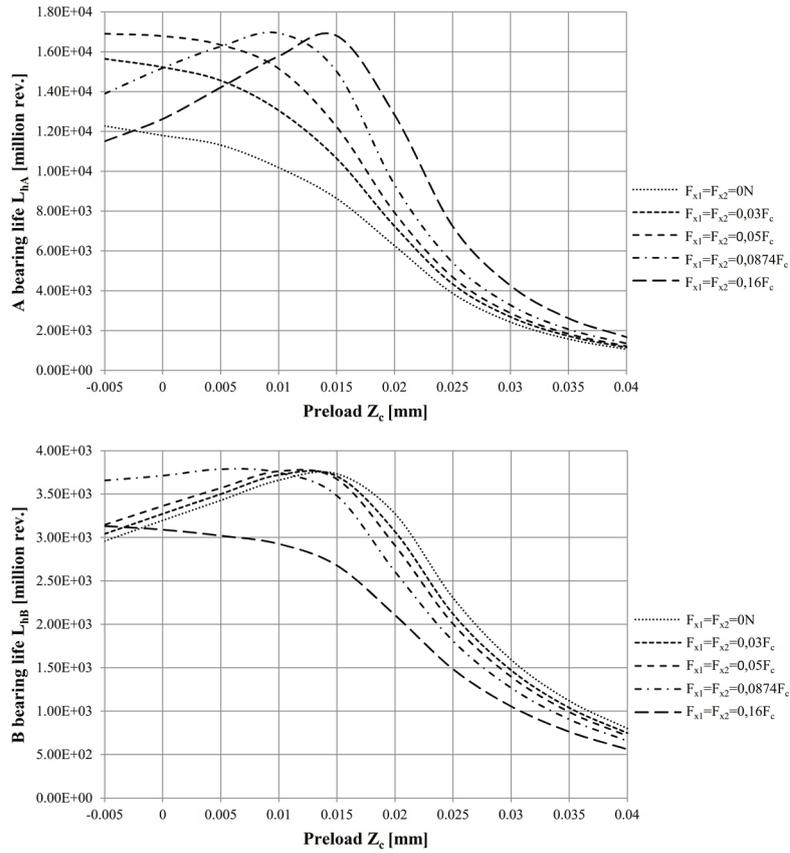
Figure 9. Life of 'A' and 'B' bearing for the circumferential load which is 0,075C for I load variants
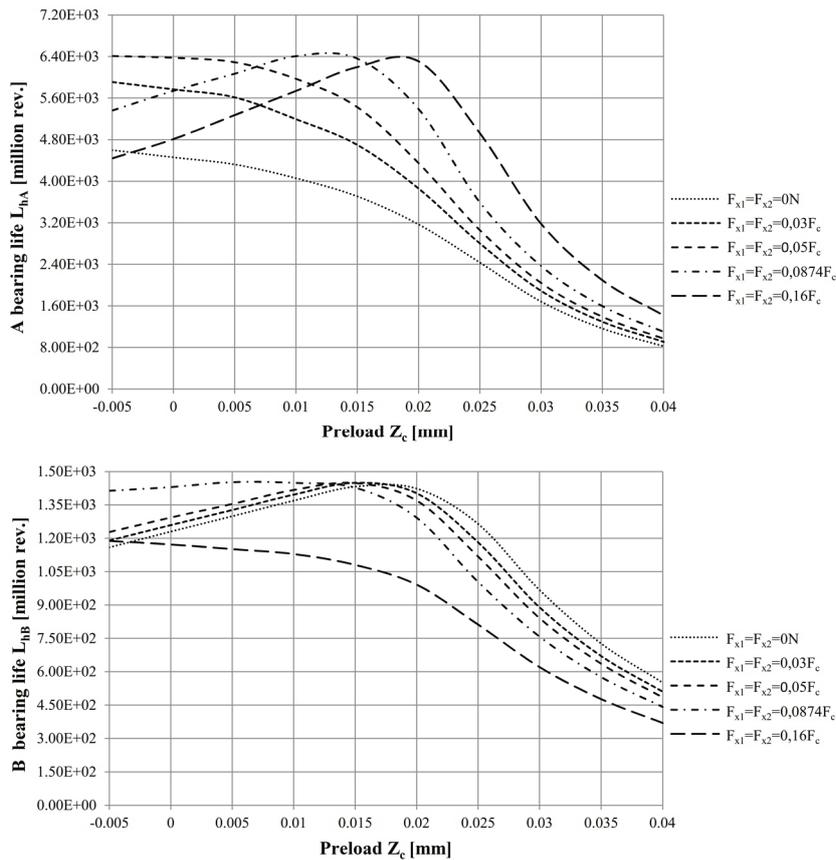
Figure 10. Life of 'A' and 'B' bearing for the circumferential load which is 0,1C for I load variant
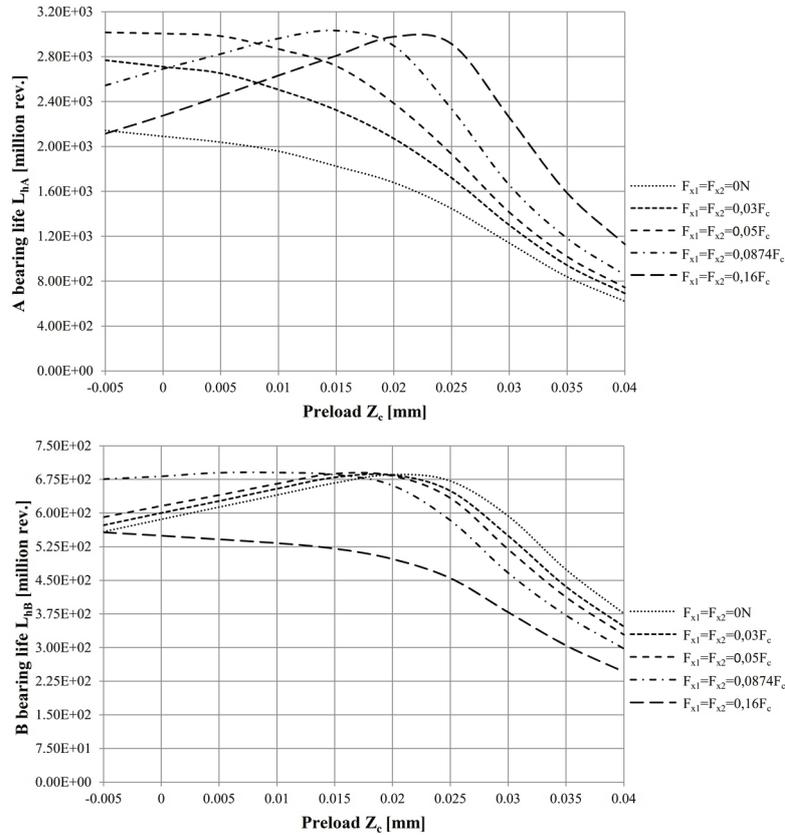
Figure 11. Life of 'A' and 'B' bearing for the circumferential load which is 0,125C for I load variant
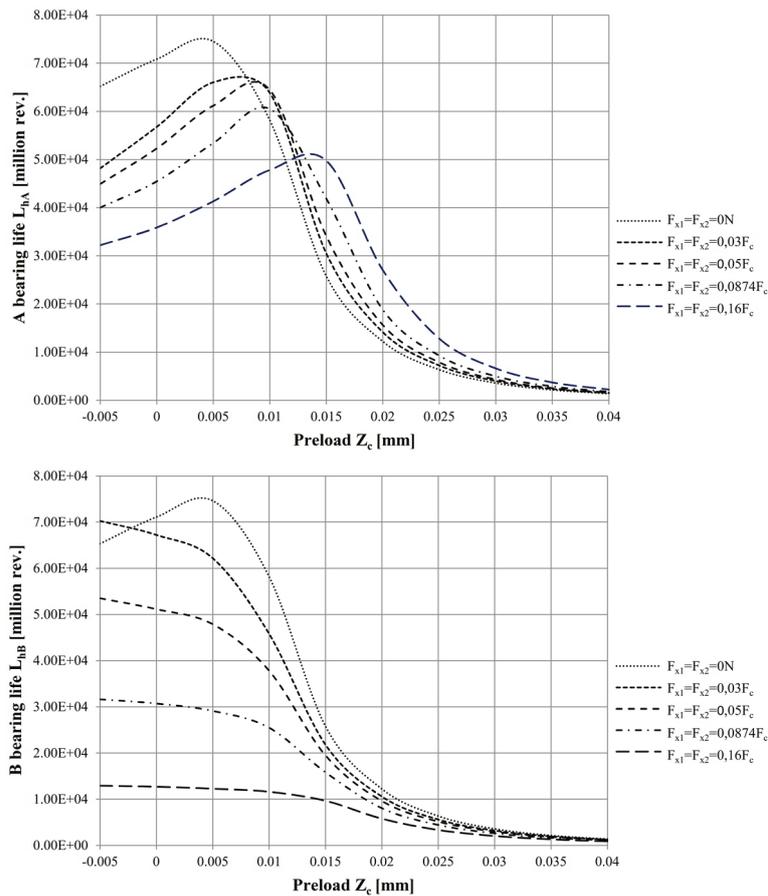
Figure 12. Life of 'A' and 'B' bearing for the circumferential load which is 0,075C for II load variant
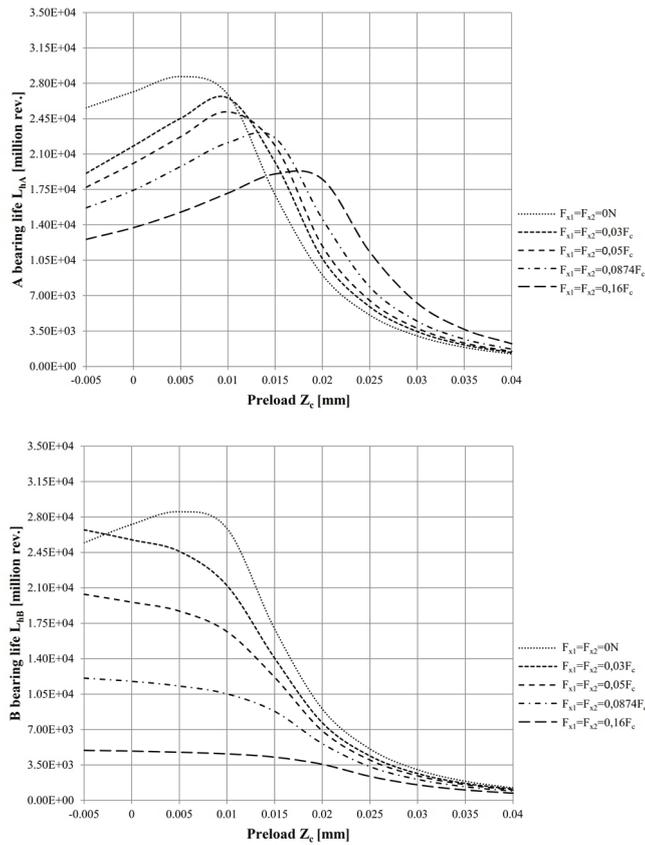
Figure 13. Life of 'A' and 'B' bearing for the circumferential load which is 0,1C for II load variant
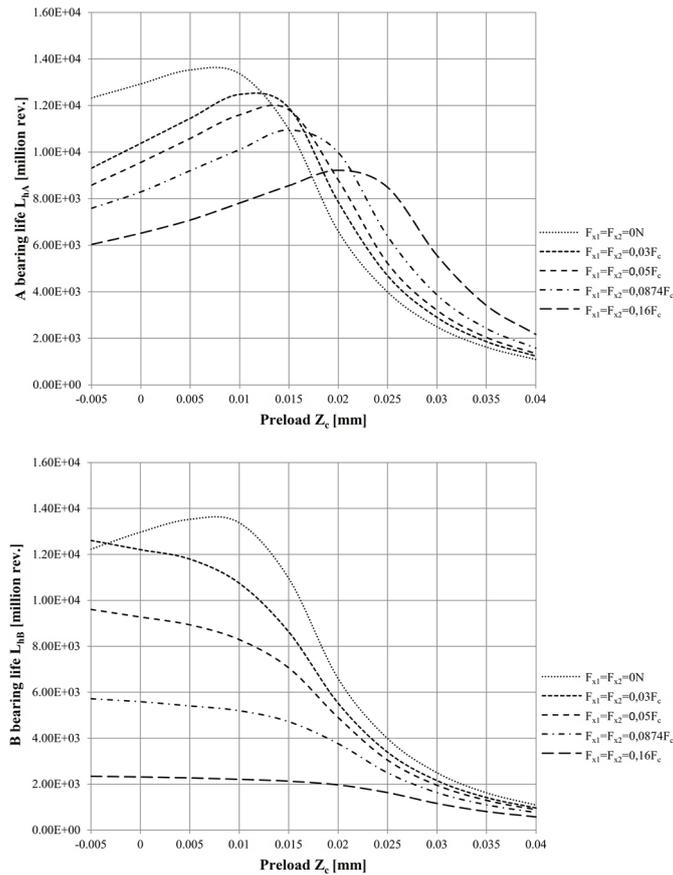
Figure 14. Life of 'A' and 'B' bearing for the circumferential load which is 0,125C for II load variant
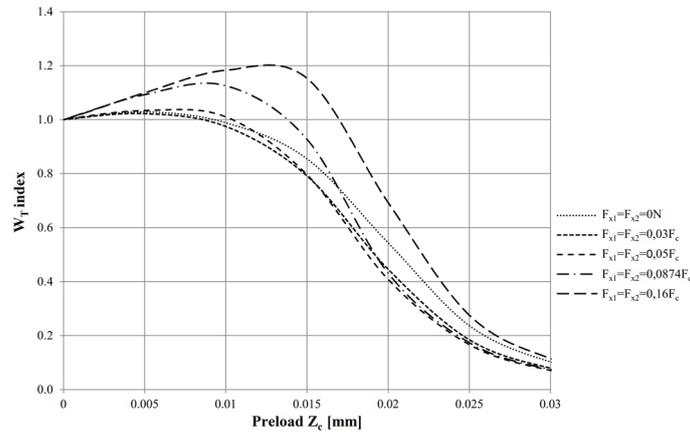
Figure 15. W_T index characteristics for the circumferential load which is 0,075C for I load variant
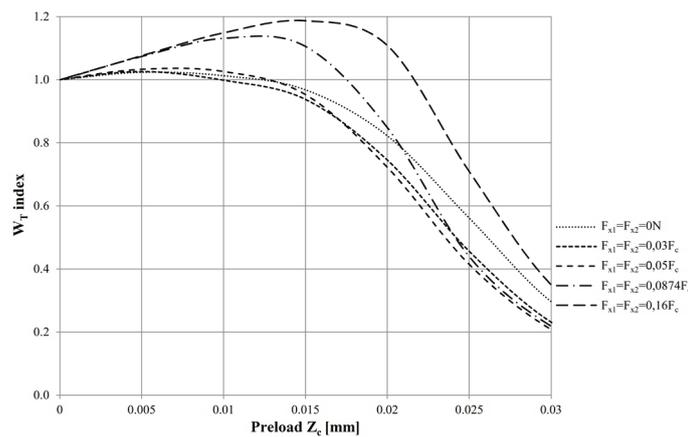
Figure 16. W_T index characteristics for the circumferential load which is 0,1C for I load variant
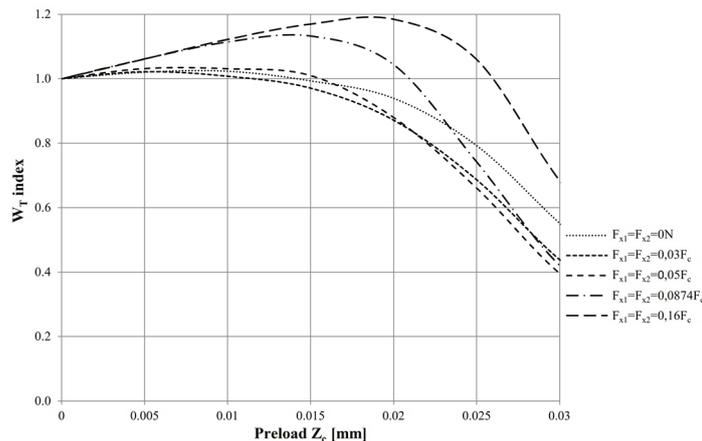
Figure 17. W_T index characteristics for the circumferential load which is 0,125C for I load variant

The W_T index is an overall parameter that applies to the simultaneous operation of both bearings. Therefore, the formulation of this indicator made it possible to observe the life of the complete bearing system.

The second important advantage of the indicator formulated in this way is that when the life of one of the bearings drastically decreases (e.g., close to zero), the value of this indicator also decreases close to zero. Moreover, an increase in the life of both bearings causes an increase of this indicator, and a decrease of one of them causes a decrease. Therefore, it gives a correct evaluation of the bearing life as a whole.

W_T indicator characteristics for the loads variants are shown in Figures 15÷20.
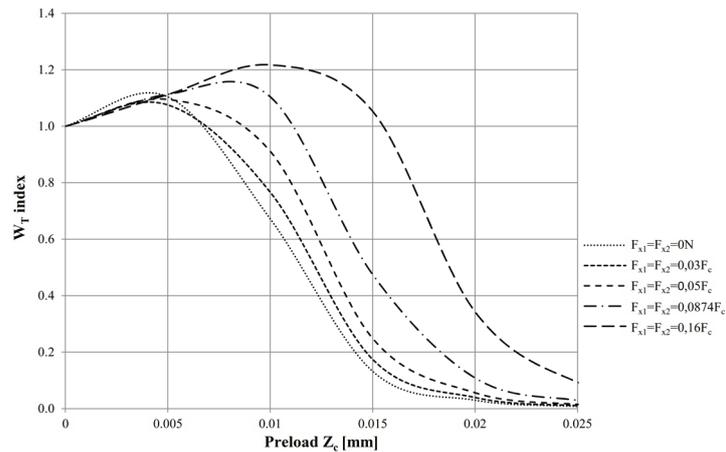
Figure 18. W_T index characteristics for the circumferential load which is 0,075C for II load variant
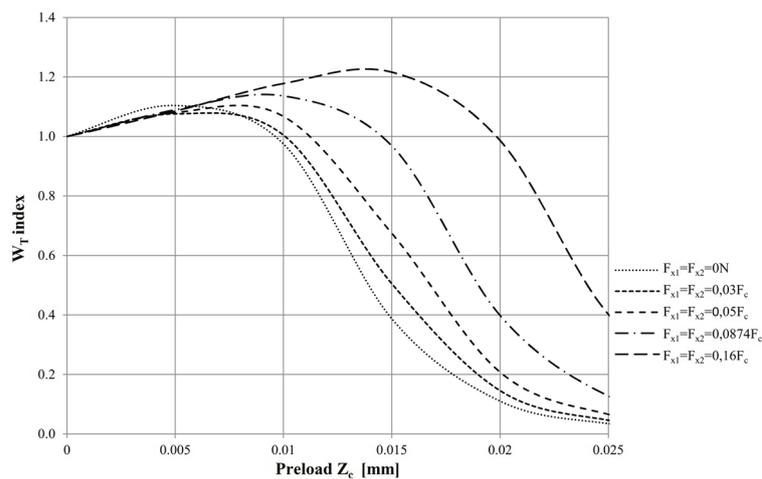
Figure 19. W_T index characteristics for the circumferential load which is 0,1C for II load variant
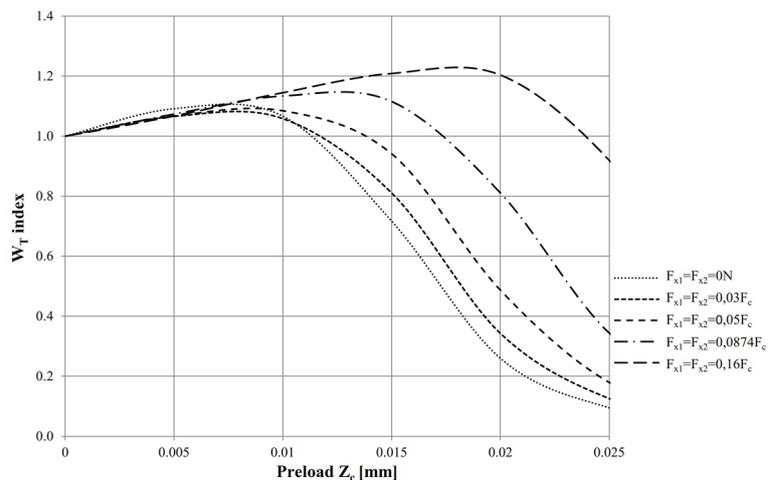
Figure 20. W_T index characteristics for the circumferential load which is 0,125C for II load variant

Based on the obtained characteristics, it can be observed:

1. If the loads are applied to the load plane $x_L = 0.6 L_w$ (I load variant), the characteristics of the durability index have different courses, but they all increase to a certain maximum, which is achieved at different values of the initial clamp. The characteristics that correspond to the greatest received axial force pile up the most. The lines corresponding to small axial forces slightly exceed the level of $W_T = 1$ and begin to drop even with a slight initial clamp. This indicates a low permissible preload in these cases, but with all data sets, the use of a limited preload does not pose a risk to the overall bearing life.

2. If the loads are applied in the load planes x_{L1} = 0.4 L_w and x_{L2} = 0.6 L_w (II load variant), then for each data set, as the initial clamp Z_c increases, the fatigue life index W_T increases. The amount of preload at which the W_T line maximum occurs is proportionally dependent on the relative axial force and the transverse load. The rate of decrease in the characteristics, noticeable after reaching the maximum, is the greater, the smaller the value of the axial force acting on the bearing arrangement. The individual characteristics are closer to each other than in the previous variant of the location of the load. General observation: The elevation of the durability index characteristics above the initial level in all cases in this series proves that the use of a limited initial clamp in this case does not endanger the overall bearing life.

5. Conclusions

The assumptions adopted for the determination of the coefficient determining the durability of the entire bearing system make it possible to determine the actual loads on the bearings. Thanks to the assumptions made, the influence of shaft rigidity on the displacements in the bearings and the influence of initial pressure on these displacements were taken into account. These displacements provided the basis for determining the internal deformations in the bearings and then the internal forces, resulting in the determination of the bearing system life.

The individually observed bearing life characteristics do not answer the question of what is the optimum initial clamp value for the bearing system. Therefore, it was decided to create an index combining the life of both bearings of the system.

The correctness of the applied mathematical model of the methodology should be verified by experimental research, which unfortunately is time-consuming. Such research is planned by the author in cooperation with FLT in Kraśnik.

6. References

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