



<https://doi.org/10.30678/fjt.127785>

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# A critical review of the effect of nano-lubricant on the performance of hydrodynamic journal bearing

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## ABSTRACT

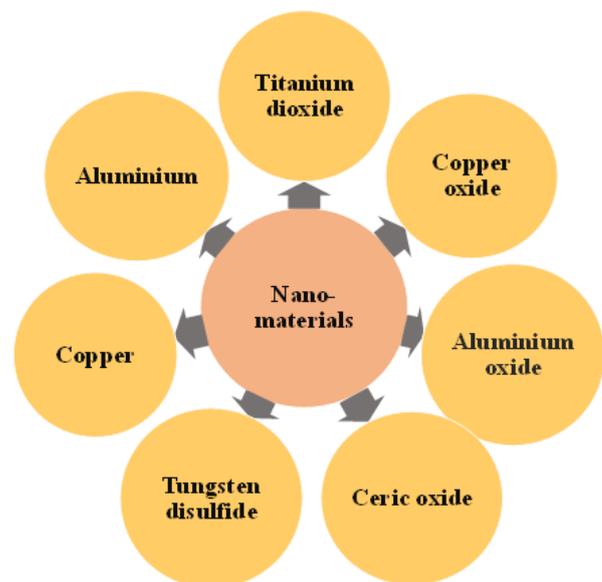
The applications of nano-lubricants in journal bearing are found to be capable of improving load-carrying capacity and reducing friction. Many numerical and experimental investigations have been performed to find the effect of nano-lubricants on the performance of the journal bearing. In this paper, a state-of-the-art review is done of the articles of numerical and experimental assessments of the effect of nano-lubricants on the performance of journal bearings. Here research articles are classified on the basis of nanoparticles used as nano-additive, which are further classified in the category of numerical and experimental study. It is seen that the application of nano  $\text{TiO}_2$ ,  $\text{CuO}$  and  $\text{Al}_2\text{O}_3$  based nano-lubricants gives much higher load-carrying capacity than the base lubricants. This article concludes by discussing the effect of different nanoparticle-based nano-lubricants on the performance characteristics of the journal bearing and mathematical models used for numerical investigations.

**Keywords:** Nano-lubricants, Journal Bearings, Coefficient of friction, Load Carrying Capacity, Viscosity models

## 1. Introduction

Journal bearing is an essential element of heavy machinery such as turbines, compressors, pumps and the automotive industry [1, 2]. Due to the widespread utilization of bearings, numerous efforts are being made by researchers in the direction of improving the performance behaviours of the bearing. Different techniques such as surface texture, use of bio-lubricants and lubricant additives have been widely investigated in order to improve the tribo-dynamic behaviours [3–8]. Nowadays, researchers are exploring the impact of the addition of nano-particles of the size of the order of 10-100 nm in the lubricants on the performance of journal bearings. These particles are made of metals, metal oxides, silicates, semiconductors and carbonates [9–12]. Researchers have investigated the effect of different types of nano-lubricants [13–23] with varying concentrations of nano-additives [14, 19, 24–26] on the performance of the bearing. Figure 1 shows different nano-particles applied in fluid film lubrication.

The presence of nanoparticles affects the rheology of the lubricant, hence understanding the rheological behaviour of nano-lubricant is required [27]. Various rheological tests of nano-lubricants confirm that the nano-additives increase the viscosity of base lubricant [28–33], control the reduction of the viscosity with increasing temperature and enhances the thermal conductivity [29–32, 34], which improves the



**Fig. 1.** Nanoparticles used as nano-additives in lubrication of journal bearings

static [13, 14, 35] and dynamic performance [20, 36] of the bearing. The inclusion of nano-additives in lubricant significantly improves the tribological properties of the lubricant [37–41]. Tribological tests performed on pin-on-

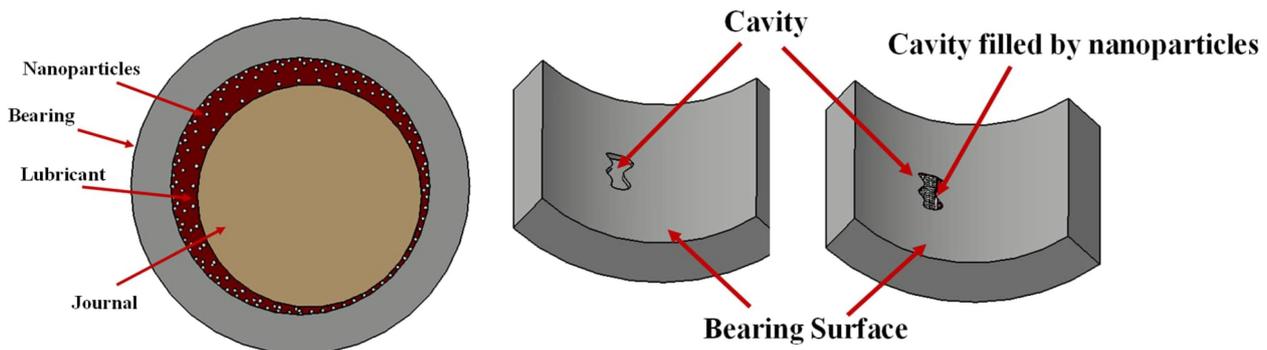


Fig. 2 (a) Rolling mechanism of nanoparticles inside bearing (b) self-repairing by nanoparticles on bearing surface.

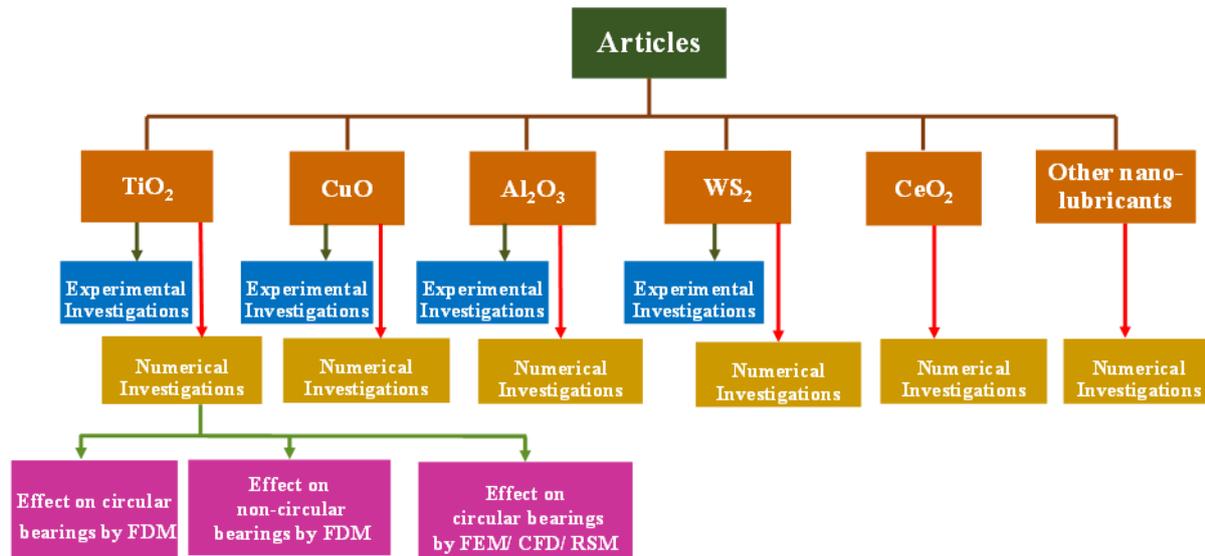


Fig. 3. Classification of reviewed articles

disk [40], four ball tribo-tester [42] reported reduced friction and wear [43–52] with the addition of nanoparticles in mineral oils due to a reduction in real area of contact of the mating parts [53]. Various experimental investigations have revealed the non-Newtonian [33, 54, 55] behaviour of lubricant due to the presence of nano additives, however, a few reported the Newtonian nature of lubricant [31]. Nano-lubricants are prepared by mixing the nanoparticles by mechanical agitator in lubricant followed by ultrasonication process [29, 31, 32, 54, 55].

The mechanism of nano-lubrication is complex due to the presence of numerous particles present in a small weight fraction of the lubricant, which affects the properties of the lubricant [56]. Nanoparticles perform a micro-rolling process [46, 57] between the mating surfaces and decrease the area of contact between the mating parts [58] as illustrated in Fig. 2(a). Moreover, by agglomeration on the mating surface, nanoparticles form a protective layer [59–61], the formation of this tribo-film is the primary wear reduction mechanism [62]. Nanoparticles create a smooth/ levelled surface [63], as illustrated in Fig.2(b), and act as a polishing element [46,58].

It is evident that nano-lubricants have better tribological and rheological properties than base lubricants. The effectiveness of nano-lubricants influences the

researchers to investigate the effect of nano-lubricants on the journal bearing's performance. Many numerical and experimental studies have been performed by researchers so far. However, the authors of the present paper could not find any article covering the review of the effects of nano-lubricants on the performance of the journal bearing.

Therefore, the objective of the present paper is to provide a comprehensive review of the published numerical and experimental studies focused on the application of different nano-additives in journal bearing lubrication. Moreover, it highlights important mathematical models that are helpful in predicting the effects of nano-lubricants. The effects of nano-additives on tribological and rheological properties of lubricants and the mechanism of nano-lubricants are also discussed in brief. Relevant research articles are classified according to the types of nano-additives used and thereafter based on the type of investigation. The classification is illustrated in Fig. 3.

## 2. Classification of nano-lubricants used for journal bearing

Researchers have employed different nanoparticles such as titanium dioxide, copper oxide, aluminium oxide, zinc oxide, etc. in their numerical and experimental

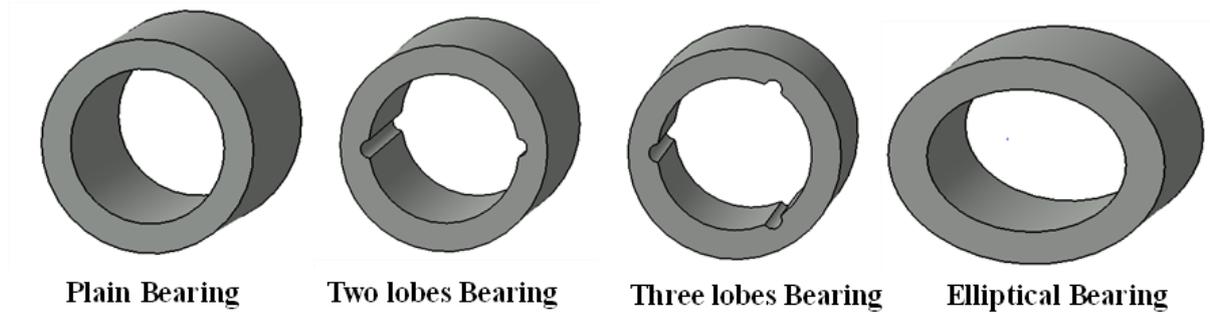


Fig. 4. Plain and non-circular Journal Bearing

investigations. Mineral oil-based lubricants [13-20, 26, 35, 66-68, 70, 73-76, 79-82], bio-lubricants [24, 25, 64, 65] and blend of mineral oil and bio-lubricant [71] have been employed in the explorations.

### 2.1 Effect of TiO<sub>2</sub>-based nano lubricant on the performance of journal bearing

Around 40% of total investigations were performed on the TiO<sub>2</sub> based nano-lubricants. These explorations have been further classified into numerical [13, 14, 16, 17, 18, 19, 20, 24-26, 64-66, 74-76] and experimental studies [24, 25, 35, 67-69]. Due to a large number of numerical investigations, these are further divided based on the type of geometry into circular [13, 15-20, 24-26, 64-73] and non-circular journal bearings, namely, elliptical bearings [14, 74] and lobed bearing [75, 76] and type of numerical method employed to solve the governing equations into

finite difference method (FDM) [13, 16, 17, 26, 64-66] and Finite Element Method (FEM), Computational Fluid Dynamics (CFD) [19, 20, 24]. Fig. 4 illustrates the schematic illustrations of circular/ plain and non-circular bearings.

### Numerical investigation of performance of the circular journal bearing with TiO<sub>2</sub> based nano-lubricant by FDM

Researchers have employed FDM to discretise the Reynolds equation. The viscosity values of nano-lubricants in the investigations are either obtained numerically by employing different mathematical models as the Modified Krieger-Dougherty model (MKD), Krieger-Dougherty model (KD model) or measured experimentally. The viscosity values are evaluated at low volume fractions of 0.01% [64, 65, 75, 76] to high volume fractions of 0.025 [13, 16] and weight fractions of 0.5 wt.% - 2 wt.% [14, 19, 24-26, 74] of nanoparticles. Few researchers have considered non-

Table 1. Summary of numerical investigations on performance of circular bearings involving TiO<sub>2</sub> based nano-lubricants by FDM

Articles	Nanomaterial properties	Input parameters	Model	Key findings
Binu et al., [13]	$\Phi=0.005-0.025$ size <100 nm	$\varepsilon=0.2-0.8$	MKD model	Increased $\bar{W}$ by 40%, $\bar{F}$ by 40% at $\Phi = 0.01$
Binu et al., [16]	$\Phi=0.005-0.025$ size <100 nm	$\varepsilon=0.2-0.8$ $C_s=0.01-0.05$	MKD model, Couple stress model	$\bar{W}$ increased by 45%, $\bar{F}$ increased by 40% at $\Phi = 0.01$ and $C_s = 0.03108$ Negligible effect on $a$ and $\bar{Q}_z$ .
Shenoy et al., [17]	$\Phi = 1\%$	Adjustable bearing $\varepsilon = 0.1-1.2$	Viscosity values from [46]	At $\varepsilon = 0.4$ Increased $\bar{W}$ by 35%, $\bar{F}$ by 37%. Reduced $\bar{Q}_z$ by 23%.
Dang et al., [26]	$\omega_{np}=0.5\%-2\%$ Size= 40 nm.	$N=2000-5000$ rpm, $\varepsilon = 0.4-0.7$	MKD model/ Temperature dependent viscosity model	Increased $P_{max}$ up to 14.23% at $\omega_{np} = 2\%$ , $N=2000$ , $\varepsilon=0.6$ , $T$ by 0.67%, $W$ by 9.39% at $\varepsilon=0.5$ . $P_f$ increased by 9.8% to 27% at $N=2000$ rpm and $\omega_{np}=0.5\%-2\%$ .
Dhanola & Garg, [64]	$\Phi = 0.01\% - 0.04\%$	$N = 500-2500$ rpm. $\varepsilon = 0.1-0.8$ . $n = 0.9, 1.0, 1.1,$	Temperature dependent viscosity model, Power law model, K D model	At $\Phi = 0.04\%$ for adiabatic case, increased $\bar{P}_{max}$ by 6%, $\bar{W}$ by 8% at $n=0.9$ , $\bar{P}_{max}$ by 10%, $\bar{W}$ by 9% at $n=1$ and $\bar{P}_{max}$ by 7%, $\bar{W}$ by 8% at $n=1.1$ .
Dhanola & Garg, [65]	$\Phi = 0.01\% - 0.04\%$	$N= 500-2500$ rpm. $\varepsilon=0.1-0.8$ $n = 0.9, 1.0, 1.1.$	Temperature dependent viscosity model, Power law model, K D model,	at $\Phi = 0.04\%$ for THD case, increased $\bar{P}_{max}$ by 11%, $\bar{F}$ by 10% and $\bar{W}$ by 10% at $n=0.9$ , for TEHD case, increased $\bar{P}_{max}$ by 10%, $\bar{F}$ by 10% and $\bar{W}$ by 8% at $n=0.9$ ,
Khan et al., [66]	$\Phi = 0.02\% - 0.2\%$	$n= 0.9, 1, 1.1.$ $\varepsilon = 0.1-0.8.$ $N=1000$ rpm	K D model, power law model	At $\Phi = 0.2\%$ and $n=1.1$ , Increased $\bar{P}_{max}$ by 82%, $\bar{W}$ by 83%, $\bar{F}$ by 83%. $a$ constant for rigid bearings For flexible bearing, $\bar{P}_{max}$ by 93%, $\bar{W}$ by 82%, $\bar{F}$ by 89% $a$ reduced by 55%

**Table 2.** Summary of numerical investigations on performance of non-circular bearings involving TiO<sub>2</sub> based nano-lubricants by FDM

Articles	Characteristics of Nanomaterial	Input parameters	Model	Key findings
Dang et al., [14]	$\omega_{np}$ =0.5-2% Size= 40	Elliptical bearing $N=2000-5000$ rpm, $\varepsilon$ =0.4- 0.7	MKD model	at 2 wt.%, $N=5000$ , $\varepsilon =0.7$ , Increased $P_{max}$ by 13.86%, $W$ by 14.24% $T$ by 1.76%, $P_f$ by 12.7%
Suryawanshi & Pattiwar, [74]	$\omega_{np}$ =0.5% Size= 40 nm	Elliptical bearing $N=500-1000$ rpm, $\varepsilon$ =0.4- 0.7. $W_a =1000$ N.	MKD model	Increased $P_{max}$ by 1.18%, $W$ by 2.93% Reduced $f$ up to 78%, $\Delta T$ up to 74.6% $W$ elliptical bearing equal to 3 times $W$ plain bearing.
Kumar & Kakoty, [75]	$\Phi =0.001- 0.02$	Three lobed bearing, $C_s = 0.4$ , $L/D =1$	K D model, Couple stress model	At $\Phi =0.02$ , Increased $\bar{P}_{max}$ up to 114%, $\bar{W}$ by 15%. Stiffness and damping coefficient increased
Kumar & Kakoty, [76]	$\Phi=0.001- 0.02$	Two lobed bearing. $\varepsilon =0.05- 0.5$ , $C_s = 0.2-0.6$ , $L/D = 1$	K D model, Couple stress model	At $C_s = 0.2$ and $\Phi = 0.02$ , Increased $\bar{W}$ by 12%, $\bar{P}_{max}$ by 108% $a$ and dynamic coefficients remained constant

Newtonian behaviour of nano-lubricants by implementing the power law model [64–66]. Some researchers have also included thermal effects and elastic deformations in their analyses [64-66].

Around 45% increase in load-carrying capacity with the application of nano-lubricants was obtained employing MKD model and Reynolds equation [13, 16]. Researchers have evaluated the static and dynamic performance of the bearing, including the vibration characteristics of the journal bearing [72]. A summary of articles involving the numerical investigations of the effect of TiO<sub>2</sub>-based nano lubricant on the performance of circular journal bearing is given in Table 1.

From Table 1 it can be concluded that nano-lubricants improved the load-carrying capacity of the bearing. With the inclusion of 0.005 to 0.025 volume fraction of TiO<sub>2</sub> load carrying capacity increased by 35%-45%. An increase of 40% is reported in the friction force. The application of viscosity derived from MKD and KD models predicts an improvement in the performance parameters of the bearing. Similar improvements are seen when experimentally evaluated viscosity values of nano-lubricants are applied for numerical investigations.

#### Numerical investigation of performance of the non-circular journal bearing involving TiO<sub>2</sub> based nano-lubricant by FDM

Few researchers explored the effects of nano-lubricant on the performance of non-circular bearings which includes elliptical bearings [14, 74], two lobed [76] and three lobed bearing [75]. The non-circular bearings provide increased shaft stability, oil flow and less power loss than circular bearings [77]. Summary of these articles is given in Table 2.

From Table 2 it can be seen that nano-lubricants improved the load-carrying capacity of the non-circular journal bearings. Moreover, a larger increase in load-

carrying capacity of non-circular bearings is reported in comparison to circular bearings. Three lobed bearing showed higher increase in load-carrying capacity than two lobed bearing.

#### Numerical investigation of performance of the circular journal bearing involving TiO<sub>2</sub> based nano-lubricant by FEM and CFD

Few researchers employed FEM and CFD [78] simulation for numerical investigations. Cavitation effects have also been considered for getting realistic results [19, 20, 24]. Baskar et al. [25] developed Response Surface Methodology (RSM) based D-optimal design for designing bearing experiments to compare the effect of engine oil and chemically modified rapeseed oil (CMRO) based nano-lubricant and compared its effects with mineral oil and found a nominal decrease in maximum pressure with the addition of TiO<sub>2</sub> based bio-lubricant in comparison to mineral oil. RSM is mathematical and statistical technique which is employed to obtain desired output on the basis of selected input parameters. Table 3 presents the summary of the effects of nano-lubricant on the bearing investigated employing FEM/CFD.

From Table 3 it can be seen that with the inclusion of 0.5 -2 wt.% TiO<sub>2</sub>, load carrying capacity is increased by 5% to 23%.

#### Experimental studies of performance of the hydrodynamic journal bearing with TiO<sub>2</sub> based nano-lubricant

Very few research articles are available on experimental studies. These experiments are performed on journal-bearing test rigs [24, 25, 35, 67–69]. Nanoparticles are added in 0.5 wt.% [24, 25, 35] and in low volume fractions of 0.01% [69] to high volume fraction of 2% [68]. Tests are performed at wide speed ranges from 250 rpm [69] to 3000 rpm [24, 25, 69] and loads varying in between 10 N [24] to

**Table 3.** Summary of numerical investigations on performance of circular bearings involving TiO<sub>2</sub> based nano-lubricants by FEM/ CFD

Articles	Nanomaterial properties	Input parameters	Method	Model	Key findings
Yasir et al., [19]	$\omega_{np}=0.5-2.0\%$ Size= 30	$L/D= 0.5$ $\varepsilon= 0.1-0.9$ . $N= 1000-3000$ rpm.	CFD	Zwart–Gerber–Balamri Cavitation model, MKD model	Increased $W$ by 23%, $T$ by 3%, $P_{max}$ by 1.7% at $\omega_{np}=2.0\%$ $T_{max}$ decreased by 15.7% $P_{max}$ decreased by 19-21% when cavitation effects considered in comparison to $P_{max}$ without cavitation.
Abass et al., [20]	$\Phi = 0\% - 3\%$	$L/D= 0.5$ $\varepsilon= 0.1-0.9$ . $N= 2000-4000$ rpm.	CFD	Temperature dependent viscosity model, MKD model, Zwart–Gerber–Balamri cavitation models	Increased $W$ by 14.3 %, $P_{max}$ by 19.8%, $F$ by 22.5%, $f$ by 7%. $P_{max}$ decreased by 32% to 40% at $\Phi = 1\%-3\%$ when cavitation effects considered.
Ramaganesh et al., [24]	$\omega_{np}=0.5\%$ Size= 30-50 nm	$W_a = 10$ kN. $L/D = 0.5$ , $N = 3000$ rpm	FEM/CFD	Viscosity values from [6]	$P_{max}$ decreased by 2.56%.
Baskar et al., [25]	$\omega_{np}=0.5\%$ Size= 30-50 nm	$W_a=2-10$ kN. $L/D=0.5$ , $N=1000-3000$ rpm	RSM	Experimental viscosity values	$P_{max}$ decreased by 2.98%.

**Table 4:** Summary of experimental studies involving TiO<sub>2</sub> based nano-lubricant

Articles	Properties of Nanomaterial	Input parameters	Apparatus	Key findings
Ramaganesh et al., [24]	$\omega_{np}=0.5\%$ Size= 30-50 nm	$W_a = 10N- 10$ kN, $L/D = 0.5$ , $N = 3000$ rpm	Journal bearing test rig	$P_{max}$ decreased by 2.26%.
Baskar et al., [25]	$\omega_{np}=0.5\%$ Size= 30-50 nm	$W_a = 2- 10$ kN. $L/D = 0.5$ , $N=1000-3000$ rpm	Journal bearing test rig	$P_{max}$ decreased by 4%.
Suryawanshi et al., [35]	$\omega_{np}=0.5\%$ Size=40 nm	$W_a = 1000$ N $L/D = 1$ , $N = 500, 750, 1000$ rpm	Journal bearing test rig	$P/P_{max}$ increased in a range of 87% to 364%. $\Delta T$ decrease in range of 6-73% in comparison to plain bearing
Singh et al., [67]	$\Phi=0.075-0.15\%$	$W_a = 600- 1000$ N. $N=500- 1500$ rpm	Journal bearing test rig	Reduced $f$ up to 73 %, $T_{max}$ up to 5.9 % at $W_a = 600$ N. $\varepsilon$ and $a$ initially increased then decreased.
Gundarneeya & Vakharia, [68]	$\Phi =0.25-2\%$ Size=20 nm	$W_a = 300-450$ N. $N=250-500$ rpm	Journal bearing test rig	Increased $P_{max}$ up to 3% to 21%.
Dhanola & Garg, [69]	$\Phi = 0.01\%- 0.04\%$ Size= 10-25 nm	$W_a = 500, 1500, 3000$ N. $N = 600-3000$ rpm	Journal bearing test rig	At $W_a =1500$ N Increased $p$ by 19%. Reduced $T$ by 15%. Newtonian fluid

10 kN [24, 25]. The lubricants applied are mineral oil based SAE30 [67], DTE 24, DTE 25, DTE 26 [35], Avalon ISO Viscosity grade 46 oil [68], bio-lubricant canola oil [69] and chemically modified rapeseed oil (CMRO) [24, 25]. The size of TiO<sub>2</sub> particles lie between 10-50 nm. The outcome of experimental studies is summarised in Table 4.

From Table 4, it can be observed that by employing nano-lubricant, temperature reduces by 5-15%, coefficient of friction decreases up to 73%. With CMRO-based nano-lubricant maximum pressure of journal bearing reduced by 2% to 4% in comparison to mineral oil.

## 2.2 Effect of CuO-based nano lubricant on the performance of journal bearing

Some studies are performed with CuO based nano-lubricants. Numerical [14, 15, 17, 18, 20, 24–26, 70, 79, 80] and experimental investigations [24, 25, 68, 71] have been carried out by researchers in order to improve the performance behaviour of journal bearing.

### Numerical investigation of performance of the journal bearing with CuO-based nano-lubricant

Numerical investigations are done by FDM [14, 17, 26, 70, 80], FEM [15, 18, 24, 79], CFD [20] and RSM [25]. Nanoparticles are added in weight fraction of 0.1-2.0 wt.% [14, 15, 17, 18, 24–26, 70, 79] and volume fraction of 1-4%

**Table 5.** Summary of numerical investigations involving CuO based nano-lubricants

Articles	Nanomaterial properties	Input parameters	Method	Model	Key findings
Dang et al., [14]	$\omega_{np}=0.5\%-2\%$ Size = 40 nm	Pure Elliptical bearing $N=2000-5000\text{rpm}$ , $\varepsilon=0.4-0.7$	FDM	MKD model	at 2 wt.%, $N=5000$ , $\varepsilon=0.7$ , Increased $P_{max}$ by 8.9%, $W$ by 9.2%, $T$ by 15.3%, $P_f$ by 8.2%.
Kalakada et al., [15]	$\omega_{np}=0.1-0.5\%$	$L/D = 1$ $\varepsilon = 0.1-0.9$	FEM	Non-dimensional regression viscosity model	At 0.5wt%, Increased $\bar{W}$ by 14.45%. $\bar{F}$ by 8.6%. $\bar{Q}$ by 6.97%. $\bar{\zeta}$ decreased by 3.47%.
Shenoy et al., [17]	$\Phi = 1\%$	Adjustable fluid film bearing $\varepsilon = 0.1-1.2$	FDM	Viscosity values from [40]	$\varepsilon = 0.4$ , Increased $\bar{W}$ by 17% $\bar{F}$ by 19.4%
Nair et al., [18]	$\omega_{np}=0.1-0.5\%$	$\varepsilon= 0.1-0.9$ $L/D = 1$	FEM	Non-dimensional regression viscosity model	$\bar{W}$ increased by 14.45%. $\bar{F}$ increased by 7%.
Abass et al., [20]	$\Phi = 0\% - 3\%$	$L/D= 0.5$ $\varepsilon= 0.1-0.9$ . $N= 2000- 4000$ rpm.	CFD	Temperature dependent viscosity model, MKD model, Zwart-Gerber-Balamri cavitation model	Increased $W$ up to 14%, $P_{max}$ by 18.6%, $F$ by 19.9%. Increase in $f$ is insignificant. $P_{max}$ decreased by 32-40% at $\Phi = 1-3\%$ when cavitation effect considered.
Ramaganesh et al., [24]	$\omega_{np}=0.5\%$ Size=40-80 nm	$W_n = 10$ kN. $L/D = 0.5$ , $N = 3000$ rpm	FEM/CFD	Viscosity values from [6]	$P_{max}$ reduced by 9.9%.
Baskar et al., [25]	$\omega_{np}=0.5\%$ Size=40-70nm	$W_n = 2- 10$ kN $N=1000-3000\text{rpm}$	RSM	Experimental viscosity values	$P_{max}$ reduced by 13.65%.
Dang et al., [26]	$\omega_{np}=0.5-2\%$ Size=40 nm	$N=2000- 5000$ rpm, $\varepsilon = 0.4-0.7$	FDM	MKD model/ Temperature dependent viscosity model	at 2 wt.%, $N=2000$ , $\varepsilon=0.6$ , Increased $P_{max}$ up to 9.23%. $T$ up to 1.04%. $W$ by 14.49% at $\varepsilon=0.5$ , $P_f$ increased by 7% to 21% at $\omega_{np}=0.5-2\%$ .
Awati and Kengangutti, [70]	$\omega_{np}=0.0-0.5\%$	$T= 30-90^\circ\text{C}$	FDM	Temperature dependent viscosity model	$\bar{W}$ increased by 4.3% at 0.25 wt.%. $\bar{F}$ increased by 4%.
Byotra and Sharma, [79]	$\omega_{np}=0.1-0.5\%$	circular bearing, $\varepsilon = 0.2-0.8$ , $L/D= 1$ Arc shaped texture depth= 0.8 $N = 3000$ rpm	FEM	Non dimensional regression viscosity model	$W$ increased by 68% in comparison to plain bearing. $W$ increased 24% in comparison to textured bearing. $f$ decreased by 5.4% at higher $\omega_{np}$
Nicoletti, [80]	$\Phi = 4\%$	$L/D = 0.5$ , $\varepsilon = 0.1-0.8$	FDM	Temperature dependent viscosity model, Einstein viscosity model	$W$ increased by 11% at $\varepsilon = 0.4$ .

[20, 80]. Nicoletti [80] compared effect of CuO based nano-lubricant with other nano-lubricants and found that CuO based lubricant has highest volumetric heat capacity and can improve the load carrying capacity of bearing. It was concluded that the higher volumetric heat capacity has a significant impact in improving the performance of the bearing along with viscosity of nano-lubricants. The comparative studies of the effect of chemically modified

rapeseed oil based (CMRO) nano-lubricant employing nanoparticles of CuO with SAE20W40 found that CMRO based nano-lubricant offers an alternative to SAE20W40 [24, 25]. The outcome of numerical studies is summarised in Table 5.

Table 5 shows that load carrying capacity improved by 5% to 18%, maximum pressure enhanced by 6% to 19% in most of the articles and a reduction of 10% to 14% is seen

**Table 6:** Summary of experimental studies involving CuO based nano-lubricant

Articles	Properties of Nanomaterial	Operating Conditions	Apparatus	Key Findings
Ramaganesh et al., [24]	$\omega_{np}$ =0.5% Size=40-80 nm	$W_a$ = 10 kN, $L/D$ = 0.5, $N$ = 3000 rpm	Journal bearing test rig	$P_{max}$ reduced by 9.45%.
Baskar et al., [25]	$\omega_{np}$ =0.5% Size=40- 70 nm,	$W_a$ = 2- 10 kN. $N$ =1000- 3000 rpm	Journal bearing test rig	$P_{max}$ reduced by 10%.
Gundarneeeya & Vakharia, [68]	$\Phi$ = 0.25% - 2% Size= 50 nm	$W_a$ = 300-450 N. $N$ = 250-500 rpm	Journal bearing test rig	Increased $P_{max}$ up to 19%
Katpatal et al., [71]	$\omega_{np}$ =1% - 2.5% Size=30-50 nm	$W_a$ = 400, 600, 800 N $N$ = 600, 1600 rpm	Journal bearing test rig	increased $P_{max}$ by 4.5-21%, $f$ by 2-8%, Thermal conductivity by 5%

only with bio-based nano-lubricants when compared with effect of mineral oil. Investigations reported an increase in friction force of approximately 20%. Due to increased friction force power loss also increased. Performance of textured bearing lubricated by nano-lubricants is better than textured bearing lubricated by base oil and smooth bearing.

#### Experimental studies of performance of the hydrodynamic journal bearing with CuO based nano-lubricant

Investigations are performed on journal bearing test rigs [24, 25, 68, 71]. The size of nano particles lies between 30 nm to 80 nm [24, 25, 68, 71]. For experimental investigation Katpatal et al. [71] selected a blend of bio-lubricant Jatropa oil and mineral oil VG46 in different ratios and added CuO nanoparticles. Baskar [25] and Ramaganesh [24] selected bio lubricant CMRO. As bio-lubricants have lower viscosity than mineral oils, addition of nanoparticles increase the viscosity [24,71]. The results of experimental studies are summarised in Table 6.

From Table 6, it is seen that experimental studies reported variation in maximum pressure of bearing with copper-based nano-lubricants in the range of 9-20%. These studies suggested that bio-based nano-lubricants can be a good alternative to mineral oil.

#### 2.3 Performance of the journal bearing with Al<sub>2</sub>O<sub>3</sub>-based nano-lubricant

Articles involving numerical [15, 18, 20, 70, 73, 79–81] and experimental studies [68, 82] are available on the application of Al<sub>2</sub>O<sub>3</sub>-based nano-lubricants.

#### Numerical investigation of performance of the journal bearing with Al<sub>2</sub>O<sub>3</sub>-based nano-lubricant

Numerical investigations are performed by the researchers with the help of FDM [70, 80], FEM [15, 18, 79] FVM [81] and CFD [20, 73]. Nanoparticles are added in 0.1-0.5 wt.% [18, 70, 79, 81] and volume fraction of 1 to 5% [20, 80]. Few studies considered cavitation effect [20, 81], surface roughness [70] and textures on bearing surface [79]. Byotra et. al [79] applied Al<sub>2</sub>O<sub>3</sub> based nano-lubricant on

textured journal bearing and reported that presence of textures with nano-lubricant improves the bearing performance by 64% than the textured bearing without nano-lubricant. A summary of these research articles is enlisted in Table 7.

It is evident from Table 7 that load carrying capacity improved by approximately 10% to 18%, maximum pressure increased by about 18%. However, inclusion of cavitation effects reduced maximum pressure. The force of friction increased in the range of 5% to 18%.

#### Experimental investigation of performance of the journal bearing with Al<sub>2</sub>O<sub>3</sub>-based nano-lubricant

Experimental studies are done on journal bearing test rigs [68, 82]. Size of nanoparticles of aluminium oxide ranged between 50 nm [68] to 80 nm [82]. The journal bearing test rig operated on a load ranging between 300 N to 800 N [68, 82]. The outcomes of these investigations are enlisted in Table 8. From Table 8 it is seen that Bui et. al [82] found a reduction in pressure of about 40% to 50% while Gundarneeeya & Vakharia, [68] reported an increase of 13%.

#### 2.4. Performance of the journal bearing with WS<sub>2</sub>-based nano-lubricant

Researchers investigated effect of WS<sub>2</sub> based nano-lubricant using both numerical and experimental method [24, 25]. For numerical investigations RSM [25] and FEM/CFD [24] is applied. Experimental investigations are performed on journal bearing test rigs. Size of nano tungsten disulphide ranged between 40-80 nm [24, 25] added in 0.5 wt.% [24, 25]. The outcomes of these research are enlisted in Table 9 and 10.

From Table 9, it can be concluded that in numerical investigation maximum pressure decreased by 3.94% [25] to 5.12% [24] when compared with pressure obtained by employing SAE20W40. From Table 10, it can be seen that in experimental studies a decrease of 5% [25] and 4.5% [24] in maximum pressure is reported.

**Table 7.** Summary of numerical investigations involving Al<sub>2</sub>O<sub>3</sub> based nano-lubricants

Articles	Properties of Nanomaterial	Input parameters	Method	Model	Key findings
Kalakada et al., [15]	$\omega_{np}=0.1-0.5\%$	$L/D = 1$ . $\varepsilon = 0.1-0.9$ .	FEM	Non dimensional regression viscosity model	Increased $\bar{W}$ by 12.5%, $\bar{F}$ by 7.6%, $\bar{\alpha}$ by 5.8%, $\bar{\zeta}$ decreased by 3%.
Nair et al., [18]	$\omega_{np}=0.1-0.5\%$	$\varepsilon = 0.1-0.9$ . $L/D = 1$ .	FEM	Non dimensional regression viscosity model	$\bar{W}$ increased by 12.5%, $\bar{F}$ increased by 5.7%
Abass et al., [20]	$\Phi = 1\% - 3\%$	$L/D= 0.5$ $\varepsilon = 0.1-0.9$ . $N = 2000-4000$ rpm.	CFD	Temperature dependent viscosity model, MKD model, Zwart-Gerber-Balamri cavitation models	Increased $W$ by 13.8%, $P_{max}$ by 17.5%, $F$ by 18.4%, $f$ by 7%. $P_{max}$ decreased by 32% and 40% at $\Phi = 1\%$ and 3% respectively when cavitation effect considered.
Awati and Kengangutti, [70]	$\omega_{np}=0-0.5\%$	$\varepsilon = 0.1-0.9$ . $T = 30-90^\circ\text{C}$	FDM	Temperature dependent viscosity model,	$\bar{W}$ increased by 6.5% $\bar{F}$ increased by 5%.
Kadhim et al., [73]	$\Phi = 0- 5\%$	$\varepsilon = 0.1-0.6$ . $L/D = 0.7$ $N = 3000$ rpm.	CFD	MKD model, Thermal effect	$P_{max}$ increased by 17.9%. $W$ increased from 1.2% to 14% at $\Phi = 0.5- 5\%$ .
Byotra and Sharma, [79]	$\omega_{np}=0.1-0.5\%$	Arc shaped textures, $\varepsilon = 0.2$ , texture depth = 0.1. $N = 3000$ rpm	FEM	Temperature dependent Viscosity model	With nano-lubricant $W$ increased by 64% in comparison to plain bearing. By 21% in comparison to textured bearing. $f$ decreased by 3% at higher $\omega_{np}$ .
Nicoletti, [80]	$\Phi = 4\%$	$L/D = 0.5$ , $\varepsilon = 0.1-0.8$ .	FDM	Temperature dependent viscosity model, Einstein viscosity model	$W$ increased by 11% at $\varepsilon = 0.4$ .
Solghar, [81]	$\Phi = 5\%$	$W_a = 6$ kN. $\varepsilon = 0.1-0.9$ $N = 4000$ rpm	FVM	Non dimensional regression viscosity model,	Increased $W$ by 17.7%. $P_{max}$ by 15.7%. $Q$ decreased by 7.5% $T_{max}$ decreased by 7.2%.

**Table 8:** Summary of experimental studies involving Al<sub>2</sub>O<sub>3</sub> based nano-lubricant

Articles	Characteristics of Nano-material	Operating conditions	Apparatus	Key Findings
Gundarneeeya & Vakharia, [68]	50 nm, spherical $\Phi = 0.25\% - 2\%$	$W_a = 300-450$ N. $N = 250-500$ rpm	Journal bearing test rig	$P_{max}$ increased up to 13%.
Bui et al., [82]	80 nm $\Phi = 0.00117$	$W_a = 300, 500$ and 800 N. $\omega = 94, 141$ and 214 rad/s	Journal bearing test rig	$P_{max}$ reduced by 40% to 50%. Pressure near boundary increased by 30%.

**Table 9.** Summary of numerical investigations involving WS<sub>2</sub> based nano-lubricants

Articles	Characteristics of Nano-material	Operating conditions	Models used	Key Findings
Ramaganesh et al., [24]	40-80 nm $\omega_{np}=0.5\%$	$W_a = 10$ kN. $L/D = 0.5$ , $N = 3000$ rpm	FEM/ CFD Viscosity values from [6]	$P_{max}$ decreased by 5.12%.
Baskar et al., [25]	40-80 nm $\omega_{np}=0.5\%$	$W_a = 2- 10$ kN. $N = 1000- 3000$ rpm	RSM, Experimental viscosity values	$P_{max}$ decreased by 3.94%.

## 2.5. Performance of the journal bearing with CeO<sub>2</sub>-based nano-lubricant

A few numerical studies on effect of CeO<sub>2</sub> based nano-lubricants were found [15, 18, 70]. These are performed by

employing FDM [70] and FEM [15, 18]. 0.1-0.5 wt.% nano cerium oxide [15, 18, 70] is applied in these studies. The summary of research articles is given in Table 11. From Table 11 it can be concluded that load carrying capacity

**Table 10:** Summary of experimental studies involving WS<sub>2</sub> based nano-lubricant

Articles	Characteristics of Nano-material	Operating conditions	Apparatus	Key Findings
Ramaganesh et al., [24]	40-80 nm $\omega_{np}=0.5\%$	$W_a = 10$ kN. $L/D = 0.5$ , $N = 3000$ rpm	Journal bearing test rig	$P_{max}$ decreased by 4.5%.
Baskar et al., [25]	40-80 nm $\omega_{np}=0.5\%$	$W_a = 2-10$ kN. $N=1000-3000$ rpm	Journal bearing test rig	$P_{max}$ decreased by 5%.

**Table 11.** Summary of numerical investigations involving CeO<sub>2</sub> based nano-lubricants

Articles	Properties of Nanomaterial	Input parameters	Method	Model	Key findings
Kalakada et al., [15]	$\omega_{np}=0.1-0.5\%$	$L/D = 1$ . $\varepsilon = 0.1-0.9$ .	FEM	Non dimensional regression viscosity model	Increased $\bar{W}$ by 13.98%, $\bar{F}$ by 7.8%, $\bar{Q}$ by 5.86%, $\bar{\zeta}$ decreased by 3.26%.
Nair et al., [18]	$\omega_{np}=0.1-0.5\%$	$\varepsilon=0.1-0.9$ . $L/D = 1$ .	FEM	Non dimensional regression viscosity model	$\bar{W}$ increased by 13.98% $\bar{F}$ increased by 3% at 0.5 wt. %.
Awati and Kengangutti,[70]	$\omega_{np}=0-0.5\%$	$T= 30-90^\circ\text{C}$	FDM	Temperature dependent viscosity model	$\bar{F}$ increased by 4.5%.

**Table 12.** Summary of numerical investigations involving other nano-lubricants

Articles	Properties of Nanomaterial	Input parameters	Method	Model	Key findings
Shenoy et al., [17]	Nano-diamond $\Phi = 1\%$	Adjustable fluid film bearing $\varepsilon = 0.1-1.2$	FDM	Viscosity values from [40]	$\varepsilon = 0.4$ , Increased $\bar{W}$ by 14%. $\bar{F}$ by 15%.
Yasir et al., [19]	ZnO Size = 30 nm $\omega_{np}=0.5-2.0\%$	$L/D=0.5$ $\varepsilon=0.1-0.9$ . $N= 1000-3000$ rpm	CFD	Zwart-Gerber-Balamri Cavitation models, MKD model	increased $W$ by 6.5-21%, $P_{max}$ by 0.5-2% at $\omega_{np}=0.5-2.0\%$ . $P_f$ increased by 0.4% to 1.3% at 2000 rpm. $P_{max}$ decreased by 19% to 21% than $P_{max}$ without cavitation. $T_{max}$ decreased by 15.7% at 2 wt. %.
Nicoletti, [80]	SiO <sub>2</sub> / Si/ Al/ Cu $\Phi = 4\%$	$L/D = 0.5$ , $\varepsilon = 0.1-0.8$ .	FDM	Temperature dependent viscosity model, Einstein viscosity model	$W$ increased by 10.3% with Cu 9.4% with Al. 8.6% with Si and SiO <sub>2</sub> at $\varepsilon = 0.4$

and force of friction increased [15, 18, 70]. The threshold speed increased and damped frequency decreased [18].

### 2.6 Performance of the journal bearing with other nano-lubricant

Few studies are done with some uncommon nanoparticles namely silicon dioxide, aluminum, copper [80], nano-diamond [17] and zinc oxide [19]. For these investigations FDM [17, 80] and CFD [19] have been employed for solution of governing equations. The summary of these investigation is given in Table 12.

From Table 12, it is evident that load carrying capacity of journal bearing improved with the application of different types of nano-lubricants.

### 3. Discussion

On the basis of the review of the numerical and experimental studies carried out by researchers in the field of applications of nano-lubricants in journal bearings, it is

revealed that the performance of journal bearing is improved by employing nano-lubricants. Due to environment concerns, some researchers have used bio-lubricants as base lubricant in their investigations. A summary of base oils used by researchers in their investigations is shown in Fig. 5 and the distribution of research articles based on base oil is shown in Fig.6. From Fig. 6 it is noticed that majority of research articles have taken mineral oil-based lubricants, however bio-based lubricants such as canola oil, CMRO, etc. have also been used. Single grade oil such as SAE30, ISO VG 46, ISO VG68, anti-wear hydraulic oils such as AW32, AW 100, DTE24, 25, 26 and multi grade oils namely SAE15W40, SAE10W50, SAE20W40 have been employed in different explorations. The distribution of research articles based on nanoparticles is shown in Fig. 7. It is noticed that around 38% investigations have been carried out by taking TiO<sub>2</sub> based nano-lubricants, 26% studies were conducted on CuO based nano-lubricants, 20% studies were performed

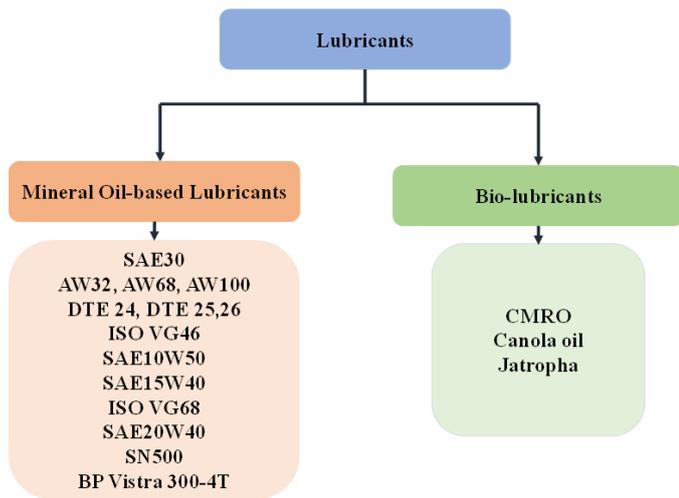


Fig 5. Lubricants applied for nano-lubrication of journal bearing

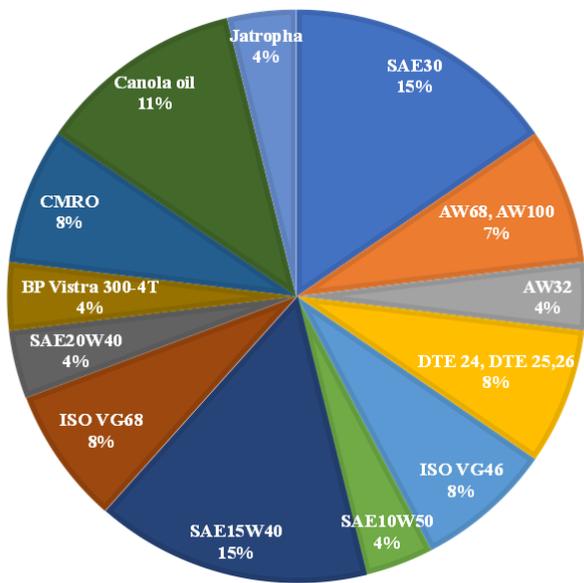


Fig.6. Categories of research articles based on base oil

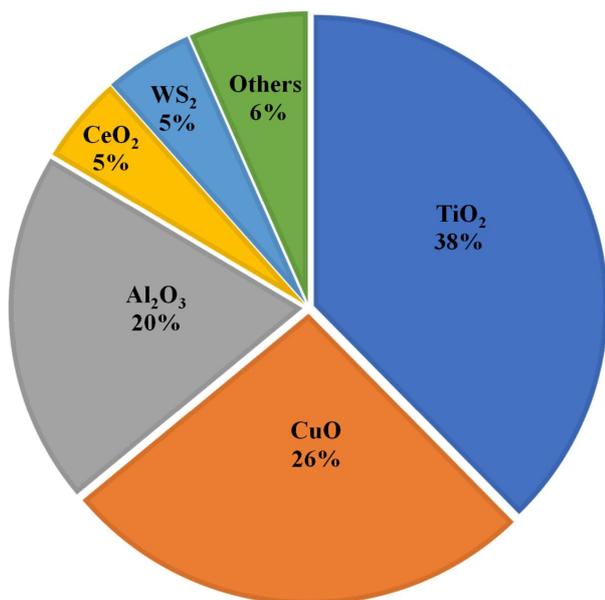


Fig.7. Categories of research articles based on nano-lubricants

on Al<sub>2</sub>O<sub>3</sub> based nano-lubricants and around 16% studies were conducted on other types of nanoparticles-based nano-lubricants.

On the basis of findings obtained from the research articles, a comparison of performance characteristics of journal bearing with respect to different nano-lubricants is done. Fig. 8 shows the effect of addition of TiO<sub>2</sub>, CuO, Al<sub>2</sub>O<sub>3</sub> and CeO<sub>2</sub> nanoparticles on percentage increase of load carrying capacity. It is seen that with the presence of nanoparticles of TiO<sub>2</sub> in the base lubricant results in maximum increase of around 45% in the load carrying capacity [13, 16, 17]. However, CuO and Al<sub>2</sub>O<sub>3</sub> based nano-lubricants improves load carrying capacity by 17% [15, 17, 18, 20, 26] and 15% [15, 18, 20, 73, 81] respectively. In Fig. 9, comparison of effects of nano-additives on percentage increase of maximum pressure is illustrated. Addition of TiO<sub>2</sub> nanoparticles enhances the maximum pressure by 20% [14, 20, 26] whereas CuO and Al<sub>2</sub>O<sub>3</sub> improves maximum pressure by 10% [14, 26] and 18% [15, 73, 81] respectively.

Effects of different nano-additives on friction force of the journal bearing is shown in Fig. 10. The presence of nano-additives increases the friction force in comparison to lubricant without nano-additives which increases the power loss. To limit the viscosity, researchers applied the nano-additives in 0.5 wt.% and in volume fraction from 1% to 5%, as higher presence of nano-additives aggravates the friction losses. The comparative studies performed by researchers [14, 17, 19, 20, 68] also concluded that TiO<sub>2</sub> based nano-lubricant give best results among all types of nano-lubricants.

The lubricating oil's properties which significantly change with nano-additives are viscosity, density, thermal conductivity and specific heat capacity. Important correlations used to evaluate these properties are listed in Table 13. Many mathematical correlations have been developed by the researchers. These models can be successfully applied for the investigations. Table 14 enlists models applied in the research related to application of nano-lubricant in journal bearing. Among all, MKD model [83] is widely applied which is an extension of Krieger Dougherty model [84]. Researchers have developed

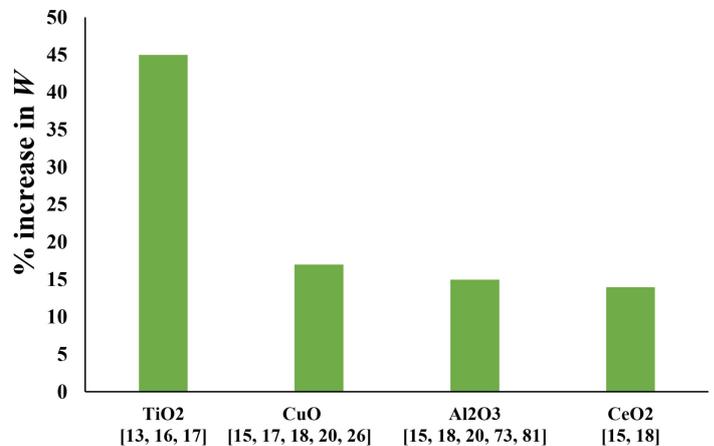


Fig. 8. Comparison of % increase of W

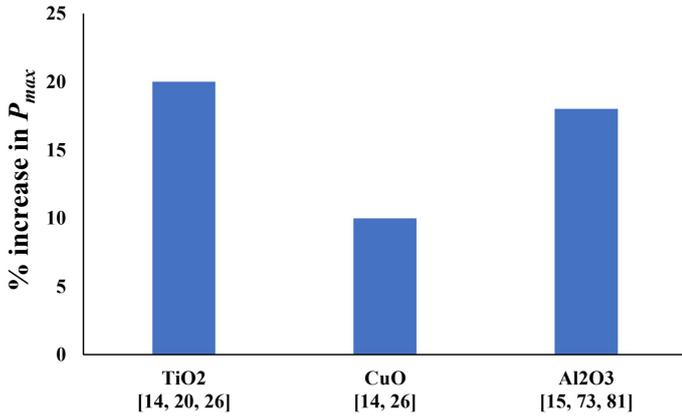


Fig. 9. Comparison of % increase of maximum pressure

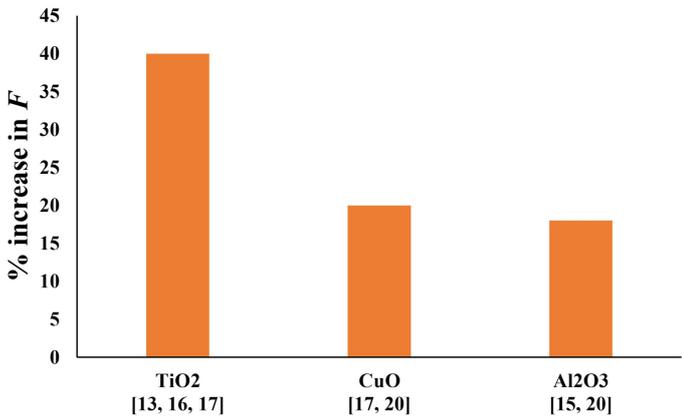


Fig. 10. Comparison of % increase of Friction force

models which consider both temperature variations and nanoparticle concentration to evaluate the viscosity of nano

Table 13: Important correlations

S. No.	Authors	Parameter	Correlation
1	Azmi et al. [85]	Volume fraction of nanoparticles $\phi$	$\frac{W_{np}}{\rho_{np}}$ $\frac{W_{np} + W_{bl}}{\rho_{np} + \rho_{bl}}$
2	Maxwell [86]	Thermal -conductivity	$\frac{K_{nl}}{K_{bl}} = 1 + \frac{3(\psi-1)\phi_{np}}{(\psi+2) - (\psi-1)\phi_{np}}$
3	Buongiorno [87]	Density of nano-lubricant $\rho_{nl}$	$(1 - \phi) \rho_{bl} + \phi \rho_{np}$
4	Buongiorno [87]	Specific heat of nano-lubricant	$(1 - \phi) C_{pbl}\rho_{bl} + \phi C_{pnp}\rho_{np}$ $\rho_{nl}$

Table 14: Mathematical models

Model	Corelation	Significance	Articles
Einstein’s viscosity model	$\bar{\mu} = \frac{\mu_{nl}}{\mu_{bl}} = 1 + 2.5 \phi$	For low particle fractions $\phi \leq 0.01$	[80]
Krieger- Dougherty model	$\mu_{nf} = \mu_0 [1 - \frac{\phi}{\phi_m}]^{-[\eta]\phi_m}$	Considers particle packing fractions of nanoparticles	[26, 65, 75, 76]
Modified Krieger- Dougherty model	$\mu_{nf} = \mu_{bl} [1 - \frac{\phi_a}{\phi_m}]^{-2.5\phi_m}$	Considers aggregate to primary particle size ratio of nanoparticles.	[13, 14, 16, 17, 20, 26, 74]
Temperature viscosity model	$\phi_a = \phi (\frac{a_a}{a})^{3-F}$ $\mu = \mu_i e^{-\gamma(T_p - T_i)}$	Temperature variation considered.	[14, 19, 20, 26]
Viscosity regression model	$\mu = \mu_{bl} e^{(K_1 - K_2 * T)}$	Temperature variation and nanoparticle concentration considered	[15, 18, 70, 79]
Power law model	$\bar{\mu} = \bar{m} * \bar{\gamma}^{n-1}$	Considering non-Newtonian behaviour	[64–66]

-lubricants [18]. The nano-additives change the rheology of the lubricant and it is reported that lubricant behaves as non-Newtonian fluid which is incorporated by power law model [64–66]. Presence of nanoparticles in the lubricant give resistance to the flow so couple stress parameter is included in the Reynolds equation to get more realistic results [16, 75, 76].

4. Conclusion

The studies on the effects of nano-lubricants on the performance of hydrodynamic journal bearing reveal that presence of nano-additives in lubricants improves the bearing performance. In comparison to base lubricants, nano-lubricants reduce the coefficient of friction in between the contacting surfaces. The application of nano-lubricants increases the load carrying capacity of bearings. The nano-lubricants have higher heat capacity.

Among all types of nanomaterials used, TiO<sub>2</sub> is found to be most effective nanomaterial because in most cases it gives better results in comparison to others. The other nanomaterials with significant impact are CuO and Al<sub>2</sub>O<sub>3</sub>. Nano based bio lubricants works as better alternatives to mineral oil-based lubricants in improving bearing performance. Nano-lubricants effectively improve the performance of circular and non-circular bearings. However, an increase in power loss was reported in research articles, which happens due to an increase in friction force which limits the usage of nanoparticles in low volume fractions only.

Future Scope:

Experimental explorations to evaluate the effect of size of nano particles on the performance behaviours of journal

bearing can be done. Investigations of the addition of nano-lubricants on textured journal bearing need to be done. Moreover, analysis of hybrid nanomaterials in base oil can be performed.

## Notations

$c$  = radial clearance, m

$C_{pbl}, C_{pnp}$  = specific heat of base lubricant and nanoparticles

$C_s$  = couple stress factor

$e$  = eccentricity, m

$f$  = coefficient of friction

$F$  = force of friction

$K_{nl}, K_{bl}, K_{np}$  = thermal conductivity of nano-lubricant, base-lubricant and nanoparticles

$L$  = bearing length, m

$\bar{m} = [1 - \frac{\phi_a}{\phi_m}]^{-2.5\phi_m} \times (\frac{U}{c})^{n-1}$

$N$  = speed of shaft, rpm

$n$  = power law index

$p$  = lubricant film pressure, N/m<sup>2</sup>.

$P_{max}$  = maximum pressure,

$\bar{P}_{max}$  = non-dimensional maximum pressure

$P_f$  = Power loss, kW

$Q$  = oil flow rate, m<sup>3</sup>/s

$Q_s$  = side leakage, m<sup>3</sup>/s

$Q_z$  = end leakage, m<sup>3</sup>/s

$R$  = journal radius, m

$R_s$  = surface roughness parameter

$T$  = operating temperature, °C

$T_{max}$  = Maximum oil film temperature, °C

$\Delta T$  = temperature rise, °C

$U$  = velocity of the shaft, m/s

$W$  = load carrying capacity, N

$W_a$  = External Load, N

$\bar{W}$  = non-dimensional load.

$W_{bl}$  = weight of lubricant without nano-additive

$W_{np}$  = weight of nanoparticles mixed in base lubricant

$U$  = tangential velocity of the journal, m/sec

$\nu$  = kinematic viscosity

$\alpha$  = attitude angle, degree

$\omega$  = angular velocity of the shaft rad/s

$\varepsilon$  = eccentricity ratio =  $e/c$

$\bar{\mu}$  = Non-dimensional relative viscosity

$\mu_r$  = ratio of nanolubricant viscosity to lubricant without nanoadditive,

$\mu_{nl}$  = viscosity of nanolubricant

$\mu_{bl}$  = viscosity of lubricant

$\mu_i$  = viscosity of lubricant at inlet

$\phi_{mp}$  = maximum particle packing fraction

$\phi$  = volume fraction of nanoparticles

$\phi^*$  = maximum particle packing fraction

$\eta$  = intrinsic viscosity = 2.5

$\omega_{np}$  = weight fraction of nanoparticles

$\rho_{np}$  = Density of nanoparticle

$\rho_{bl}$  = density of lubricant without nano additive

$\Omega$  = Threshold Speed

$\bar{\Omega}$  = non-dimensional threshold speed

$\gamma$  = temperature viscosity coefficient

$\dot{\gamma}$  = shear strain rate, s<sup>-1</sup>

$\Psi$  =  $K_{np}/K_{bl}$

$\zeta$  = damped frequency

$\bar{\zeta}$  = non-dimensional damped frequency

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