SIMULATION OF PISTON RING – CYLINDER LINER LUBRICATION CONSIDERING LAYERED FLUID FILMS

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ABSTRACT

During the operation of hydrodynamically lubricated devices a fully formulated lubricant has the ability to form layers at the surfaces. A friction modifier's task is to adjust the interaction between lubricant and the surface so that friction is lowered. An antiwear additive creates a protective layer on the surface and this definitely influence the performance of the lubricated device. To gain fundamental understanding, models that address the modified liquid – solid interaction due to the formation of layers, but also models that may be used to study the effects of layers already formed on the contacting surfaces are required. In this paper, two non-Newtonian lubricant rheology models that may be used to simulate reacted layers resembling those created by lubricant additives are adopted for the simulation of the piston ring – cylinder liner lubrication problem. The possibility of layer to layer interaction, which is likely to occur in the convex conjunction between the ring and the liner, is considered and this extends the models found in the literature. The effects induced by this type of layering are studied by using a modified Reynolds' equation where the coefficients have been corrected with factors that accounts for the layer properties. This enables, effectively, studies of layers resembling those created by lubricant additives during the operation of the lubricated conjunction between a piston ring and a cylinder liner.

Keywords and phrases: Homogenization, Time dependent, Reynolds equation, surface roughness, surface topography.

1 INTRODUCTION

During the operation of hydrodynamically lubricated devices a fully formulated lubricant has the ability to form layers at the surfaces. A lubricated interface with interacting surfaces exhibiting a film resulting from a friction modifier reacted to the surface may show a significant reduction in friction/traction force. An antiwear additive creates a protective layer on the surface and this definitely influence the performance of the lubricated device, in terms of hydrodynamic film formation and pressure build up. To gain fundamental understanding, models that address the formation of layers but also models that may be used to study the effects of existing layers are required.

Chan and Horn [1] used the surface force apparatus developed by Israelachvili [2] – with which they studied the drainage of thin liquid films between solid surfaces – to test the Reynolds theory right down to nanometre thick films. By considering simple, non-polar, Newtonian liquids they showed that the Newtonian Reynolds theory was very accurate down to film thickness of about 50 nm but then predicts a slightly faster drainage. By assuming an immobile layer about two molecular layers thick they could account for this slight but distinct increase in viscosity empirically. Georges et al. [3] probed into the same type of problem by assessing the influence of surface roughness, liquid formulation and elastic deflection on the layer thickness. The scope of the abovementioned work is perhaps not directly applicable to piston ring - cylinder liner lubrication. However, they show that by assuming a layered lubricant it is possible to model the lubrication problem subject to examination.

In this paper, a non-Newtonian model of the lubricant rheology that may be used to resemble layers of variable shear strength is adopted for the simulation of piston ring – cylinder liner lubrication. This model was first presented by Tichy [4] and it models the effects induced by the layers through a modified Reynolds' equation where the coefficients have been corrected with factors that accounts for layer properties. More precisely, the model is based on assuming a piece-wise constant viscosity (piece-wise Newtonian) in which the layers properties are specified by a viscosity and a thickness and this model may be used to predict the results observed in [1]. The main purpose for Tichy developing the model was to attain predictive ability for studying immobile layers, i.e. highly viscous layers, in nanometre thin film lubrication. The possibility of studying wall-slip, i.e. layers with vanishingly small viscosity, by utilizing the same model was not considered.

Clear evidence of wall-slip in a Newtonian liquid was reported by e.g. Craig et al. [5]. In that work they used the slip-length parameter; Petroff [6] (for later work see e.g. [7, 8, 9]), to characterize the flow in the conducted experiments. Modelling slip was also part of the motivation for the work by Meurisse and Morales-Espejel [10] in which they employed the surface layer model presented by Tichy [4]. They addressed a linear slider bearing and concluded that plug flow caused by wall-slip as well as a slip layer, in the middle of the film due to immobile layers at the walls, display significant reduction friction force in comparison to a flow corresponding to a mono-layered film. It is worth noting that during these test the clearance was held fixed.

The current work utilize the aforementioned models and implements such to enable studies of immobile layers as well as wall-slip, in an attempt to resemble the effects induces by lubricant additives, in the lubricated conjunction between a piston ring and a cylinder liner. It extends the model presented in [4], which was revisited in [10], to allow for the possibility of layer-to-layer interaction. This is essential in the present analysis as the piston ring - cylinder liner problem studied exhibits convex conjunction. The convex geometry of a piston ring also requires cavitation to be considered. The cavitation algorithm proposed by Vijayaraghavan [11, 12] was considered adequate for carrying out the task. This is basically the same algorithm as the one presented by Elrod [13], but with a discretization of the shear flow term in the Reynolds' equation describing compressible flow throughout the solution domain. It should be noted however that this treatment might not be the most appropriate from a physical point of view. More precisely, based on experimental observations using laser induced fluorescence (LIF) techniques; Hoult et al, and Richardson and Borman, [14, 15], a different boundary condition based on the assumption that flow separation from the piston ring occurs rather than cavitation, see Taylor et al. [16]. Priest et al. [17] took this a step further by reviewing various potential boundary conditions to be applied undertaking piston ring lubrication analysis.

Our main result is that we have successfully applied a model that considers the lubricant to be layered, also accounting for layer-to-layer interaction, to the piston ring - cylinder liner problem. The well-known model based on slip-length theory was considered for comparative purpose.

By using the surface layer model it is possible to study the effects of immobile layers, wall-slip, as well as layers of variable shear strength, simulating either the reacted film accredited to a friction modifier component or an immobile layer such as an antiwear film. This was demonstrated in an example connected to piston ring – cylinder liner lubrication, where the effects of the layer thickness and the layer viscosity and layer-to-layer interaction on film formation and frictional force were investigated. When analyzing the results, it was observed that there were more solutions to reduce friction force than the most intuitive ones, which are always connected to a deteriorated film formation. For example, assuming an interface exhibiting layering of the type described above, it was shown that an increasing layer shear strength, i.e., increase in layer viscosity, can reduce friction force and improve film build-up at the same time. This highlights the relevance of the present study and expresses the need of further research into this subject; experimental as well as theoretical.

2 THEORY

The theoretical foundation of the model relies on the well-known thin film flow approximation. Section 2.1 reports on the derivation of a generalized Reynolds' equation accounting for a viscosity variation across the film, see also [4]. Extending this model, the possibility of layer-to-layer interaction likely to occur in the convex conjunction between the ring and the liner, was added in the present work. The specific layered lubricant model employed to enable simulation of layers resembling the action of antiwear films and friction modifiers is described in Section 2.2. In Section 2.3 an effort is made to resemble the action induced by a friction modifier by using the slip-length theory introduced by Petroff [6]. This is actualized by deriving a modified form of the Reynolds' equation.

The aforementioned modifications of the Reynolds' equation consist in correction factors that accounts for the influence of a layered lubricant and the slip-length respectively. A comparison of the correction factors corresponding to the two different models is also being made here; showing e.g. that the friction force correction factors becomes identical in a restricted parameter space. The piston ring – cylinder liner model is detailed in Section 2.4, stating the modified Reynolds' equation, the film thickness equation, the force balance equation, boundary conditions and expressions for the friction force. Finally, a set of scaling parameters is introduced in Section 2.5 in order to transform the governing system of equations to a corresponding non-dimensional counterpart.

2.1 A modified Reynolds equation accounting for cross film varying viscosity

Let us start with the thin film flow approximation, which relates the velocity of the fluid $u = u(x, z)$ (for a one-dimensional problem) and the hydrodynamic pressure $p = p(x)$ as shown below

$$\frac{\partial}{\partial z}\left(\mu(z)\frac{\partial u}{\partial z}\right) = \frac{\partial p}{\partial x}, \quad 0 \le z \le h. \tag{2.1}$$

Integrating twice with respect to z and applying the boundary conditions

$$u(x, 0) = 0, \quad \text{and} \quad u(x, h) = U,$$

where U is the linear speed of the piston and $h = h(x)$ is the film thickness, gives

$$u = \left(f_1(z) - \frac{f_1(h)}{f_0(h)} f_0(z)\right)\frac{\partial p}{\partial x} + \frac{f_0(z)}{f_0(h)} U, \tag{2.2}$$

where

$$f_k(z) = \int_0^z \frac{\zeta^k d\zeta}{\mu(\zeta)}. \tag{2.3}$$

Preservation of mass flow requires that

$$\frac{d}{dx}\int_0^h \rho u(x, z)\ dz = 0. \tag{2.4}$$

Thus inserting (2.2) into the continuity equation above (2.4) gives the modified Reynolds equation

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu_0\sigma^R}\frac{\partial p}{\partial x}\right) = \frac{U}{2}\frac{\partial}{\partial x}\left(\eta^R \rho h\right), \quad x \in \Omega, \tag{2.5}$$

where σ^R the correction factor for the pressure driven flow, η^R is the correction factor for shear driven flow, μ_0 is a viscosity reference value to be defined (in the following section) as the dynamic viscosity of the bulk layer and $\Omega = [x_1,\ x_2]$. Expressed in terms of the functions $f_k(z)$ the correction factors may be stated as

$$\frac{1}{\sigma^R(x)} = -\frac{12\mu_0}{h^3(x)} \int_0^{h(x)} \left(f_1(z) - \frac{f_1(h(x))}{f_0(h(x))} f_0(z) \right) dz \tag{2.6}$$

and

$$\frac{1}{\eta^R(x)} = -\frac{2}{h(x)\, f_0(h(x))} \int_0^{h(x)} f_0(z)\, dz. \tag{2.7}$$

The following (modified) expression

$$\begin{aligned} f_f &= -2\pi R \int_\Omega \left(\mu \frac{\partial u}{\partial z} \right)\Bigg|_{z=0} dx = \\ &= 2\pi R \int_\Omega \eta^f \frac{\mu_0 U}{h} + \frac{1}{\sigma^f} \frac{h}{2} \frac{\partial p}{\partial x} dx, \end{aligned} \tag{2.8}$$

where

$$\sigma^f(x) = \frac{2}{h(x)} \frac{f_1(h(x))}{f_0(h(x))} \tag{2.9}$$

and

$$\eta^f(x) = \frac{h(x)}{\mu_0 f_0(h(x))}, \tag{2.10}$$

are utilized for computation of the friction force.

2.2 Modelling viscosity

The expressions derived in Section 2.1 are general, and can be applied to any viscosity variation. Here it is assumed that the lubricant is layered in terms of viscosity and the results from the previous section is applied to the specific example shown in Figure 1,

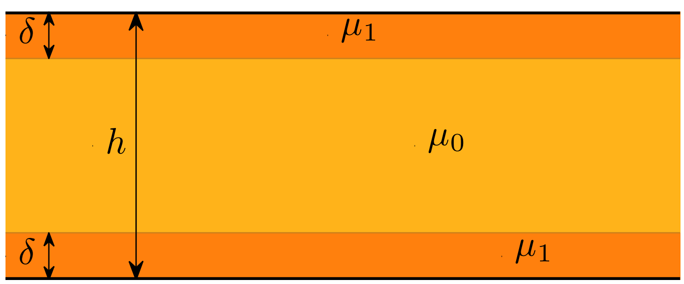

Figure 1: Schematics of the layered lubricant rheological model.

and Newtonian rheology for each layer is assumed. Considering the possibility of layer-to-layer interaction, the expression for the viscosity reads

$$\mu(z) = \begin{cases} \begin{array}{ll} \mu_1 & 0 \le z < \delta \\ \mu_0 & \delta \le z < h-\delta \\ \mu_1 & h-\delta \le z \le h \end{array} & , \ \delta \le h/2 \\ \mu_1 & , \ \delta > h/2 \end{cases}, \tag{2.11}$$

where μ_1 is the viscosity of the layer and δ the layer thickness. Both μ_1 and δ are a function of the spatial coordinate x, i.e. $\mu_1 = \mu_1(x)$ and $\delta = \delta(x)$. In this case (from (2.3))

$$f_0(z) = \frac{1}{\mu_0} \begin{cases} \begin{array}{ll} z/\mu^* & 0 \le z < \delta \\ z + (1/\mu^* - 1)\,\delta & \delta \le z < h-\delta \\ z/\mu^* + (1/\mu^* - 1)(2\delta - h) & h-\delta \le z \le h \end{array} & , \ \delta \le h/2 \\ z/\mu^* & , \ \delta > h/2 \end{cases}, \tag{2.12}$$

$$f_1(z) = \frac{1}{2\mu_0} \begin{cases} \begin{cases} z^2/\mu^* & 0 \le z < \delta \\ z^2 + (1/\mu^* - 1)\,\delta^2 & \delta \le z < h-\delta \\ z^2/\mu^* + (1/\mu^* - 1)\,h\,(2\delta - h) & h-\delta \le z \le h \end{cases} & ,\ \delta \le h/2 \\ z^2/\mu^* & ,\ \delta > h/2 \end{cases}, \tag{2.13}$$

where $\mu^* = \mu_1/\mu_0$. Moreover, the correction factors are given by

$$\frac{1}{\sigma^R_{\mu^*,\Delta}(x)} = \begin{cases} \left(1 + 2\Delta\,(1/\mu^* - 1)\left((2\Delta)^2 - 3\,(2\Delta) + 3\right)\right) & ,\ \Delta \le 1/2 \\ 1/\mu^* & ,\ \Delta > 1/2 \end{cases}, \tag{2.14}$$

$$\eta^R_{\mu^*,\Delta}(x) = \sigma^f_{\mu^*,\Delta}(x) \equiv 1 \tag{2.15}$$

and

$$\eta^f_{\mu^*,\Delta}(x) = \begin{cases} 1/(1 + 2\Delta\,(1/\mu^* - 1)) & ,\ \Delta \le 1/2 \\ \mu^* & ,\ \Delta > 1/2 \end{cases}. \tag{2.16}$$

where $\Delta = \delta/h$ is a dimensionless representation of the layer thickness. In Figures 2 and 3 the mathematical expressions for $\sigma^R_{\mu^*,\Delta}$ and $\eta^f_{\mu^*,\Delta}$ treated as functions of Δ and μ^* are visualized, respectively.

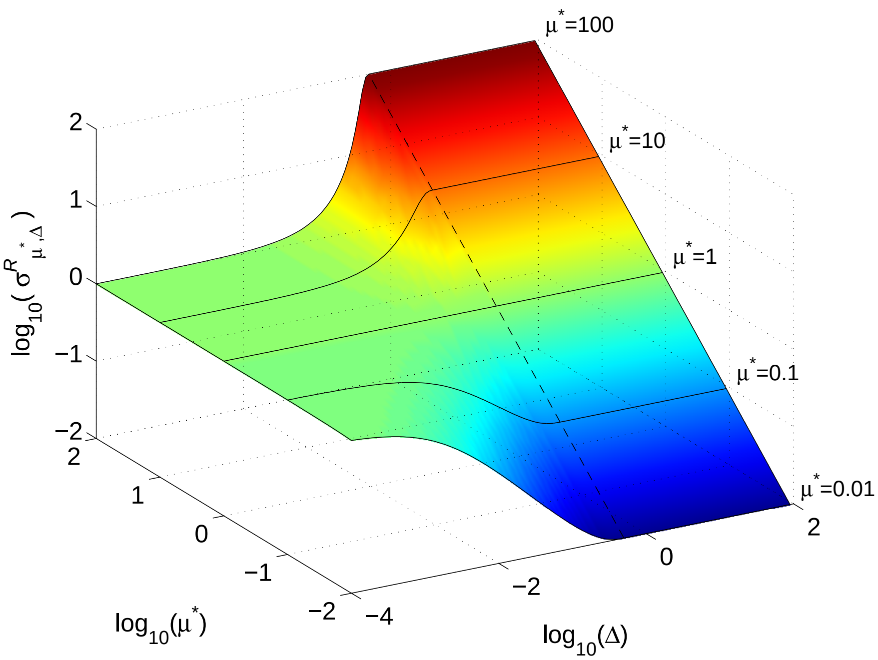

Figure 2: The Reynolds equation correction factor $\sigma^R_{\mu^*,\,\Delta}$ as a function of μ^* and Δ. Dashed line, at the function's surface, indicates $\Delta = 1/2$.

Of note is that the symmetry in the description of the layers in (2.11), i.e., each layer has identical properties, effectively reduces the number of significant correction factors in both the Reynold's equation and the equation for the friction force (from 4 to 2). For example, if it is assumed that only one of the surfaces exhibits a layer, then three correction would differ from unity, i.e., $\sigma^R_{\mu^*,\Delta}$, $\eta^f_{\mu^*,\Delta}$ and also $\eta^R_{\mu^*,\Delta}$. That is, the assumed asymmetry introduce a correction of the Couette flow contribution in the Reynold's equation. If analyzed further it may be seen that it is due to the sliding motion of the piston ring and that it becomes unity under the assumption of pure rolling. Further, in the case of two layers with different descriptions all four correction factors differs from unity.

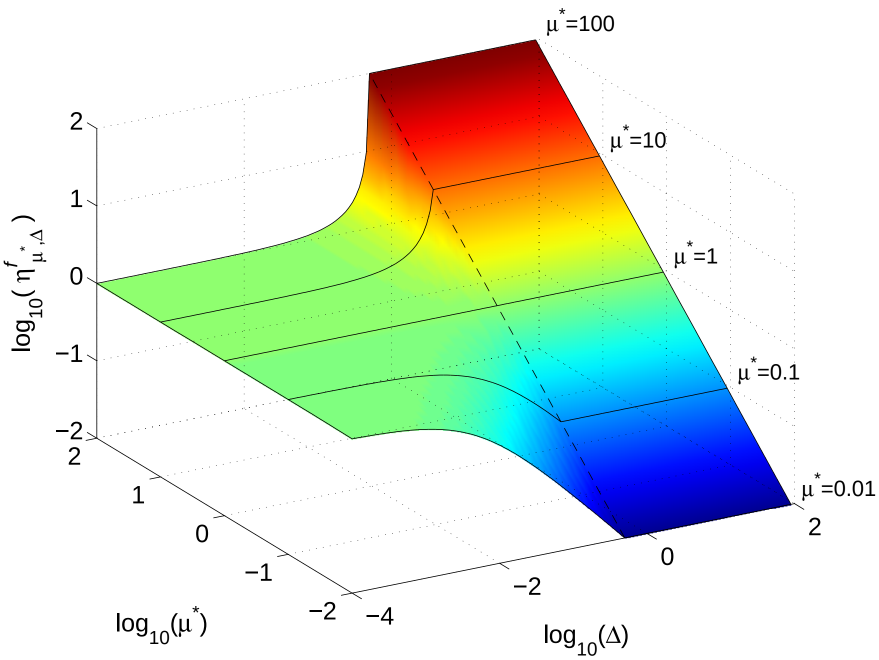

Figure 3: The friction force equation correction factor $\eta^R_{\mu^*,\ \Delta}$ as a function of μ^* and Δ. Dashed line, at the function's surface, indicates $\Delta = 1/2$.

2.3 A modified Reynolds equation based on slip-length theory

In this section an already established model, for simulating wall-slip based on the slip-length theory [6], is revisited to compare with the model described in the previous section.

Assuming $\mu = \mu_0 = const$ and utilizing (2.1) as starting point the following equation for the velocity field is obtained

$$u = \frac{z^2}{2\mu_0}\frac{\partial p}{\partial x} + \frac{z}{\mu_0}C_1 + C_2. \tag{2.17}$$

Applying the slip-length theory to the piston ring – cylinder liner problem we assume the following boundary conditions for the velocity field

$$u\left(x, -s\right) = 0 \tag{2.18}$$

and

$$u\left(x, h+s\right) = U \tag{2.19}$$

where $s = s\left(x\right)$ is the slip-length parameter. Thus, this model simulates wall-slip effectively. For an illustrative definition of the slip-length, see [9]. After solving this problem (for the unknowns C_1 and C_2) an explicit expression for the velocity field is obtained

$$u\left(x, z\right) = \frac{z^2 - zh - \left(s^2 + sh\right)}{2\mu_0}\frac{\partial p}{\partial x} + \frac{z+s}{h+2s}U, \tag{2.20}$$

and for the shear strain rate

$$\frac{\partial u}{\partial z} = \frac{2z-h}{2\mu_0}\frac{\partial p}{\partial x} + \frac{1}{h+2s}U. \tag{2.21}$$

Utilizing (2.4) to ensure preservation of mass flow and after some calculations a modified Reynold's equation of the form (2.5) may be devised. Also, by inserting (2.21) into expression for the friction force

(2.8) the correction factors may now summarized:

$$\sigma_S^R(x) = \frac{1}{1+6S+6S^2}, \tag{2.22}$$

$$\eta_S^R(x) = \sigma_S^f(x) \equiv 1 \tag{2.23}$$

and

$$\eta_S^f(x) = \frac{1}{1+2S}. \tag{2.24}$$

Here the dimensionless representation, $S = s/h$ of the slip-length has been introduced.

In the figures 4 and 5 the, two non-trivial, correction factors, σ_*^R and η_*^f, are compared to the ones obtained in Section 2.2.

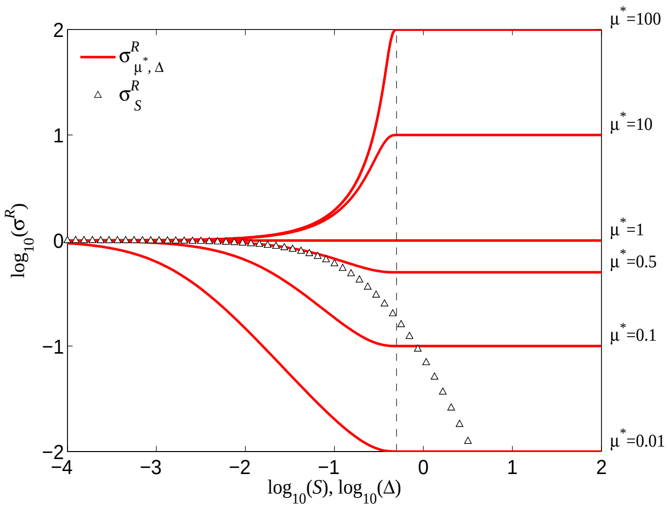

Figure 4: The Reynolds equation correction factor σ_*^R for the slip-length based model as a function of S (triangles) and the layered lubricant model for some different values of μ^* as a function of Δ (continuous lines). Dashed vertical line indicates $\Delta = S = 1/2$.

Figure 5 visualizes the fact that the correction factors $\eta_{\mu^*,\ \Delta}^f$, and η_S^f are identical when $\mu^* = 1/2$, for $\Delta \wedge S \leq 1/2$. It is also clear, from Figure 4 that this is not the case for the Reynold's equation correction factors σ_*^R.

2.4 Piston ring – cylinder liner model

In this work we adopt the cavitation algorithm presented by Vijayaraghavan [11, 12]. It is thus assumed that fluid density and pressure are related as

$$\rho = \rho_c e^{(p-p_c)/\beta} \quad \iff \quad p = p_c + \beta \log(\rho/\rho_c). \tag{2.25}$$

Here ρ is lubricant density, p_c is the cavitation pressure, ρ_c the corresponding density and β is the lubricant bulk modulus. This relation transforms (2.5) to

$$\frac{\partial}{\partial x}\left(\frac{\beta h^3}{12\mu_0\sigma^R}\frac{\partial \theta}{\partial x}\right) = \frac{U}{2}\frac{\partial}{\partial x}\left(\eta^R \theta h\right), \quad x \in \Omega, \tag{2.26}$$

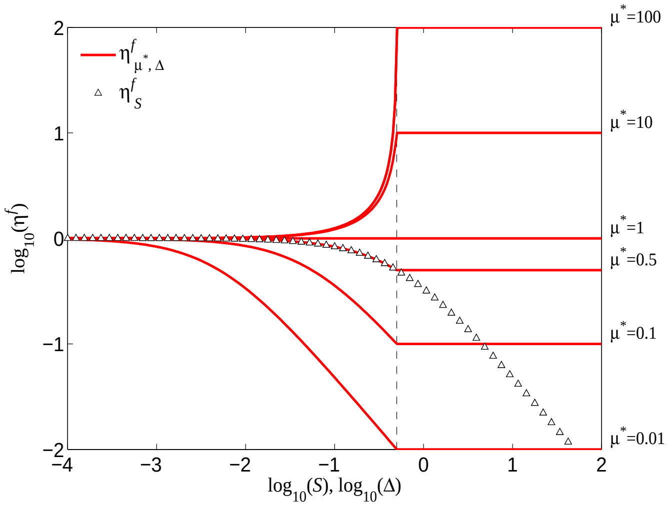

Figure 5: The friction force correction factor η_*^f for the slip-length based model as a function of S (triangles) and the layered lubricant model for some different values of μ^* as a function of Δ (continuous lines). Dashed vertical line indicates $\Delta = S = 1/2$.

where $\theta = \rho/\rho_c$. Pressure boundary conditions $p(x_1) = p_1$ and $p(x_2) = p_2$ are applied, p_1 is the gas pressure upstream and p_2 is the pressure downstream the piston ring.

Since we consider the smooth, piston ring - cylinder liner, problem the film thickness equation may be stated as

$$h(x) = h_{00} + \frac{x^2}{2R}, \quad x \in \Omega, \tag{2.27}$$

where R is the radius of the piston ring and h_{00} is a solution parameter that adjusts in order to meet the force-balance requirement. Accounting for starvation, the force-balance criterion yields

$$(x_1 - (-b))\, p_1 + \int_\Omega p(x)\ dx + (b - x_2)\, p_2 - 2b\,(t_{\text{ring}} + p_1) = 0, \tag{2.28}$$

where t_{ring} is the piston ring tension and b is the ring semi-width. Please see [17] for a schematic illustration. Remark that the parameters x_1 and x_2 are regarded as solution parameters and more precisely, x_1 is the location where the film (first) fully floods the contact and $x_2\,(> 0)$ is the point (in the outlet) where the film ruptures. For the starved problem $x_1 > -b$ and we employ the following equation in terms of mass flow

$$Uh_1 = \frac{U}{2} h(x_2)\, \eta^R(x_2)\,, \text{ i.e., } h(x_2)\, \eta^R(x_2) - 2Uh_1 = 0, \tag{2.29}$$

where h_1 is the (specified) thickness of the lubricant film on the liner, sufficiently far from the inlet zone to be realistic, to fully specify the problem.

2.5 Non-dimensional forms

Here the non-dimensional forms applicable to the slip-length based and the surface layer model(s) described above are considered. This is accomplished by introducing the non-dimensional parameters

$$X = x/x_r,\ \overline{\mu} = \mu/\mu_r,\ H = h/h_r,\ P = p/p_r,\ F_f = f_f/f_{f\ r}$$

and then by choosing the scaling parameters as

$$x_r = b,\ \mu_r = \frac{\beta b^2}{UR^2},\ h_r = \frac{b^2}{R},\ p_r = \beta,\ f_{f\ r} = b^2\beta.$$

Incorporating the above transforms (2.26) to

$$\frac{\partial}{\partial X}\left(\frac{H^3}{12\sigma^R}\frac{\partial\theta}{\partial X}\right) = \frac{1}{2}\frac{\partial}{\partial X}(\theta H), \quad X \in \overline{\Omega}, \tag{2.30}$$

resorting to the fact that $\eta^R \equiv 1$, in the present analysis. Note that the non-dimensional pressure may be retrieved from the non-dimensional form of (2.25), i.e.

$$P = p_c/\beta + \log(\theta). \tag{2.31}$$

The non-dimensional film thickness (that corresponds to (2.27)) reads

$$H(X) = H_{00} + X^2, \quad X \in \overline{\Omega}, \tag{2.32}$$

and from (2.29) we have that

$$H(X_2)\,\eta^R(X_2) - 2H_1 = 0, \tag{2.33}$$

for considering a starved inlet.

The force-balance condition (2.28) reads, in it's non-dimensional form

$$(X_1 - (-1))\,P_1 + \int_{\Omega} P(X)\ dX + (1 - X_2)\,P_2 - 2\,(T_{\text{ring}} + P_1) = 0, \tag{2.34}$$

where $T_{\text{ring}} = t_{\text{ring}}/\beta$, and from the expression (2.8), for friction force, we obtain

$$F_f = -2\pi\int_{\Omega}\overline{\mu_0}\eta^f\frac{1}{H} + \frac{H}{2}\frac{\partial P}{\partial X}dX, \tag{2.35}$$

for the non-dimensional counterpart. Note that the fact that $\sigma^f \equiv 1$ for both the models investigated here has been adopted. In summary, for the problem with a layered surface film, the non-dimensional system of equations is fully determined by the seven parameters

$$H_{in},\ P_1,\ P_2,\ T_{\text{ring}},\ \overline{\mu_0},$$

plus

$$\overline{\mu}_1\ (\text{or}\ \mu^*),\ \text{and}\ \Delta$$

for the layered fluid model, or plus

$$S$$

for the slip-length based one.

3 RESULTS AND DISCUSSION

The input parameters chosen for the specific problem studied here, held fixed during the simulations, may be found in Table 1.

It was deduced that the inlet film thickness (h_1) specified was large enough not to starve the inlet while conducting the present analysis. Let us first compare the two different models. In Figure 6

the normalized minimum film thickness considered as a function of normalized layer thickness $\delta/\left.h_{00}\right|_{\mu^*=1}$ and slip-length $s/\left.h_{00}\right|_{\mu^*=1}$, for the different two different models, is displayed. For relatively small values of the normalized layer thickness and slip-length, the two models gives more or less similar predictions

Table 1: Input parameters

Parameter	Description	Value	Unit
$\overline{\mu}_0$	Fluid viscosity	$2.6 \cdot 10^{-3}$	Pa s
R	Piston ring radius	$5 \cdot 10^{-2}$	m
T_{ring}	Piston ring tension	$1.71 \cdot 10^{5}$	Pa
h_1	Inlet film thickness	$1 \cdot 10^{-5}$	m
β	Bulk modulus	$1 \cdot 10^{9}$	Pa

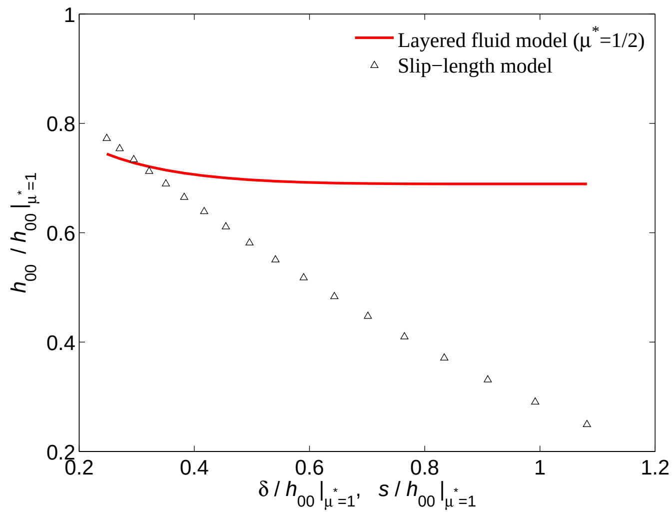

Figure 6: Normalized minimum film thickness $h_{00}/\ h_{00}|_{\mu^*=1}$ considered as a function of normalized layer thickness $\delta/\ h_{00}|_{\mu^*=1}$, continuous red line, and slip-length $s/\ h_{00}|_{\mu^*=1}$, black and white triangles.

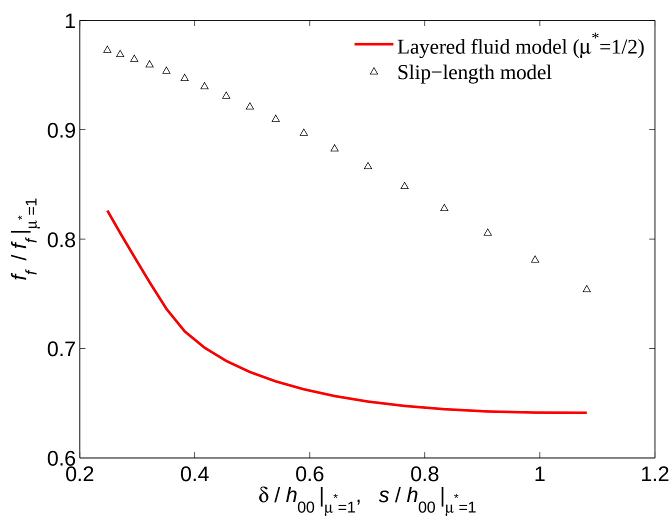

Figure 7: Normalized friction force $f_f/\ f_f|_{\mu^*=1}$ considered as a function of normalized layer thickness $\delta/\ h_{00}|_{\mu^*=1}$, continuous red line, and slip-length $s/\ h_{00}|_{\mu^*=1}$, black and white triangles.

of the lubricant film. However, as the slip-length increase this induce a pronounced reduction of the film comparing to an increasing layer thickness (in the layered fluid model) that yields a convergent film, i.e. approximately 0.7 $h_{00}|_{\mu^*=1}$ for a normalized thickness of approximately 0.6. Next the normalized friction force is considered and in Figure 7 this quantity regarded as a function of normalized layer thickness $\delta/\ h_{00}|_{\mu^*=1}$ and slip-length $s/\ h_{00}|_{\mu^*=1}$ is made visual.

Also when regarding the friction force, a completely different behaviour is observed for the two different models. An increasing slip-length yields a reduction in friction force, which is in-line with the model that with an increasing slip-length effectively simulates an increasing wall-slip. The layered fluid model shows, if not a convergent behaviour, a tendency of converging towards a constant value. This will be addressed in the preceding analysis.

In Figure 8 the film thickness parameter, $h_{00}/\ h_{00}|_{\mu^*=1}$, treated as a function of μ^* and $\delta/\ h_{00}|_{\mu^*=1}$, is depicted. Obviously, for $\mu^* = 1$, and for $\delta = 0$ as well, there is no layer and the plotted normalized

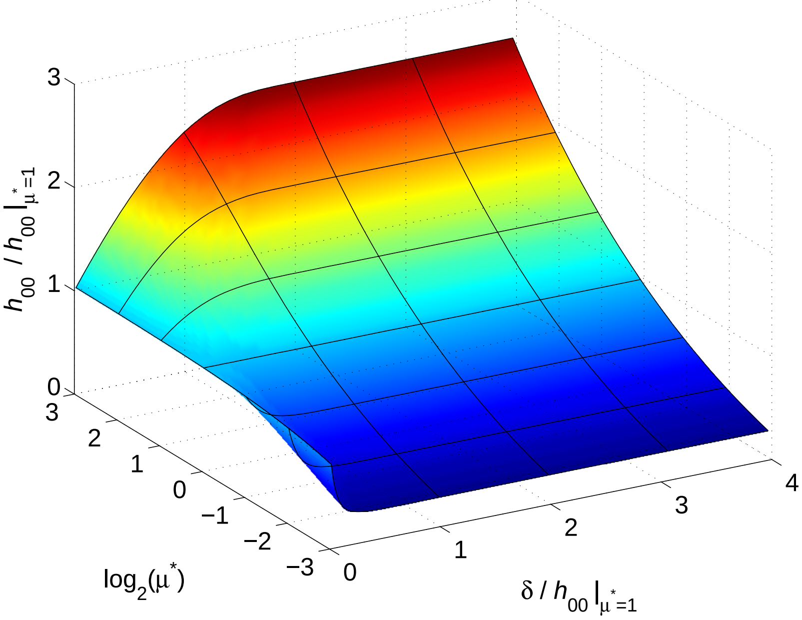

Figure 8: Normalized minimum film thickness as a function of μ^* and $\delta/\ h_{00}|_{\mu^*=1}$.

quantity equals unity. For any fixed μ^*, we see that the film thickness attains a constant value within the range of values of $\delta/\ h_{00}|_{\mu^*=1}$ studied. For an easily sheared layer, i.e., a layer described by a $\mu^* \ll 1$, the convergence is fast and the limiting value is reached already for very thin layers. When a more or less immobile layer is examined, i.e., a layer described by a $\mu^* \gg 1$, a converged value is first observed when the layers completely occupy the lubricated conjunction. For example, for $\mu^* = 8\ \left(= 2^3\right)$, this occurs for $\delta/\ h_{00}|_{\mu^*=1} \approx 1.5$, whereas for $\mu^* = 2$ it is realized at a layer of thickness $\delta/\ h_{00}|_{\mu^*=1} < 1$. It is clear that already very thin layers impede film formation when they exhibit a comparably low shear strength ($\mu^* < 1$). In contrast, the layer thickness must be much larger to induce a significant effect in the case of a layer with a higher shear strength ($\mu^* > 1$) than the corresponding non-layered film itself.

Figure 9 visualize the variation in frictional behaviour with layer composition, by displaying the parameter $f_f/\ f_f|_{\mu^*=1}$ treated as a function of μ^* and $\delta/\ h_{00}|_{\mu^*=1}$. As for the film thickness parameter $\left(h_{00}/\ h_{00}|_{\mu^*=1}\right)$ reported in Figure 8, the normalized friction force shows asymptotes in the $\delta/\ h_{00}|_{\mu^*=1}$-direction (within the parameter space considered). From the figure it is clear that the friction force attains a local maximum in the close vicinity of $\left(\mu^*,\ \delta/\ h_{00}|_{\mu^*=1}\right) = (4, 1)$ which is a behaviour not found in

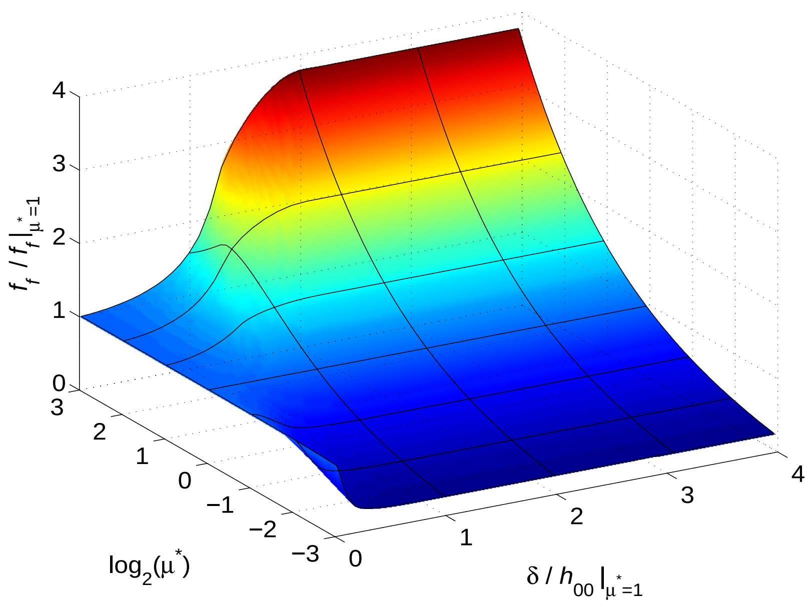

Figure 9: Normalized friction force as a function of μ^* and $\delta/\ h_{00}|_{\mu^*=1}$.

studying the film thickness parameter. An example interpretation would be that; for a layer with relative thickness $\delta/\ h_{00}|_{\mu^*=1} \approx 1$ the highest friction is observed for a layer described by a relative viscosity of approximately 4 times the viscosity of the bulk fluid. This also is true in a diagonally oriented elliptical region stretching from $\left(\mu^*,\ \delta/\ h_{00}|_{\mu^*=1}\right) \approx (2, 0.5)$ to $\left(\mu^*,\ \delta/\ h_{00}|_{\mu^*=1}\right) \approx (8, 1.5)$. It is noted that this introduce more solutions leading to a friction reduction than the most intuitive ones. For example, for a film with characteristics $\left(\mu^*,\ \delta/\ h_{00}|_{\mu^*=1}\right) \approx (8, 1)$ the friction force is lower and the lubricant film is at the same time thicker than for a film with characteristics $\left(\mu^*,\ \delta/\ h_{00}|_{\mu^*=1}\right) \approx (4, 1)$. This provides a novel solution that is better from a lubrication point-of-view.

4 CONCLUSIONS

Our main result is that we have successfully applied models addressing the influnce of lubricant additives to the piston ring - cylinder liner problem.

The well-known model based on slip-length theory was considered for comparative purpose. By comparison it was found that $\mu^* = 1/2$ (and for applicable choices of Δ, i.e. $\Delta \leq 1/2$) resulted in identical expressions of the friction force correction factor η^f for the proposed layered lubricant model and the slip-length based one. However, the (full) expression for friction force consist also of a contribution from the Poiseuille flow - determined by the pressure gradient that is native to the Reynolds equation, that in its modified form, exhibit different correction factors for the two different models. Therefore the different models gives different predictions of the friction force as well as the minimum film thickness, which was also confirmed by numerical simulations.

By using the layered lubricant model it is possible to study the effects of wall-slip (realized by choosing a very thin layer with extremely low viscosity), as well as layers of variable shear strength, simulating either the reacted film accredited to a friction modifier component (low shear strength) or an immobile layer such as an antiwear film. This was demonstrated in an example connected to piston ring – cylinder liner lubrication, where the effects of the layer thickness and the layer viscosity on film formation and frictional force were investigated. When analyzing the results, it was observed that there was more solutions to reduce friction force than the most intuitive ones that are always connected to a deteriorated film formation. For example, it was shown that increasing the effective shear strength of the layer may reduce the friction force and improve the film formation at the same time. This highlights the relevance of the present study and expresses the need of future research, experimental as well as theoretical, into this subject.

ACKNOWLEDGEMENTS

The European Commission, Marie Curie Host Fellowships - Transfer of Knowledge (ToK), Sustainable Fuelube, 509777.

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A Appendix

Parameter

x	Spatial coordinate	m
z	Spatial coordinate	m
u	Velocity field $u = u(x, z)$	$\mathrm{m\,s^{-1}}$
p	Hydrodynamic pressure $p = p(x)$	$\mathrm{Pa\,m^{-1}}$
U	Linear speed of the piston	$\mathrm{m\,s^{-1}}$
h	Film thickness $h = h(x)$	m
σ^R	Correction factor for pressure driven flow	$-$
η^R	Correction factor for shear driven flow	$-$
μ_0	Viscosity reference value - dynamic viscosity of the bulk layer	Pa s
μ_1	Viscosity of the layer	Pa s
δ	Layer thickness	m
μ^*	Viscosity of the layer	Pa s
Δ	Dimensionless layer thickness, $\Delta = \delta/h$	$-$
s	Slip-length, $s = s(x)$	m
ρ	Lubricant density	$\mathrm{kg\,m^{-3}}$
β	Lubricant bulk modulus	Pa
θ	Non-dimensional density, $\theta = \rho/\rho_c$	$-$
p_1	Pressure upstream piston ring	Pa
p_2	Pressure downstream piston ring	Pa
R	Radius of the piston ring (Determining the shape of the conjunction)	m
h_{00}	Minimum film thickness	m
t_{ring}	Piston ring tension	Pa
b	Ring semi-width	m
h_1	Inlet film thickness (Specified)	m
X	Non-dimensional spatial coordinate, $X = x/x_r$	$-$
$\overline{\mu}$	Non-dimensional, $\overline{\mu} = \mu/\mu_r$	$-$
H	Non-dimensional spatial coordinate, $H = h/h_r$	$-$
P	Non-dimensional spatial coordinate, $P = p/p_r$	$-$
F_f	Non-dimensional spatial coordinate, $X = x/x_r$	$-$
Ω	Open bounded subset of $\mathbb{R}^2$	**Subscript**

Subscript

$_r$	Denoting a reference parameter
$_c$	Denoting a parameter evaluated at cavitation pressure